Partial Vertical Integration, Ownership Structure and Foreclosure*

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Abstract

We study the incentive to acquire a partial stake in a vertically related firm and then foreclose rivals. We show that whether such partial acquisitions are profitable depends crucially on the initial ownership structure of the target firm and on corporate governance.

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1 Introduction

One of the main antitrust concerns that vertical mergers raise is that the merger will result in the foreclosure of either upstream or downstream rivals. Although most of the discussion has focused on full vertical mergers, in reality, many firms acquire partial stakes in suppliers (partial backward integration) or in buyers (partial forward integration). A case in point is the cable industry, where several operators acquired partial ownership stakes in cable or television networks (see Waterman and Weiss, 1997, p. 24-32). This situation has raised the concern that non-integrated networks will be denied access to cable systems or will obtain access at unfavorable terms.[1] More broadly, policymakers seem to be increasingly concerned about the potential anticompetitive effects of partial vertical integration.[2]

The Industrial Organization literature has mostly studied full vertical mergers and it implicitly assumes that vertical integration occurs if and only if the profit of the merged entity exceeds the sum of the pre-merger profits of the two merging firms. But when integration is partial and the target firm (the upstream supplier under partial backward integration or the downstream firm under partial forward integration) is initially held by more than one shareholder, then this is no longer true, because the merger also affects the remaining, passive, shareholders of the target firm. If it lowers their payoff, then the passive shareholders effectively subsidize the merger, while if it raises their payoff, then the merger effectively subsidizes the passive shareholders. This suggests that the initial ownership structure of the target firm has important implications for the incentive to partially integrate and foreclose rival firms after integration. The goal of this paper is to ex-

[1] Recent prominent examples include News Corp.'s (a major owner of TV broadcast stations and programming networks) acquisition of a 34% stake in Hughes Electronics Corporation in 2003, which gave it a de facto control over DirecTV Holdings, LLC (a direct broadcast satellite service provider which is wholly-owned by Hughes), and the 2011 joint venture agreement between Comcast, GE, and NBCU, which gave Comcast (the largest cable operator and Internet service provider in the U.S.) a controlling 51% stake in a joint venture that owns broadcast TV networks and stations, and various cable programming. In the UK, BSkyB (a leading TV broadcaster) acquired in 2006 a 17.9% stake in ITV (UK's largest TV content producer). The UK competition commission concluded that "BSkyB had acquired the ability materially to influence the policy of ITV which gives rise to common control" and argued that BSkyB would use it to "reduce ITV's investment in content" and "influence investment by ITV in high-definition television (HDTV) or in other services requiring additional spectrum." For examples from other inudstries, see EC (2013b) and Gilo and Spiegel (2011).

[2] See for example EC (2013a), where the commission states that "...non-horizontal acquisitions of minority shareholdings that also provide material influence may raise competitive concerns of input foreclosure. For some minority shareholdings, foreclosure may even be more likely than when control is acquired..."

plore these implications in the context of a model that explicitly takes into account the acquisition process.

To this end, we consider a model in which two downstream firms buy inputs from several upstream suppliers. In most of the paper we study the conditions under which partial integration between one downstream firm, D_1, and one upstream supplier, U_1, leads to input foreclosure, i.e., after integration, U_1 forecloses the nonintegrated downstream rival. Our model, however, can be also modified to consider customer foreclosure, under which D_1 stops buying from nonintegrated rival suppliers after integration.

Input foreclosure weakens the downstream rival and hence boosts the profit of D_1. But then, it also lowers the profit of U_1, who now forgoes sales to the downstream rival. Under partial backward integration, part of the resulting upstream loss is borne by the passive shareholders of U_1. We show that partial backward integration, which leads to input foreclosure, is particularly profitable when U_1 is initially held by dispersed shareholders. In that case, D_1 can acquire the minimal stake that ensures control and hence minimize its share in the upstream loss from foreclosure. The rest of the loss is borne by the remaining shareholders of U_1, who effectively subsidize the input foreclosure. When U_1 has initially a controlling shareholder, then D_1 needs to compensate him for the reduction in the value of his entire stake in order to induce him to relinquish control. Since this stake may well exceed the minimal stake needed for control, partial backward integration, which is followed by input foreclosure, is more costly in this case and therefore less likely to occur.

Under partial forward integration by contrast, U_1 bears the entire upstream loss from foreclosure, but needs to share the associated downstream gain with the passive shareholders of D_1. We show that the resulting transfer of wealth to D_1's passive shareholders renders foreclosure unprofitable when D_1 is initially held by dispersed shareholders, but it may still be profitable when D_1 has a controlling shareholder, whose controlling stake is sufficiently large (i.e., D_1 has relatively few passive shareholders who receive a subsidy).

We also use our model to study additional ownership structures. In particular, we consider the incentive to partially integrate and then foreclose rivals when the target has initially two controlling shareholders, the incentive to backward integrate when D_1 initially holds a non-controlling stake in U_1 (i.e., a toehold), and the incentive of a controlling shareholder of D_1 to acquire a stake in U_1 either directly or through some other firm that he controls rather through D_1 itself.

A key driving force in our analysis is that the passive shareholders of the foreclosing firm (U_1 under backward integration and input foreclosure and D_1 under forward integration and costumer foreclosure) subsidize the foreclosure of rivals and hence foreclosure arises for a larger set of parameters when there are more passive shareholders. While this concern would vanish if the rights of the passive shareholders were effectively protected, in reality, policymakers appear to be skeptical about the ability of corporate governance to satisfactorily address this concern.[3]

Our paper merges ideas from Corporate Finance and from Industrial Organization. From a Corporate Finance perspective, our paper is related to the literature on takeovers (e.g., Grossman and Hart (1980), Bebchuk (1994), Burkart, Gromb, and Panunzi (1998, 2000), Burkart, Gromb, Mueller, and Panunzi (2014), Israel (1992), and Zingales (1995)). This literature, however, abstracts from the interaction between the market for corporate control and competition in the product market and how this interaction depends on the initial ownership structure of the target.[4] A main contribution of our paper is to develop a general framework that allows us to study this interaction and examine its potential antitrust implications. In particular, we explore how the ownership structure of the target and acquirer affect the likelihood and competitive effects of partial vertical mergers.

The Industrial Organization literature on input foreclosure has three strands.[5] In Bolton and Whinston (1993), vertical integration strengthens the incentive of the integrated downstream firm to invest and weakens the incentive of the nonintegrated firm to invest. As a result, the latter is less likely to buy the input when its supply is limited. Bolton and Whinston interpret this situation as foreclosure. In our paper by contrast, foreclosure is due to a deliberate refusal to deal with a nonintegrated rival rather than a by-product of the effect of integration on downstream investments.

The second strand of the literature, due Hart and Tirole (1990), shows that an upstream supplier may prefer to deal exclusively with one downstream firm in order to alleviate an opportunism problem that prevents the supplier from extracting profits from the downstream firms. In

[3] For example, in its decision regarding News Corp.'s acquisition of a 34% stake in Hughes in 2003, the FCC wrote: "We therefore discount the likelihood that corporate governance, corporate law or securities laws in general may be relied upon to adequately protect MVPD and video programming competitors from potential anti-competitive vertical foreclosure behavior on the part of Applicants." See FCC, 2014, Paragraph 100.

[4] There are additional differences: backward integration in our paper lowers the target's value rather than increases it as in most of the corporate finance literature. Under forward integration, the acquisition increases the target's value, but this is because it affects the acquirer's strategy rather than the target's strategy. For this reason, the acquisition does not necessarily involve a controlling stake. Moreover, instead of enjoying private benefits as most of the literature assumes, the acquirer in our model sustains a loss due to the forgone sales to a downstream rival.

[5] See Rey and Tirole (2007) and Riordan (2008) for literature surveys.

our paper there is no opportunism problem and the role of foreclosure is to shift profits from one downstream firm to another.

Our model is closely related to the third strand due to Ordover, Saloner and Salop (1990) and Salinger (1988). In this strand, input foreclosure raises the costs of downstream rivals and hence benefits integrated downstream firm.[6] The idea in our model is similar although foreclosure lowers the value that a downstream rival can offer consumers rather than raises its cost. More importantly, we examine the incentive to acquire a partial stake in a vertically related firm and examine how this incentive depends on the initial ownership structure of the target firm and on corporate governance. Our model also generalizes to the case of customer foreclosure with minimal modifications and hence it differs from Ordover, Saloner and Salop (1990), Salinger (1988), or Hart and Tirole (1990), which consider input foreclosure, but cannot be naturally adapted to explain customer foreclosure.

There are only a few papers which consider the competitive effects of partial vertical integration. Riordan (1991), Greenlee and Raskovitch (2006) and Hunold and Stahl (2016) consider passive acquisitions, which affect the competitive strategy of the acquirer, but do not affect the target's strategy as in the case of backward integration in our paper.[7] We are aware of only two papers which consider the acquisition of a controlling stake in a vertically related firm. Baumol and Ordover (1994) show that a downstream firm which controls a bottleneck owner with a partial ownership stake has an incentive to divert business to itself, even if downstream rivals are more efficient.[8] Spiegel (2013) examines a model in the spirit of Bolton and Whinston (1993), in which (partial) vertical integration affects the incentives of downstream firms to invest in the quality of

[6]The assumption that the upstream supplier can commit to foreclose the downstream rival has been criticized as problematic, see Hart and Tirole (1990) and Reiffen (1992) and see Ordover, Salop, and Saloner (1992) for a reply. Several papers, including Ma (1997), Chen (2001), Choi and Yi (2001), Church and Gandal (2000), and Allain, Chambolle, and Rey (2011) have proposed models that are immune to this criticism. Moresi and Schwartz (2015) show that when a vertically integrated input monopolist supplies a differentiated downstream rival and sets a linear price for the input, it has an incentive to induce expansion by the rival irrespective of whether downstream competition involves prices or quantities.

[7]Fiocco (2014) considers passive partial forward integration and shows that it allows the manufacturer to capture some of the information rents that accrue to a privately informed retailer and hence affects the contracts that the manufacturer offers the retailer and consequently the resulting competition in the downstream market.

[8]Reiffen (1998) finds that the stock price of Chicago Northwestern (CNW) railroad reacted positively, rather than negatively, to events that made it more likely that Union Pacific (UP) Railroad will gain effective control over CNW with a partial ownership stake. This finding is inconsistent with the idea that UP would have used its control over CNW to foreclose competing railroads.

their products. He shows that relative to full integration, partial vertical integration may either alleviate or exacerbate the concern for input foreclosure and examines the resulting implications for consumers. Neither paper, however, studies the takeover game, nor examines how the incentive to partially integrate depends on the ownership structure of the target, which is the main focus of the current paper.

The rest of the paper is organized as follows. The basic model is presented in Section 2. In Section 3 we study two benchmarks: non integration and full integration. Our main results appear in Section 4, where we examine how the incentives to engage in "input foreclosure" following partial backward or forward integration depend on the initial ownership structure of the target firm. In particular, we consider two polar cases: (i) the target has a single controlling shareholder, and the (ii) the target is owned by atomistic shareholders. In Section 5 we examine three additional ownership structures: (i) the target has two large shareholders, (ii) the acquirer holds a non-controlling stake (i.e., a toehold) in the target, and (iii) the acquisition is made by the controlling shareholder of the firm (or another firm he controls) rather than by the firm itself. In Section 6 we show that our model also applies, with minimal modifications, to "customer foreclosure," and we also study two extensions of our basic setup. In Section 7 we conclude. The Appendix includes technical proofs and an example that shows how our reduced form profits can be derived from an explicit model of downstream competition.

2 The model

Consider two downstream firms, D_1 and D_2, that provide a final good/service to consumers. The downstream firms use up to $N \geq 1$ differentiated inputs, each of which is produced by a single upstream supplier U_i, $i = 1, 2, \ldots, N$. The cost that an upstream supplier incurs when it serves a downstream firm is c.[9] Let $\pi(k, l)$ denote the (reduced form) profit of a downstream firm when it uses k inputs and its rival uses l inputs, before any payments to upstream suppliers. In Appendix B we show an example for how $\pi(k, l)$ can be derived from an explicit model of downstream competition.[10]

[9] It is straightforward to modify this assumption and assume instead that the cost of serving two downstream firms is more or less than twice the cost of serving only one downstream firm.

[10] In a technical Appendix, available at https://www.tau.ac.il/~spiegel/papers/partialVI-tech-appendix.pdf, we show that the main implications of our basic setup can be also derived from variants of Ordover, Salop and Saloner (1990) and Salinger (1988), which are two of the leading "raising your rivals costs" models of input foreclosure.

Throughout the analysis we will impose the following assumption:

A1 $\pi(k, l)$ *is increasing with* k *at a decreasing rate and decreasing with* l

For example, D_1 and D_2 can be cable or satellite TV providers, which deliver TV channels to viewers. Assumption A1 then says that, other things being equal, the demand that each TV provider faces is higher when it offers more channels and when the rival offers fewer channels. Likewise, if D_1 and D_2 are retailers, then Assumption A1 says that each retailer faces a higher demand when it carries more brands, while its rival carries fewer brands.

The sequence of events is as follows. At the outset, all firms are independently owned. Then, either one downstream firm, D_1, acquires a stake α in upstream supplier U_1 (backward integration), or U_1 acquires a stake α in D_1 (forward integration). We will say that integration is partial if $\alpha < 1$. In most of the paper we will assume that the acquisition gives the acquirer full control over the target if $\alpha \geq \underline{\alpha}$, where $\underline{\alpha}$ is the minimal stake needed for de facto control.[11] In Section 6.2, we will relax this assumption, and show that our main results generalize to the case where a stake $\alpha < 1$ gives the acquirer only partial control over the target.

Given the new ownership structure, each of N upstream suppliers decides whether to supply its input to both downstream firms or to only one. These decisions are publicly observable and irreversible.[12] Then, $U_1, \ldots, U_N$ make simultaneous take-it-or-leave-it offers to D_1 and D_2, and once the offers are accepted or rejected, the final product is produced and payoffs are realized.[13]

3 Non-integration and input foreclosure under full integration

In this section we study the non-integration and full integration benchmarks. In later sections we will consider the incentive to partially integrate, taking the merger process explicitly into account.

[11] Typically the assumption in the literature is that $\underline{\alpha} = 50\%$, but in reality, $\underline{\alpha}$ can be well below 50%. For example, as Footnote 1 mentions, News Corp.'s acquisition of a 34% stake in Hughes Electronics Corporation gave it de facto control, while BSkyB's acquisition of a 17.9% stake in ITV gave rise to "common control."

[12] This assumption can be justified as in Church and Gandal (2000) and Choi and Yi (2000), where each upstream firm needs to adapt the input to the special needs of each downstream firm. The assumption allows us to sidestep the "commitment problem," which arises for example in Ordover, Salop, and Saloner (1990).

[13] The assumption that the upstream firms can make take-it-or-leave-it offers is not essential: in a technical Appendix, available at https://www.tau.ac.il/~spiegel/papers/partialVI-tech-appendix.pdf, we show that our results generalize to the case where input prices are determined by a more general bargaining process.

We begin by solving the last stage of the game in which the upstream suppliers make simultaneous take-it-or-leave-it offers to D_1 and D_2 for the sale of their inputs. To this end, suppose that D_i already buys $k-1$ inputs and D_j buys l inputs. Then, the marginal willingness of D_i to pay for the k'th input is

$$\Delta_1(k,l) \equiv \pi(k,l) - \pi(k-1,l).$$

This expression represents the incremental profit from adding the k'th input, given that the rival uses l inputs. Assumption A1 implies that $\Delta_1(k,l)$ is positive but decreasing with k. For later use, we denote the (negative) externality that an increase in l imposes on D_i's profit by

$$\Delta_2(k,l) \equiv \pi(k,l) - \pi(k,l-1).$$

By Assumption A1, $\Delta_2(k,l) < 0$ for all k and l.

To ensure that selling N inputs is profitable, we will make the following assumption:

A2 $\Delta_1(N,N) > c$

While Assumption A2 ensures that selling inputs is profitable if both downstream firms buy all N inputs, it is possible that an upstream supplier may prefer to sell its input to only one downstream firm. To see why, note that the change in D_i's marginal willingness to pay for input k when D_j increases the number of its inputs from $l-1$ to l, is given by

$$\Delta_{12}(k,l) \equiv \Delta_1(k,l) - \Delta_1(k,l-1).$$

Although in general $\Delta_{12}(N,N)$ could be either positive or negative, the example in Appendix B shows that it is reasonable to assume that $\Delta_{12}(N,N) < 0$. That is, D_i's marginal willingness to pay for inputs decreases when D_j uses one more input. For the sake of concreteness, we will assume that this is indeed the case:

A3 $\Delta_{12}(k,l) \leq 0$ for all k,l

Given Assumption A3, it is possible, at least in principle, that an upstream firm may be unwilling to supply both downstream firms. The following assumption rules out this possibility and ensures that under non-integration, both downstream firms buy all N inputs:

A4 $\Delta_1(k,l) - c > -\Delta_{12}(l,k)$ for all k, l

Assumption A4 implies that the maximal profit that an upstream supplier can make by selling an extra input to D_i, $\Delta_1(k,l) - c$, exceeds $-\Delta_{12}(l,k)$, which is the associated loss of profit from selling to D_j.

3.1 The non-integration benchmark

With Assumption A4 in place, we now characterize the equilibrium behavior of non-integrated upstream suppliers:

Lemma 1: *In equilibrium, non-integrated upstream suppliers sell to both D_1 and D_2, irrespective of whether D_1 and U_1 are partially or fully integrated and irrespective of whether U_1 forecloses D_2 or not. If D_1 and U_1 are integrated and U_1 forecloses D_2, then upstream suppliers $2, \ldots, N$ charge D_1 an amount $\Delta_1(N, N-1)$ for the input and charge D_2 an amount $\Delta_1(N-1, N)$. If D_2 is not foreclosed, all upstream suppliers charge D_2 an amount $\Delta_1(N,N)$ and all non-integrated upstream suppliers charge D_1 an amount $\Delta_1(N,N)$.*

Given that $\Delta_1(N, N-1) > \Delta_1(N,N)$ by Assumption A3, Lemma 1 implies that when D_2 is foreclosed, D_1 ends up paying a higher price for the N inputs. Note however that the payments that downstream firms make for inputs are total payments rather than per-unit prices. Hence, there is no double marginalization in our model.[14]

The following corollary follows immediately from Lemma 1:

Corollary 1: *Under non-integration, as well as absent foreclosure, both D_1 and D_2 buy all N inputs at a price of $\Delta_1(N,N)$. The resulting profit of each downstream firm is*

$$V_0^D \equiv \pi(N,N) - N\Delta_1(N,N),$$

while the profit of each upstream supplier is

$$V_0^U \equiv 2(\Delta_1(N,N) - c),$$

where $V_0^D > 0$ and $V_0^U > 0$.

[14] With double marginalization, vertical integration can harm consumers both because D_2 does not use one of the inpupts and because D_1 pays a higher price for the inputs it uses.

3.2 The full integration benchmark

Now suppose that D_1 and U_1 fully integrate. Absent foreclosure, the sum of D_1 and U_1's profits is $V_0^D + V_0^U$. When U_1 forecloses D_2, the sum of the profits becomes $V_1^D + V_1^U$, where

$$V_1^D \equiv \pi (N, N-1) - N\Delta_1 (N, N-1),$$

and

$$V_1^U \equiv \Delta_1 (N, N-1) - c.$$

These profits reflect the fact that when D_2 is foreclosed, D_1's downstream profit is $\pi (N, N-1)$ and it pays each upstream supplier (including U_1) an amount $\Delta_1 (N, N-1)$ for inputs.[15]

Foreclosing D_2 is profitable for the vertically integrated firm if and only if

$$V_1^D + V_1^U \geq V_0^D + V_0^U.$$

We will refer to an equilibrium in which integration between U_1 and D_1 leads to the foreclosure of D_2 as a "foreclosure equilibrium."

Proposition 1: *Suppose that D_1 and U_1 are fully integrated. Then a foreclosure equilibrium exists and is unique if and only if*

$$G \geq L,$$

where

$$G \equiv V_1^D - V_0^D = -\underbrace{[\pi (N, N) - \pi (N, N-1)]}_{\Delta_2(N,N)} + N\underbrace{[\Delta (N, N) - \Delta (N, N-1)]}_{\Delta_{12}(N,N)},$$

is the downstream gain from foreclosure, and

$$L \equiv V_0^U - V_1^U = \Delta_1 (N, N) - c + \underbrace{[\Delta (N, N) - \Delta (N, N-1)]}_{\Delta_{12}(N,N)},$$

is the associated upstream loss.

Proposition 1 simply says that foreclosure is profitable if the downstream gain from foreclosing D_2 exceeds the associated upstream loss.[16] The latter consists of $\Delta_1 (N, N) - c$, which is

[15] We assume for simplicity that D_1 pays U_1 the same amount it pays all other suppliers. This assumption is without loss of generality since under full integration, D_1's payment to U_1 is merely a transfer within the same organization, and hence is irrelevant.

[16] Interestingly, Proposition 1 does not require more than one upstream supplier. Hence, input foreclosure can arise in our framework even if there is only one upstream supplier. This is in contrast to Ordover, Saloner, and Salop (1990), where the existence of two upstream suppliers is crucial for input foreclosure.

the forgone profit from not selling to D_2, plus $\Delta_{12}(N,N) < 0$, which is due to the fact that when D_2 is foreclosed, U_1 charges D_1 for the input $\Delta_1(N,N)$ instead of $\Delta_1(N,N-1)$). This increase moderates somewhat the upstream loss from foreclosure, though by Assumption A4, we still have $L > 0$. As for the downstream gain G, note that $-\Delta_2(N,N) > 0$ is the extra downstream profit that D_1 makes when D_2 is foreclosed, while $N\Delta_{12}(N,N) < 0$, reflects the higher payments of D_1 for the N inputs.

As far as know, the adverse effect of foreclosure on the foreclosing firm's payment for inputs has not been identified earlier. While this effect is extreme in our model because we assume that upstream suppliers have all the bargaining power vis-a-vis D_1 and D_2, the effect would not disappear completely unless D_1 and D_2 can make take-it-or-leave-it offers to the upstream suppliers.

4 Input foreclosure under partial integration

Our analysis in the previous section shows that vertical integration leads to the foreclosure of D_2 if and only if $G > L$. Although at first blush it would seem that vertical integration is profitable and would take place in this case, our analysis in this section and the next shows that this is not necessarily true when the target firm has passive shareholders.

To study the incentive for vertical integration, we assume that D_1 and U_1 are initially independent and then ask whether D_1 would like to acquire a controlling stake, $\alpha \geq \underline{\alpha}$, in U_1 (backward integration), or U_1 would like to acquire a stake α, not necessarily controlling, in D_1 (forward integration). It turns out that the answer depends heavily on the initial ownership structure of the target firm (U_1 in the case of backward integration and D_1 in the case of forward integration). In this section we will consider two extreme cases:

(i) Initially, the target (U_1 in the case of partial forward integration and D_1 in the case of partial backward integration) has a single controlling shareholder whose stake is $\alpha_C \in [\underline{\alpha}, 1]$; the remaining $1 - \alpha_C$ stake in U_1 (if any) is held by passive shareholders.

(ii) Initially, the target is owned by a mass 1 of atomistic shareholders.

In Section 5 below we will consider additional possibilities.

Before we start, note that when D_1 partially controls U_1 (partial backward integration), it would like to pay U_1 as little as possible (and thereby expropriate the wealth of U_1's passive shareholders), while when U_1 partially controls D_1 (partial forward integration), U_1 would like to charge D_1 as much as possible (and thereby expropriate the wealth of D_1's passive shareholders). The incentive to distort D_1's payment for U_1's input is often referred to as "tunneling" (see e.g., Johnson et al, 2000, and Bae, Kang, and Kim, 2002).[17] To model tunneling, we will assume that under partial integration, D_1 pays for U_1's input the same amount it pays under non-integration, but minus a discount t if D_1 controls U_1, and plus a premium t if U_1 controls D_1. The parameter t measures the extent of tunneling and is larger when the protection of minority shareholders is weaker.

To simplify the analysis, we will assume that the effect of tunneling on the payoffs of D_1 and U_1 is smaller than the effect of foreclosure:

A5 $t \leq \min\{G, L\}$

To reduce the number of cases we need consider, we will also make the following assumption:

A6 $G > \underline{\alpha} L$

Assumption A6 holds trivially when $G > L$; when $G < L$, Assumption A6 imposes an upper bound on $\underline{\alpha}$, which is the minimal stake that ensures control. As we shall see, without this assumption, foreclosure never arises in our model under backward integration.

4.1 Backward integration when U_1 has initially a single controlling shareholder

Suppose that U_1 has initially a single controlling shareholder, whose controlling stake is α_C. To acquire a controlling stake $\alpha \in [\underline{\alpha}, \alpha_C]$ in U_1, D_1 makes U_1's controller a take-it-or-leave-it offer b. If the offer is accepted, D_1 becomes the new controlling shareholder in U_1. If the offer is rejected, the two firms remain independent. As we shall see below, the assumption that D_1 has all the bargaining power vis-a-vis U_1's controller is not essential.[18]

[17] Although the foreclosure of D_2 also tunnels wealth from U_1 to D_1 under partial backward integration, we will only refer to the manipulation of input prices as "tunneling" to distinguish it from vertical foreclosure. Note that tunneling can arise even if D_1 is a monopoly in the downstream market.

[18] The assumption that the acquirer (here D_1) makes a take-it-or-leave-it offer is natural when the target's ownership is dispersed. We wish to make the same assumption when the target has an initial controller to ensure that the two scenarios differ only with respect to the target's ownership structure and not the relative bargaining powers of the acquirer and the target. In any event, this assumption is not essential.

Conditional on acquiring a controlling stake α in U_1, D_1 would use its control to foreclose D_2 if foreclosure increases its post-acquisition payoff:

$$V_1^D + t + \alpha\left(V_1^U - t\right) \geq V_0^D + t + \alpha\left(V_0^U - t\right), \qquad \Longrightarrow \qquad \underbrace{V_1^D - V_0^D}_{G} \geq \alpha\underbrace{\left(V_0^U - V_1^U\right)}_{L}.$$

Hence, foreclosure arises if and only if the downstream gain from foreclosure, G, exceeds D_1's share in the associated upstream loss, αL. Recalling that under partial backward integration D_1's payment for U_1's input is discounted by t, and noting that Assumption A6 ensures that $\underline{\alpha} < \frac{G}{L}$, and that by definition, $V_1^D \equiv V_0^D + G$ and $V_1^U \equiv V_0^U - L$, we can express the payoffs of D_1 and U_1 under backward integration as functions of α:

$$V_{BI}^D(\alpha) = \begin{cases} V_0^D, & \alpha < \underline{\alpha}, \\ V_1^D + t \equiv V_0^D + G + t, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^D + t, & \alpha > \frac{G}{L}, \end{cases} \tag{1}$$

and

$$V_{BI}^U(\alpha) = \begin{cases} V_0^U, & \alpha < \underline{\alpha}, \\ V_1^U - t \equiv V_0^U - L - t, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^U - t, & \alpha > \frac{G}{L}. \end{cases} \tag{2}$$

Since $\alpha \leq 1$, the last line in $V_{BI}^D(\alpha)$ and $V_{BI}^U(\alpha)$ is irrelevant when $G > L$.

Given $V_{BI}^U(\alpha)$, the minimal acceptable payment b^U that D_1 must offer U_1's controller should leave the controller indifferent between accepting and rejecting the offer:

$$\underbrace{b^U + (\alpha_C - \alpha)\, V_{BI}^U(\alpha)}_{\text{Accepting the offer}} = \underbrace{\alpha_C V_0^U}_{\text{Rejecting the offer}}, \qquad \Longrightarrow \qquad b^U = \alpha V_{BI}^U(\alpha) + \alpha_C\left(V_0^U - V_{BI}^U(\alpha)\right). \tag{3}$$

That is, b^U is equal to the post-acquisition value of the acquired stake, $\alpha V_{BI}^U(\alpha)$, plus a premium, $\alpha_C\left(V_0^U - V_{BI}^U(\alpha)\right)$, that compensates the initial controller of U_1 for the change in the value of his entire initial stake.

To find out if D_1 will offer b^U for a controlling stake in U_1, note that D_1's payoff is equal to its post-acquisition payoff, $V_{BI}^D(\alpha) + \alpha V_{BI}^U(\alpha)$, minus b^U. Using (1)-(3) and rearranging terms, we can rewrite D_1's payoff as a function of the size of the acquired stake α:

$$Y^D(\alpha) = \begin{cases} V_0^D, & \alpha < \underline{\alpha}, \\ V_0^D + G - \alpha_C L + (1 - \alpha_C)\, t, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^D + (1 - \alpha_C)\, t, & \alpha > \frac{G}{L}. \end{cases} \tag{4}$$

As (4) shows, $Y^D(\alpha)$ is maximized at $\alpha \in \left[\underline{\alpha}, \frac{G}{L}\right]$ when $G \geq \alpha_C L$, and at $\alpha > \frac{G}{L}$ when $G < \alpha_C L$.[19] Since the initial controller of U_1 can sell at most a stake of α_C, D_1 will acquire in the latter case a controlling stake $\alpha \in \left[\frac{G}{L}, \alpha_C\right]$. Hence,

Proposition 2: *Suppose that initially, U_1 has a single controlling shareholder. Then, in equilibrium, D_1 will acquire a controlling stake $\alpha \in [\underline{\alpha}, \alpha_C]$ and will use it to foreclose D_2 if*

$$G \geq \alpha_C L. \tag{5}$$

When this condition fails, D_1 will acquire a controlling stake $\alpha \in \left[\frac{G}{L}, \alpha_C\right]$ in U_1 but will not use it to foreclose D_2 after the acquisition.

Proposition 2 implies that D_1 would always acquire a controlling stake in U_1 because it allows him to buy U_1's input at a discount, and thereby effectively expropriate the wealth of U_1's passive shareholders. If in addition condition (5) holds, i.e., the downstream gain from foreclosure exceeds the stake of U_1's initial controller in the associated upstream loss, then the acquisition leads to the foreclosure of D_2. Interestingly, the condition for foreclosure is independent of α, which is the size of the acquired stake, because D_1 needs to compensate the initial controller of U_1 for the loss to his entire initial stake, α_C, even if it does not fully acquire this stake.

The passive shareholders of U_1 effectively subsidize foreclosure since they bear a fraction $1 - \alpha_C$ of the loss from foreclosure. Not surprisingly then, condition (5) is more likely to hold when their stake in U_1, $1 - \alpha_C$, gets larger. Recalling that under full integration foreclosure arises if and only if $G \geq L$, Proposition 2 suggests that antitrust authorities should be more concerned with backward integration than with full vertical integration, particularly when the controlling stake of the initial controller is relatively small. These concerns are alleviated to some extent when the protection of minority shareholders is effective, in which case D_1 may find it harder to use it control over U_1 to foreclose downstream rivals, as well as engage in tunneling.

It should be emphasized that Proposition 2 continues to hold even if D_1 does not have all the bargaining power vis-a-vis U_1's initial controller. To see why, note from (1) and (2) that the

[19] To the extent that tunneling and foreclosure are easier when the initial controller of U_1 is out of the picture, D_1 might wish to acquire the entire stake α_C of U_1's initial controller.

joint payoff of the initial controllers of D_1 and U_1 under integration is

$$V_{BI}^{D}(\alpha)+\alpha_C V_{BI}^{U}(\alpha)=\begin{cases} V_0^D+\alpha_C V_0^U, & \alpha<\underline{\alpha}, \\ V_0^D+\alpha_C V_0^U+G-\alpha_C L+(1-\alpha_C)\,t, & \underline{\alpha}\leq\alpha\leq\frac{G}{L}, \\ V_0^D+\alpha_C V_0^U+(1-\alpha_C)\,t, & \alpha>\frac{G}{L}. \end{cases}$$

Since the joint payoff absent integration is $V_0^D+\alpha_C V_0^U$, transferring control to D_1 is jointly profitable, and it leads to foreclosure if and only if $G\geq\alpha_C L$, exactly as stated in Proposition 2.[20]

4.2 Backward integration when U_1's ownership is initially dispersed

We now turn to the case where U_1 is initially held by a mass 1 of atomistic shareholders. To acquire a controlling stake $\alpha\geq\underline{\alpha}$ in U_1, D_1 makes a restricted tender offer $(\overline{\alpha},V)$, where $\overline{\alpha}\leq 1$ is the maximal stake it offers to acquire, and V is the price for the entire firm . Given the tender offer, each of U_1's shareholders decides whether to tender his shares. If more than $\overline{\alpha}$ shares are tendered, the submitted shares are prorated. We will say that the tender offer succeeds if D_1 manages to acquire at least a stake of $\underline{\alpha}$ and gains control over U_1, and it fails otherwise. When the offer succeeds, D_1 pays $\overline{\alpha}V$ for the acquired shares.

To characterize the equilibrium of the tender offer game, note that the post-acquisition values of D_1 and U_1 are given by (1) and (2) and also note that $V_{BI}^{U}(\alpha)\leq V_0^U$. Whenever $V<V_0^U$, it is optimal for each shareholder to tender his shares if the tender offer succeeds (and get V for the tendered shares instead of $V_{BI}^{U}(\alpha)$), but hold on to his shares if the tender offer fails (in which case the shareholder gets V for the tendered shares, instead of V_0^U).[21] Hence, the tendering subgame admits two equilibria: (i) all shareholders tender and the offer succeeds, and (ii) no shareholder tenders, so the offer fails. Since $V_0^U\geq V_{BI}^{U}(\alpha)$, equilibrium (ii) Pareto dominates equilibrium (i). We will therefore assume that whenever $V_{BI}^{U}(\alpha)\leq V<V_0^U$, equilibrium (ii) is played.[22] With this equilibrium selection criterion in place, we prove the following lemma.

Lemma 2: *Suppose that if $V_{BI}^{U}(\alpha)\leq V<V_0^U$, then U_1's initial shareholders do not tender their shares. Then in equilibrium, $V=V_0^U$.*

[20] The relative bargaining power of D_1 vis-a-vis U_1's initial controller would matter however if D_1 has some fixed cost associated with initiating a takeover. Then, the lower D_1's bargaining power, the less likely the takeover is.

[21] If the offer is conditional on success, the shareholders are indifferent between submitting and not submitting shares when the offer fails.

[22] The same equilibrium selection criterion is also used in Grossman and Hart (1980), Burkart, Gromb, and Panunzi (1998), and Burkart, Gromb, Mueller, and Panunzi (2014)). It rules out the implausible scenario where the target's shareholders tender at prices below the status quo value of the target.

Lemma 2 implies that in order to acquire shares, D_1 needs to pay U_1's dispersed shareholders the pre-acquisition value of the shares. This value exceeds the post-acquisition value of shares whenever D_1 gains control. As we shall see shortly, D_1 will therefore prefer to acquires the minimal stake, $\underline{\alpha}$, needed for control.

To examine D_1's incentive to acquire a stake α in U_1, note that given that D_1 needs to pay αV_0^U for the acquired shares, its post-acquisition payoff is $V_{BI}^D(\alpha) + \alpha V_{BI}^U(\alpha) - \alpha V_0^U$. Using (1) and (2) and rearranging terms, we can rewrite D_1's payoff, as a function of α, as follows:

$$Y^D(\alpha) = \begin{cases} V_0^D, & \alpha < \underline{\alpha}, \\ V_0^D + G - \alpha L + (1-\alpha)t, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^D + (1-\alpha)t, & \alpha > \frac{G}{L}. \end{cases} \tag{6}$$

Since $G > \underline{\alpha}L$ by Assumption A6, $Y^D(\alpha)$ jumps upward at $\alpha = \underline{\alpha}$; given that $Y^D(\alpha)$ is continuous at $\alpha > \frac{G}{L}$ and decreasing for all $\alpha \geq \underline{\alpha}$, we obtain the following result:

Proposition 3: *Suppose that initially, U_1's ownership is dispersed. Then, in equilibrium, D_1 will acquire the minimal stake, $\underline{\alpha}$, needed to control U_1. D_1 will use its control over U_1 to foreclose D_2 if and only if*

$$G \geq \underline{\alpha}L. \tag{7}$$

Since $\underline{\alpha} \leq \alpha_C \leq 1$, Propositions 1-3 imply that a foreclosure equilibrium exists for a wider range of parameters under partial backward integration than under full integration, especially when U_1 is initially owned by dispersed shareholders. In that sense, our results suggest that antitrust authorities should be particularly concerned about partial backward integration when the initial ownership of the upstream supplier is dispersed. Intuitively, foreclosing D_2 diverts profits from U_1 to D_1. When U_1 has an initial controller, D_1 must compensate him for his entire stake in the upstream loss, $\alpha_C L$, even if D_1 buys only part of this stake. By contrast, when U_1's ownership is dispersed, D_1 can acquire a minimal stake, $\underline{\alpha}$, that ensures control and hence it internalizes only a fraction $\underline{\alpha} \leq \alpha_C$ of the upstream loss, L. Put differently, when U_1 has an initial controller, U_1's passive shareholders bear a fraction $1 - \alpha_C$ of the upstream loss from foreclosure, while under dispersed ownership this fraction increases to $1 - \underline{\alpha}$.

Note that unlike Proposition 2, where D_1 is indifferent about the size of the acquired controlling stake, here D_1 wishes to acquire the minimal stake that ensures control. The reason is that in Proposition 2, the amount that D_1 pays for the acquired stake depends on the controller's initial stake, α_C, irrespective of how large α is, whereas here, the amount paid is increasing with α (note that D_1 pays for each share its pre-acquisition value which exceeds its post-acquisition value and hence would like to acquire as few shares as possible).

To conclude this subsection, five remarks are in order. First, t does not affect the incentive to foreclose. Hence in our model, the incentive to foreclose rivals is independent of whether (partial) vertical integration leads to an upward or a downward distortion in D_1's payment for U_1's input. This feature allows us to separate the issue of tunneling, which can also arises when D_1 and U_1 do not have rivals, from the issue of foreclosure and its potential anticompetitive effects, which is our main focus.[23]

Second, when Assumption A6 is violated, the downstream gain from foreclosure always falls short of D_1's share in the corresponding upstream loss; hence, D_1 will never use its control over U_1 to foreclose D_2.

Third, suppose that contrary to our equilibrium selection criterion, all of U_1's shareholders tender shares when $V_{BI}^{U}(\alpha) \leq V < V_0^U$. While this allows D_1 to pay less for a controlling stake $\underline{\alpha}$ in U_1, the acquisition price is already sunk when D_1 decides whether or not to use its control to foreclose D_2. Hence, foreclosure still arises if and only if $G \geq \underline{\alpha}L$, as stated in Proposition 3.

Fourth, one may wonder whether an external investor may wish to acquire a sufficiently large stake from U_1's dispersed shareholders and use it to oppose D_1's decision to lower D_1's payment for the input and to foreclose D_2. Such an acquisition would raise U_1's value from $V_{BI}^{U}(\alpha)$ to V_0^U. But since the dispersed shareholders of U_1 are atomistic, then as in Grossman and Hart (1980), the investor would have to pay them the post-acquisition value of their shares to induce them to submit their shares. As a result, such an acquisition is not profitable for the investor.

Finally, so far we implicitly assumed that when U_1 has a controlling shareholder, D_1 must acquire his stake, α_C, to gain control over U_1. If we relax this assumption and assume in addition that $\alpha_C < \frac{1}{2}$, then D_1 can also gain control over U_1 by acquiring a stake $\alpha > \alpha_C$ from U_1's dispersed shareholders. However this strategy gives D_1 a payoff of $G - \alpha L$, which is less than $G - \alpha_C L$, which is D_1's payoff when it acquires the controlling stake of U_1's controller.[24] Hence, bypassing the

[23]This feature of our model depends on the assumption that the demand of downstream firms for inputs is inelastic. When the demand for inputs is elastic, the distrortion in D_1's payment for U_1's input may affect the quantity of the input that D_1 uses and hence the competition with D_2 in the downstream market.

[24]This conclusion is only strengthened if the controlling shareholder of U_1 makes a counter offer to the dispersed shareholders.

controlling shareholder of U_1 is not profitable, even if it allows D_1 to gain full control over U_1.[25]

4.3 Forward integration

Next, suppose that U_1 wishes to integrate forward. Unlike backward integration, control is no longer needed for foreclosure, since U_1 can foreclose D_2 regardless of whether it controls D_1. However, control allows U_1 to tunnel wealth from D_1 by inflating its payment for U_1's input by t.

Conditional on acquiring a non-controlling stake α in D_1, U_1 would choose to foreclose D_2 if foreclosure increases its overall payoff:

$$V_1^U + \alpha V_1^D \geq V_0^U + \alpha V_0^D, \qquad \Longrightarrow \qquad \alpha \underbrace{\left(V_1^D - V_0^D\right)}_{G} \geq \underbrace{V_0^U - V_1^U}_{L}.$$

That is, U_1 forecloses D_2 if and only if its stake in the downstream gain from foreclosure exceeds its associated upstream loss. Put differently, foreclosure is profitable if and only if α is sufficiently high in the sense that $\alpha \geq \frac{L}{G}$. When U_1 acquires a controlling stake $\alpha \geq \underline{\alpha}$ in D_1, it can also inflate D_1's payment for the input, so U_1's profit increases by t, while D_1's profit decreases by t. Together with the fact that $V_1^D \equiv V_0^D + G$ and $V_1^U \equiv V_0^U - L$, the payoffs of D_1 and U_1 under forward integration, as functions of α, are given by

$$V_{FI}^D(\alpha) = \begin{cases} V_0^D, & \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\}, \\ V_1^D \equiv V_0^D + G, & \frac{L}{G} < \alpha < \underline{\alpha}, \\ V_0^D - t, & \underline{\alpha} \leq \alpha < \frac{L}{G}, \\ V_1^D + t \equiv V_0^D + G - t, & \alpha \geq \max\left\{\underline{\alpha}, \frac{L}{G}\right\}, \end{cases} \tag{8}$$

and

$$V_{FI}^U(\alpha) = \begin{cases} V_0^U, & \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\}, \\ V_1^U \equiv V_0^U - L, & \frac{L}{G} < \alpha < \underline{\alpha}, \\ V_0^U + t, & \underline{\alpha} \leq \alpha < \frac{L}{G}, \\ V_1^U + t \equiv V_0^U - L + t, & \alpha \geq \max\left\{\underline{\alpha}, \frac{L}{G}\right\}. \end{cases} \tag{9}$$

The second lines in (8) and (9) are relevant only if $\underline{\alpha} > \frac{L}{G}$, while the third lines are relevant only if $\underline{\alpha} < \frac{L}{G}$.

[25] Burkart, Gromb, and Panunzi (2000) also show that when the target has both a large shareholder and dispersed, atomistic, shareholders, the acquirer prefers to deal with the large shareholder. Their setting however differs from ours in several respects; in particular, they consider a value-increasing acquisition while in our case the acquisition is value-decreasing.

Having computed $V_{FI}^{D}(\alpha)$ and $V_{FI}^{U}(\alpha)$, we now study U_1's incentive to acquire a stake α in D_1 in the first place. We begin with the case where initially, D_1's shareholders are atomistic. Then, (8) implies that when $\alpha < \frac{L}{G}$, the acquisition either lowers D_1's value or does not affect it. As in Lemma 2, U_1 then needs to pay the atomistic shareholders of D_1 the pre-acquisition value of their shares, V_0^D. By contrast, when $\alpha \geq \frac{L}{G}$, U_1 forecloses D_2 after the acquisition, and by Assumption A5, D_1's value increases even if U_1 gains control and inflates D_1's payment for the input. Consequently, U_1 faces the well-known free-rider problem of Grossman and Hart (1980), and must set V equal to the post-acquisition value of D_1, which is either V_1^D absent tunneling or $V_1^D - t$ with tunneling.[26] Using these prices, we prove the following result:

Proposition 4: *Suppose that initially, D_1's ownership is dispersed. Then, in equilibrium, U_1 will acquire the minimal stake, $\underline{\alpha}$, needed to control U_1 if*

$$\underline{\alpha} G < L. \tag{10}$$

The acquisition, if it takes place, does not lead to the foreclosure of D_2. When the condition fails, U_1 has no incentive to acquire a stake in D_1.

Intuitively, U_1 has no incentive to foreclose D_2 because foreclosure boosts the value of D_1, and this forces U_1 to pay the atomistic shareholders of D_1 a price equal to the post-acquisition value of D_1. But then, U_1 breaks even on the acquisition, and since it bears an upstream loss, L, due to foreclosure, it has no incentive to pursue the acquisition.[27] The acquisition is profitable only when U_1 acquires control over D_1 and can use it to inflate D_1's payment for the input, but the controlling stake is sufficiently low to ensure that U_1 does not foreclose D_2 after the acquisition (and hence there is no free-rider problem).[28] The important implication of Proposition 4 is that when D_1's ownership is dispersed, forward integration does not lead to a foreclosure equilibrium.

[26] When the acquisition is value increasing, the atomistic shareholders of the target have a dominant strategy to hold on to their shares so long as $V < V_1^D$. It should be noted that this conclusion hinges on the assumptions that D_1's shareholders are atomistic and the post-acquisition value of D_1 is common knowledge. See Bagnoli and Lipman (1988) and Holmstrom and Nalebuff (1992) for analysis of value-increasing takeovers when the target is held by a finite number of shareholders and the acquirer has private information about the post-acquisition value of the target.

[27] As in Grossman and Hart (1980), the acquirer forgoes the acquisition because it needs to pay atomostic shareholders the entire increase in the target's value. However, unlike in Grossman and Hart, here the acquisition does not necessarily involve control and it increases the target's value by affceting the acquirer's bahavior (the foreclosure of D_2) rather than the target's own behavior.

[28] In a sense, tunelling serves the same role as dilution in Grossman and Hart (1980).

Next, we turn to the case where D_1 has a controlling shareholder whose stake is $\alpha_C \in [\underline{\alpha}, 1]$. Then, the minimal payment b^D that U_1 needs offer D_1's controller to induce him to sell a stake $\alpha \leq \alpha_C$ in D_1 (this stake may or may not be controlling) is given by

$$\underbrace{b^D + (\alpha_C - \alpha) V_{FI}^D(\alpha)}_{\text{Accepting the offer}} = \underbrace{\alpha_C V_0^D}_{\text{Rejecting the offer}}, \qquad \Longrightarrow \qquad b^D = \alpha V_{FI}^D(\alpha) - \alpha_C \left(V_{FI}^D(\alpha) - V_0^D\right). \quad (11)$$

That is, b^D equals the post-acquisition value of the acquired shares, $\alpha V_{FI}^D(\alpha)$, minus $\alpha_C \left(V_{FI}^D(\alpha) - V_0^D\right)$, which is the change in the value of the initial stake of D_1's initial controller due to forward integration. Using b^D, we prove the following result:

Proposition 5: *Suppose that initially, D_1 has a single controlling shareholder. Then, in equilibrium, U_1 will acquire a controlling stake, $\alpha \in \left[\max\left\{\underline{\alpha}, \frac{L}{G}\right\}, \alpha_C\right]$ in D_1 and will use it to foreclose D_2 if*

$$\alpha_C G \geq L. \quad (12)$$

When this condition fails, U_1 will acquire a controlling stake $\alpha \in [\underline{\alpha}, \alpha_C]$ in D_1, but will not foreclose D_2 after the acquisition.

Proposition 5 shows that unlike the case where D_1's ownership is dispersed, when D_1 has initially a controlling shareholder, forward integration can lead to a foreclosure equilibrium, but only if the stake of the initial controller is sufficiently large to satisfy (12). The reason is that under forward integration, U_1 bears the entire loss from foreclosure, so foreclosure can be profitable only if U_1's share in the associated gain is sufficiently large. Put differently, under forward integration, input foreclosure subsidizes the passive shareholders of D_1. When the initial shareholders of D_1 are atomistic, they demand the entire subsidy in order to sell their shares. The resulting free-rider problem renders forward integration unprofitable whenever the acquisition leads to input foreclosure. When D_1 has an initial controller, U_1 can negotiate with him a mutually beneficial price and hence the acquisition goes through, provided that there are not too many passive shareholders who continue to be subsidized by foreclosure.

Combined, Propositions 1-5 show that relative to full integration, partial backward integration facilitates foreclosure, while partial forward integration hinders it. In particular, forward integration never leads to input foreclosure when D_1's ownership is initially dispersed, and leads to foreclosure when D_1 has an initial controller only when $\alpha_C G \geq L$. This condition is harder to satisfy than the corresponding condition under full integration, which is $G \geq L$. Under partial backward integration, the conditions for input foreclosure are easier to satisfy and are given by $G \geq \alpha_C L$ or $G \geq \underline{\alpha} L$, depending on whether U_1 has an initial controller or dispersed ownership. In particular, Proposition 5 implies that if full integration does not lead to foreclosure, i.e., $G < L$, then partial forward integration will not lead to a foreclosure either, and if full integration leads to foreclosure, i.e., $G \geq L$, then partial forward integration may lead to foreclosure only when D_1 is initially controlled by a single shareholder whose stake is sufficiently large.

As mentioned above, Proposition 5 continues to hold when U_1 does not have all the bargaining power vis-a-vis D_1's initial controller. To see why, note that given (8) and (9), the joint payoff of U_1 and D_1's initial controller if an acquisition goes through is

$$V_{FI}^{U}(\alpha) + \alpha_C V_{FI}^{D}(\alpha) = \begin{cases} V_0^U + \alpha_C V_0^D, & \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\}, \\ V_0^U - L + \alpha_C\left(V_0^D + G\right), & \frac{L}{G} < \alpha < \underline{\alpha}, \\ V_0^U + t + \alpha_C\left(V_0^D - t\right), & \underline{\alpha} \leq \alpha < \frac{L}{G}, \\ V_0^U - L + t + \alpha_C\left(V_0^D + G - t\right), & \alpha \geq \max\left\{\underline{\alpha}, \frac{L}{G}\right\}. \end{cases}$$

Since their joint payoff without an acquisition is $V_0^U + \alpha_C V_0^D$, transferring α_C to U_1 is always jointly profitable and foreclosing D_2 is jointly profitable if and only if $\alpha_C G \geq L$, exactly as in Proposition 5. Hence, the relative bargaining powers of the two parties only determines how the joint surplus is divided, but not whether the acquisitions will take place.

4.4 Backward or forward integration?

Suppose that initially, both D_1 and U_1 have controlling shareholders whose respective stakes are α_C^D and α_C^U. Suppose that the two controllers can get together and decide to integrate their firms. Will they agree that U_1's controller will sell his stake to D_1 (partial backward integration), or that D_1's controller will sell his stake to U_1 (partial forward integration)?

To address this question, suppose that U_1's controller sells his stake to D_1 for a price b^U. Then, the resulting joint post-acquisition payoff of the two controllers is

$$\alpha_C^D\left(V_{BI}^D\left(\alpha_C^U\right) + \alpha_C^U V_{BI}^U\left(\alpha_C^U\right) - b^U\right) + b^U,$$

where $V_{BI}^D\left(\alpha_C^U\right)$ and $V_{BI}^U\left(\alpha_C^U\right)$ are given by (1) and (2). Similarly, if D_1's controller sells his stake to U_1 for a price b^D, then the joint post-acquisition payoff of the two controllers is

$$\alpha_C^U\left(V_{FI}^U\left(\alpha_C^D\right) + \alpha_C^D V_{FI}^D\left(\alpha_C^D\right) - b^D\right) + b^D,$$

where $V_{FI}^D\left(\alpha_C^U\right)$ and $V_{FI}^U\left(\alpha_C^U\right)$ are given by (8) and (9).

Proposition 6: *Suppose that initially, both D_1 and U_1 have single controlling shareholders and assume further that $\alpha_C^U \leq \frac{G}{L}$ and $\alpha_C^D \geq \frac{L}{G}$, so that both partial backward integration and partial forward integration lead to the foreclosure of D_2. Then the two controllers will decide to pursue partial backward integration, regardless of the size of their initial controlling stakes.*

To see the intuition, note that the controlling shareholders of D_1 and U_1 need to share the downstream gain, G, and the upstream loss, L, from foreclosure with the passive shareholders of D_1 and U_1, and likewise they need to share the profit from tunneling with the passive shareholders of the acquiring firm. Backward integration is more profitable than forward integration because the price paid to U_1's controlling shareholder under backward integration, b^U, exceeds the post-acquisition value of the acquired shares, so the passive shareholders of D_1 subsidize part of the deal. Under forward integration, the price paid to D_1's controlling shareholder, b^D, falls short of the post-acquisition value of the acquired shares, so the passive shareholders of U_1 receive a subsidy.

5 Input foreclosure under additional ownership structures

In the previous section, we considered the incentive to vertically integrate under two polar cases: the target has a single controlling shareholder or is owned by atomistic shareholders. In this section we examine three more cases: (i) initially the target has two controlling shareholders, (ii) backward integration when the acquirer, D_1, already holds a non-controlling stake in U_1 (i.e., a toehold), and (iii) backward integration when the acquisition is made by the controlling shareholder of D_1 rather than by D_1 itself.

5.1 Backward integration when U_1 has initially two large shareholders

Suppose that initially, U_1 has two large shareholders, each with a stake of $\frac{\alpha_C}{2}$, where $\frac{\alpha_C}{2} \leq \underline{\alpha} \leq \alpha_C < 1$. That is, neither shareholder alone has control, but together they do. As we shall see, this case is more involved than the two polar cases we examined in Section 4. To simplify the analysis, we shall assume that $\underline{\alpha} \geq 1/2$. This assumption implies that $\alpha \geq \underline{\alpha}$ is necessary and sufficient for control.[29] We will also assume that $G \geq \alpha_C L$, so once D_1 acquires the stakes of the two large shareholders, it would use its control over U_1 to foreclose D_2 and to buy U_1's input at a discount.

[29] Whenever $\underline{\alpha} < 1/2$, a stake $\underline{\alpha}$ is necessary for control, but it is not sufficient, because another shareholder may acquire a stake $\alpha > \underline{\alpha}$ and gain control over U_1.

Consequently, the post-acquisition profits of D_1 and U_1 are $V_1^D + t$ and $V_1^U - t$; if D_1 fails to gain control over U_1, the profits are V_0^D and V_0^U.

The game evolves in three stages. First, D_1 makes a take-it-or-leave-it offer (α_1, b_1) to shareholder 1, where α_1 is the stake that D_1 offers to buy and b_1 is the associated payment. Second, given shareholder 1's decision to accept or reject the offer, D_1 makes a take-it-or-leave-it offer (α_2, b_2) to shareholder 2, where $\alpha_1 + \alpha_2 \geq \underline{\alpha}$. If both offers are rejected, D_1 fails to gain control over U_1 and if both offers are accepted, D_1 gains control over U_1 and uses it to foreclose D_2. However if one offer is accepted and the other is rejected, the game proceeds to the third stage in which D_1 makes a restricted tender offer to the dispersed shareholders of U_1 for a stake $\underline{\alpha} - \frac{\alpha_C}{2}$, which ensures D_1 control.[30] Given this offer, shareholder j, who rejected D_1's offer, can make a counter tender offer to the dispersed shareholders of U_1. D_1 gains control over U_1 in this case only if its tender offer is accepted.

The following Proposition characterizes the resulting equilibrium:

Proposition 7: *Suppose that initially, U_1 has two large shareholders, each with a stake of $\frac{\alpha_C}{2}$, such that $\frac{\alpha_C}{2} \leq \underline{\alpha} \leq \alpha_C \leq 1$. Then, in equilibrium, D_1 will acquire a controlling stake α_C, and given our assumption that $G \geq \alpha_C L$, will use it to foreclose D_2.*

Proposition 7 shows that when U_1 has two large shareholders, each with a stake $\frac{\alpha_C}{2}$, partial backward integration leads to a foreclosure equilibrium only when $G \geq \alpha_C L$. Although this is similar to the case of a single controlling shareholder with a controlling stake of α_C, the acquisition is now more profitable. To see why, notice from the proof of Proposition 7 that D_1's post-acquisition payoff when U_1 has two large shareholders can be written as

$$Y_2^D = \begin{cases} V_0^D + G + t, & G + t \geq \left(\underline{\alpha} + \frac{\alpha_C}{2}\right)(L + t), \\ V_0^D + 2(G + t) - (\alpha_C + \underline{\alpha})(L + t), & G + t < \left(\underline{\alpha} + \frac{\alpha_C}{2}\right)(L + t). \end{cases}$$

Using (4), D_1's post-acquisition payoff when U_1 has a single controlling shareholder and $G \geq \alpha_C L$ can be written as

$$Y_1^D = V_0^D + G + t - \alpha_C (L + t).$$

[30] D_1 does not acquire a larger stake in U_1 because it is better off exploiting as many passive shareholders of U_1 as possibe after the acquisition.

Now,

$$\begin{aligned} Y_2^D &\geq V_0^D + 2(G+t) - (\alpha_C + \underline{\alpha})(L+t) \\ &= Y_1^D + G - \underline{\alpha} L + (1-\underline{\alpha})\, t \\ &\geq Y_1^D, \end{aligned}$$

where the first inequality follows since the expression in the top line in Y_2^D exceeds that in bottom line, and the last inequality follows by Assumption A6.

Intuitively, when U_1 has two large shareholders, D_1 can credibly threaten shareholder 2 that if he does not tender his shares, D_1 will make a tender offer to the dispersed shareholders of U_1, which would be costly for shareholder 2 to block. Proposition 7 is closely related to the "naked exclusion" result of Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000), where an incumbent supplier can costlessly exclude rival suppliers by signing downstream buyers to exclusive supply contracts. Likewise, D_1 can buy the stakes of the large shareholders of U_1 at their post-acquisition values, so the foreclosure of D_2 is costless for D_1. Our result differs from the naked exclusion result in that the "naked exclusion" result requires the existence of two or more buyers, while in our case foreclosure occurs with two large shareholders for the same set of parameters that it occurs with one large shareholder.

5.2 Forward integration when D_1 has initially two large shareholders

Now suppose that D_1 has initially two large shareholders, each with a stake $\frac{\alpha_C}{2}$. For the sake of concreteness, we will assume that $\alpha_C \geq \underline{\alpha}$, so if U_1 acquires the stakes of the two large shareholders, it gains control over D_1 and then inflates D_1's payment for the input. This assumption is not essential though and the results go through even when $\alpha_C \geq \underline{\alpha}$. The sequence of events is similar to that in the previous section: U_1 makes a take-it-or-leave-it offer (α_1, b_1) to shareholder 1, then it makes a take-it-or-leave-it offers (α_2, b_2) to shareholder 2, and finally it can make a restricted tender offer to the dispersed shareholders of D_1.

There are now three cases to consider. First, if $\alpha_C G < L$, then foreclosure arises only if in addition to the stakes of shareholders 1 and 2, U_1 also acquires shares from D_1's dispersed shareholders. Foreclosure though makes the acquisition value increasing, so due to the free-rider problem, U_1 needs to pay the dispersed shareholders the post-acquisition value of their shares,

$V_1^D - t$. Hence, its payoff if its final stake α is such that $\alpha G \geq L$ is given by

$$\begin{aligned}\alpha\left(V_1^D - t\right) + V_1^U + t - (\alpha - \alpha_C)\left(V_1^D - t\right) &\equiv \alpha_C\left(V_0^D - t\right) + \left(V_0^U + t\right) + \alpha_C G - L \\ &\leq \alpha_C\left(V_0^D - t\right) + \left(V_0^U + t\right),\end{aligned}$$

where the last expression is U_1's payoff if it does not acquire shares from the dispersed shareholders of D_1. Hence, U_1 will not acquire additional shares, and since its final stake in D_1 is at most α_C, it will not foreclose D_2.

Second, if $\alpha_C G \geq L > \frac{\alpha_C}{2}G$, U_1 will foreclose D_2 after acquiring the stakes of shareholders 1 and 2, but not if it acquires the stake of only one shareholder. Now suppose that shareholder i accepted U_1's offer, while shareholder j rejected it. If U_1 acquires additional shares from D_1's dispersed shareholders, such that its final stake is $\alpha G \geq L$, then it would foreclose D_2; due to the free-rider problem, U_1 pays $(\alpha - \alpha_i)\, V_1^D$ for the additional shares, so his resulting payoff is

$$\begin{aligned}\alpha\left(V_1^D - t\right) + V_1^U + t - (\alpha - \alpha_i)\left(V_1^D - t\right) &\equiv \alpha_i\left(V_0^D - t\right) + V_0^U + t + \alpha_i G - L \\ &\leq \alpha_i\left(V_0^D - t\right) + V_0^U + t + \frac{\alpha_C}{2}G - L \\ &< \alpha_i\left(V_0^D - t\right) + V_0^U + t,\end{aligned}$$

where the first inequality follows because $\alpha_i \leq \frac{\alpha_C}{2}$ and the second follows because $L > \frac{\alpha_C}{2}G$. The acquisition of additional shares from dispersed shareholders is not profitable, however, because the expression in the last line is U_1's payoff if it does not acquire additional shares.

Since acquiring additional shares is unprofitable, foreclosure can arise only if both shareholders 1 and 2 sell their stakes to U_1. Noting that both shareholders are pivotal, U_1 must offer them a total payment of $\alpha_C V_0^D$. The acquisition is profitable because the U_1's resulting payoff is

$$\underbrace{\alpha_C\left(V_1^D - t\right) + V_1^U + t}_{\text{Post-acquisition payoff}} - \alpha_C V_0^D \equiv V_0^U + \alpha_C G - L + (1 - \alpha_C)\, t > V_0^U,$$

where the inequality follows because $\alpha_C G \geq L$.

Third, suppose that $\frac{\alpha_C}{2}G \geq L$ and suppose that shareholder 1 rejected U_1's offer. Then, shareholder 2 will accept an offer to sell his shares at $\frac{\alpha_C}{2}V_0^D$. Anticipating that shareholder 2 would accept U_1's offer, shareholder 1 would demand a price equal to the post-acquisition value of his shares. At this price, U_1 has no incentive to acquire shareholder 1's stake. Since U_1 forecloses D_2 after acquiring shareholder 2's stake, its post-acquisition payoff is

$$\underbrace{\frac{\alpha_C}{2}\left(V_1^D - t\right) + \left(V_1^U + t\right)}_{\text{Post-acquisition payoff}} - \frac{\alpha_C}{2}V_0^D \equiv V_0^U + \frac{\alpha_C}{2}G - L + \left(1 - \frac{\alpha_C}{2}\right) t \geq V_0^U.$$

Hence, the acquisition is profitable.

In sum, foreclosure equilibrium arises whenever $\alpha_C G \geq L$, exactly as in the case where D_1 initially has a single controlling shareholder whose stake is α_C.

5.3 Toeholds

We now examine what happens when, at the outset, D_1 already holds a non-controlling stake, $\alpha_1 < \underline{\alpha}$, in U_1 (i.e., a toehold). To gain control over U_1, D_1 must acquire an additional stake $\alpha - \alpha_1$ in U_1, such that after the acquisition, its controlling stake in U_1 is $\alpha \geq \underline{\alpha}$.

Proposition 8: *Suppose that initially, D_1 holds a non-controlling stake (toehold), α_1, in U_1. The toehold has no effect on the equilibrium if U_1 is initially held by dispersed shareholders. When U_1 has initially a single controlling shareholder, a foreclosure equilibrium arises if and only if $G \geq (\alpha_C + \alpha_1) L$. An increase in D_1's toehold shrinks the range of parameters for which D_2 is foreclosed.*

Intuitively, when D_1 has a toehold in U_1, it internalizes part of the upstream loss from foreclosure, L. When U_1 has an initial controller, D_1 still needs to compensate this controller for the loss to his entire stake so D_1 internalizes a larger fraction of L. Under dispersed ownership by contrast, the toehold allows D_1 to buy fewer shares from the dispersed shareholder of U_1 and hence, D_1 internalizes only a fraction $\underline{\alpha}$ of L, exactly as in the case where it does not own a toehold. Put differently, in the single controller's case, there are $1 - \alpha_C - \alpha_1$ passive shareholder in U_1 who subsidize the foreclosure of D_2, while under dispersed ownership, D_1 can acquire only $\underline{\alpha} - \alpha_1$ additional shares to gain control, so there are $1 - \underline{\alpha} > 1 - \alpha_C - \alpha_1$ passive shareholders in U_1 who can be exploited.

5.4 Acquisition by a controller

So far we have assumed that vertical integration arises when D_1 buys a controlling stake in U_1, or U_1 buys a controlling stake in D_1. However, cases exist in which the controlling shareholder of a firm, rather than the firm itself, buys a controlling stake in a vertically related firm, either directly or through other firms that he controls. For example, in 2000, Vivendi, which already held a controlling 49% stake in Canal+ (a major European producer of pay-TV channels, with a significant presence in the distribution of films and the licensing of broadcasting rights) acquired

Seagram, which owned Universal Studios Inc.[31] Among other things, the acquisition raised a concern for the foreclosure of Canal+ rivals in the pay-TV market. Another example is the 2009 offer of International Petroleum Investment Company (IPIC), which was the controlling shareholder of Agrolinz Melamine International (AMI) (one of the leading melamine producers world-wide), to acquire a 70% stake in MAN Ferrostaal, which held a controlling 30% stake in Eurotecnica Melamine (the sole supplier and licensor of high pressure technology used in melamine production). The European Commission expressed the concern that after the acquisition, IPIC would foreclose AMI's competitors from Eurotecnica's technology.[32]

In order to study how acquisitions by controllers affect the concern for foreclosure, suppose that the controlling shareholder of D_1 also controls $m-1$ other firms which operate in other industries, and let β_1 denote the controller's stake in D_1 and $\beta_2, \ldots, \beta_m$ denote his stakes in firms $2, \ldots, m$. D_1's controller can acquire a controlling stake $\alpha \geq \underline{\alpha}$ in U_1 either directly, through D_1, or through firms $2, \ldots, m$.

Proposition 9: *Suppose that D_1 has a single controlling shareholder who also owns controlling stakes in $m-1$ firms from other markets. Then, the controller would acquire a controlling stake in U_1 through firm i in which he holds the lowest stake among all firms under his control.*

(i) *If initially, U_1 has a single controlling shareholder, then in equilibrium, D_1's controller will acquire a controlling stake, $\alpha \in [\underline{\alpha}, \alpha_C]$ in U_1 through firm i and will use it to foreclose D_2 if $\beta_1 G \geq \beta_i \alpha_C L$; if $\beta_1 G < \beta_i \alpha_C L$, the controller will acquire a controlling stake $\alpha \in \left(\frac{\beta_1}{\beta_i}\frac{G}{L}, \alpha_C\right]$ though firm i but will not use it to foreclose D_2.*

(ii) *If initially, U_1's ownership is dispersed, then in equilibrium, D_1's controller will acquire a controlling stake, $\alpha = \underline{\alpha}$ through firm i and will use it to foreclose D_2 if and only if $\beta_1 G \geq \beta_i \underline{\alpha} L$.*

Since $\beta_i \leq \beta_1$, the ability of D_1's controller to choose whether to acquire a controlling stake in U_1 through D_1 or through another firm which he controls expands the range of parameters for which D_2 is foreclosed (unless D_1 happens to be the firm in which the controller has the lowest controlling stake). Moreover, so long as $\beta_i < 1$, the controller will not acquire a controlling stake in U_1 directly, but rather through firm i. Intuitively, when the controller has a small stake in firm i, a

[31] See http://ec.europa.eu/competition/mergers/cases/decisions/m2050_en.pdf

[32] See http://ec.europa.eu/competition/mergers/cases/decisions/m5406_20090313_20212_en.pdf

large fraction of the upstream loss from foreclosing D_2 is borne by the passive shareholders of i who effectively subsidize the foreclosure of D_2. And, when β_1 is large, a large fraction of the associated downstream gain accrues to the controller. Hence, a foreclosure equilibrium is more likely when β_i is small and β_1 is large.

6 Extensions

6.1 Customer foreclosure

So far we considered the effect of partial vertical integration on input foreclosure, but with minimal modifications, our model also applies to customer foreclosure. In that respect, our model differs from existing theories of input foreclosure like Ordover, Saloner and Salop (1990), Salinger (1988), or Hart and Tirole (1990), which cannot be naturally adapted to explain customer foreclosure.[33]

To consider "customer foreclosure," we will assume, without a serious loss of generality, that there are only two upstream suppliers (i.e., $N = 2$) and will also assume that while the cost of producing two units is once again $2c$, the cost of the first unit, denoted $c(1)$, is above the cost of the second unit: $c(1) > 2c - c(1)$ or $c(1) > c$.

We now consider the possibility that after D_1 and U_1 integrate (fully or partially), D_1 stops buying the input from U_2. Then, U_2 can only sell to D_2. When D_2 buys the input from both U_1 and U_2, its marginal willingness to pay for inputs is $\Delta_1(2,1) \equiv \pi(2,1) - \pi(1,1)$. Since U_1 and U_2 make take-it-or-leave-it offers to D_1 and D_2, the equilibrium profit of U_2 is at most

$$\Delta_1(2,1) - c(1).$$

If $\Delta_1(2,1) < c(1)$, then U_2 is better off exiting the market even if it can fully extract D_2's profit from selling its input.

When U_2 exits, D_1 and D_2 buy the input only from U_1, so their marginal willingness to pay is $\Delta_1(1,1) \equiv \pi(1,1) - \pi(0,1)$. If U_1 charges $\Delta_1(1,1)$ for the input, and assuming for simplicity

[33] In Ordover, Saloner and Salop (1990), the foreclosure of D_2 gives D_1 a strategic advantage in the downstream market since D_2 is forced to buy exclusively from U_2 and hence it pays more for the input. By contrast, the foreclosure of U_2 by D_1 does not give U_1 any advantage in the upstream market since U_1 and U_2 still engage in Bertrand competition for the supply of the input to D_2. Likewise, in Hart and Tirole (1990), foreclosure solve an opportunism problem that arises when D_1 and U_1 renegotiate their supply contract in a way that diverts downstream sales from D_2 to D_1. There is no equivalent diversion in the opposite case where D_1 forecloses U_2 (indeed, the Hart and Tirole model does not even require more than a single upstream supplier).

that $t = 0$, its profit is

$$2\Delta_1(1,1) - 2c = 2(\Delta_1(1,1) - c).$$

This profit is positive by Assumption A2. To ensure that U_2 cannot undercut U_1 and supply D_2 at $\Delta_1(1,1)$, we will assume that $\Delta_1(1,1) < c(1)$. Since $\Delta_1(2,1) < \Delta_1(1,1)$ by Assumption A1, the assumption that $\Delta_1(1,1) < c(1)$ also ensures that $\Delta_1(2,1) < c(1)$, so indeed U_2 exits the market after being foreclosed by D_1.

The profit of U_1 when U_2 is not foreclosed is also $2(\Delta_1(2,2) - c)$. Since $\Delta_1(1,1) > \Delta_1(2,1)$, and since by Assumption A3, $\Delta_1(2,1) > \Delta_1(2,2)$, then

$$2(\Delta_1(2,2) - c) < 2(\Delta_1(2,1) - c) < 2(\Delta_1(1,1) - c).$$

That is, U_1 makes more money when U_2 is foreclosed. The resulting gain of U_1 from customer foreclosure is

$$G_c = 2(\Delta_1(1,1) - \Delta_1(2,2)).$$

This gain comes from the increase in the willingness of D_1 and D_2 to pay for U_1's input.

As for D_1, recall that when U_2 is foreclosed, D_1 pays $\Delta_1(1,1)$ for the input, so its profit is

$$\pi(1,1) - \Delta_1(1,1) = \pi(1,1) - (\pi(1,1) - \pi(0,1)) = \pi(0,1).$$

Absent foreclosure, D_1's profit is

$$\pi(2,2) - 2\Delta_1(2,2).$$

D_1's resulting loss from customer foreclosure is

$$\begin{aligned} L_c &= \pi(2,2) - 2\Delta_1(2,2) - \pi(0,1) \\ &= \underbrace{\pi(2,2) - \pi(1,2)}_{\Delta_1(2,2)} + \underbrace{\pi(1,2) - \pi(0,2)}_{\Delta_1(1,2)} - 2\Delta_1(2,2) + \underbrace{\pi(0,2) - \pi(0,1)}_{\Delta_2(0,2)} \\ &= \Delta_1(1,2) - \Delta_1(2,2) + \Delta_2(0,2) \\ &= -\Delta_{11}(2,2) + \Delta_2(0,2). \end{aligned}$$

We will assume that $L_c > 0$.

Using G_c and L_c, we now report the following proposition which is analogous to the results from Section 4:

Proposition 10: *Suppose that the cost of the first unit is higher than the cost of the second unit, i.e.,* $c(1) > c$*, and suppose moreover, that* $c(1) > \Delta_1(1,1)$ *and* $-\Delta_{11}(2,2) + \Delta_2(0,2) > 0$*. Then,*

(i) *Under forward integration, U_1 will acquire a controlling stake $\underline{\alpha}$ in D_1 and will foreclose U_2 if and only if $G_c \geq \underline{\alpha} L_c$ when D_1's ownership is initially dispersed, and will acquire a controlling stake α_C in D_1 and will foreclose U_2 if and only if $G_c \geq \alpha_C L_c$, when D_1 has initially a controlling shareholder.*

(ii) *Under backward integration, D_1 can foreclose U_2 unilaterally, so when U_1's has initially a single controlling shareholder, a foreclosure equilibrium arises if and only if $\alpha_C G_c \geq L_c$. Backward integration is not profitable when U_1's ownership is initially dispersed.*

6.2 Backward integration under partial control

Up to now we have assumed that control is an all-or-nothing parameter: an ownership stake $\alpha \geq \underline{\alpha}$ gives a shareholder full control over the target, while an ownership stake $\alpha < \underline{\alpha}$ gives the shareholder no influence over the target. We now relax this assumption and assume instead that when D_1 holds a stake α in U_1, the management of U_1 maximizes a weighted average of the payoffs of U_1's passive shareholders and the payoff of D_1:

$$(1 - \omega(\alpha)) V^U + \omega(\alpha) \left(V^D + \alpha V^U\right), \tag{13}$$

where $\omega'(\alpha) \geq 0$, with $\omega(0) = 0$ and $\omega(1) = 1$. Notice that our setup so far is a special case of (13) and arises when $\omega(\alpha) = 0$ for $\alpha < \underline{\alpha}$ and $\omega(\alpha) = 1$ for $\alpha \geq \underline{\alpha}$.[34] The objective function (13) can also be expressed as

$$V^D + \underbrace{\left(\frac{1}{\omega(\alpha)} + \alpha - 1\right)}_{\delta(\alpha)} V^U. \tag{14}$$

Note that $\delta(0) = \infty$ and $\delta(1) = 1$, so when D_1 does not acquire any stake in U_1, the management of U_1 simply maximizes V^U (as in Section 4 when $\alpha < \underline{\alpha}$), while under a full integration, it maximizes $V^D + V^U$ (as in Section 4 when $\alpha \geq \underline{\alpha}$). Moreover, note that

$$\delta'(\alpha) = 1 - \frac{\omega'(\alpha)}{\omega(\alpha)^2}.$$

Assuming that $\omega'(0)$ is finite and recalling that $\omega(0) = 0$, it follows that $\delta'(\alpha) < 0$ when α is small. If we assume in addition that $\omega(\alpha)$ is increasing with α at a decreasing rate, i.e., $\omega'(\alpha) \geq 0 \geq \omega''(\alpha)$, then $\delta''(\alpha) > 0$. Intuitively, an increase in α raises the weight that U_1's management assigns to D_1's payoff and this lowers $\delta(\alpha)$, but when α is large, U_1's profit has a bigger effect on D_1's payoff,

[34] For alternative ways to capture partial control, see Salop and O'Brien (2000).

which in turn raises $\delta(\alpha)$. If the first effect dominates the second, $\delta(\alpha)$ is decreasing with α; otherwise, $\delta(\alpha)$ is U-shaped.

In what follows, we will assume, again for simplicity, that $t = 0$. Let $\widehat{\alpha}$ denote the ownership stake of D_1 in U_1 at which $\delta(\alpha)$ is minimized and let $\widehat{\delta}$ denote the minimum of $\delta(\alpha)$. Then, assuming that $G > \widehat{\delta}L$, the equation $G = \delta(\alpha) L$ defines either a unique value of α, denoted α^*, if $G > L$, or it generically defines two values of α, denoted α^* and α^{**}, where $\alpha^* < \widehat{\alpha} < \alpha^{**}$, if $G \leq L$. This is illustrated in Figure 1.

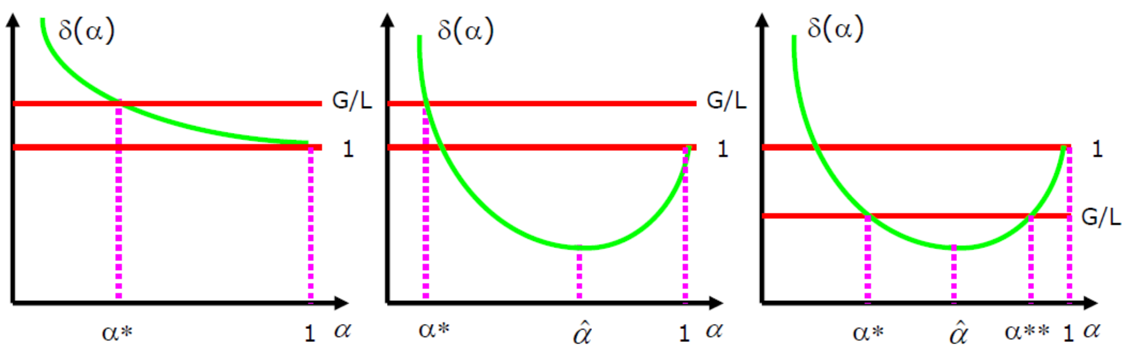

Figure 1: The $\delta(\alpha)$ function

Given (14), U_1's management decides to foreclose D_2 if and only if:

$$V_1^D + \delta(\alpha) V_1^U \geq V_0^D + \delta(\alpha) V_0^U \qquad \Longrightarrow \qquad G \geq \delta(\alpha) L.$$

If $G < \widehat{\delta}L$, then U_1's management will never foreclose D_2, so D_1 will have no incentive to integrate backward, regardless of the initial ownership structure of U_1. The next proposition shows that whenever $G \geq \widehat{\delta}L$, we always have a foreclosure equilibrium and it also characterizes the equilibrium.

Proposition 11: *Suppose that when D_1 holds a stake α in U_1, the objective function of U_1's management is given by (14).*

(i) *If U_1 has initially a single controlling shareholder whose equity stake is α_C and if $G > L$, then a foreclosure equilibrium exists. In equilibrium, D_1 acquires a stake $\alpha \in [\alpha^*, \alpha_C]$ if $\alpha^* < \alpha_C$ and α^* otherwise. If $G \leq L$, then a foreclosure equilibrium exists if only if $G \geq \max\left\{\widehat{\delta}, \alpha_C\right\} L$ and in equilibrium, D_1 acquires a stake $\alpha \in [\alpha^*, \min\{\alpha^{**}, \alpha_C\}]$.*

(ii) *If initially U_1's ownership is dispersed, then a foreclosure equilibrium exists if only if $G \geq \widehat{\delta}L$. In equilibrium D_1 acquires an ownership stake $\alpha = \alpha^*$.*

Proposition 11 shows that our results in Section 4 carry over to the case where the acquirer obtains only partial control over the target, with some modifications. Still the main point is that partial ownership affects the conditions under which foreclosure arises and foreclosure is (weakly) easier when U_1 is initially held by dispersed shareholders.

6.3 A mandatory bid rule (MBR)

A main insight of this paper is that partial backward integration promotes input foreclosure, while partial forward integration promotes customer foreclosure. The reason for this is that in both cases, foreclosure is effectively subsidized by the passive shareholders of the target. Therefore, it is quite obvious that a strong protection of minority shareholders alleviates, at least to some extent, the concern that partial integration will lead to foreclosure. In this section, we consider the effect of a mandatory bid rule (MBR), which applies in many countries, including most European countries (though not in the U.S.), and requires the acquirer of a sufficiently large controlling stake, typically $30\% - 33\%$ (see Marccus Partners, 2012), to extended the offer to the target's remaining shareholders.[35] We now briefly discuss how our theory might change under an MBR.

Consider first backward integration and suppose that $\underline{\alpha}$ is above the MBR threshold (otherwise the MBR has no bite). If U_1 has an initial controller and D_1 acquires his stake, then D_1 must extend the offer to U_1's passive shareholders. If the passive shareholders accept the offer, D_1 becomes the sole owner of U_1, so foreclosure arises if and only if $G \geq L$. To find out if the acquisition is worthwhile, note that as in Section 4.1, D_1 needs to pay U_1's initial controller $b^U = \alpha V^U_{BI}(\alpha) + \alpha_C \left(V^U_0 - V^U_{BI}(\alpha)\right)$ for his stake α_C. This payment implies a value of $\frac{b^U}{\alpha_C}$ for the

[35] The rule is also known as the Equal Opportunity Rule (EOR). The EU Directive 2004/25/EC on takeover bids requires all EU member states to adopt the rule, although it allows states to maintain exceptions from the rule, see Annex 3 in EC (2007). For analysis of the effect of the MBR or EOR on takeovers, see Bebchuk (1994) and Burkart and Panunzi (2004).

entire firm, so D_1's post-acquisition payoff is

$$\begin{aligned} Y^D &= V_{BI}^D + V_{BI}^U - b^U - (1-\alpha_C)\frac{b^U}{\alpha_C} \\ &= V_{BI}^D + V_{BI}^U - \left(1+\frac{1-\alpha_C}{\alpha_C}\right)\underbrace{\left(\alpha_C V_{BI}^U + \alpha_C\left(V_0^U - V_{BI}^U\right)\right)}_{b^U} \\ &= V_{BI}^D + V_{BI}^U - V_0^U. \end{aligned}$$

Since foreclosure arises if and only if $G \geq L$ and since there is no tunneling under full integration, it follows that when $G < L$, $V_{BI}^D + V_{BI}^U = V_0^D + V_0^U$, so $Y^D = V_0^D$. Hence, D_1 has no incentive to pursue the acquisition. When $G \geq L$, $V_{BI}^D + V_{BI}^U = V_0^D + G + V_0^U - L$, so $Y^D = V_0^D + G - L \geq V_0^D$, implying that the acquisition is profitable.

When U_1's ownership is dispersed, Section 4.1 shows that D_1 acquires the lowest stake needed for control, $\underline{\alpha}$, and pays V_0^U for the entire U_1. If D_1 needs to acquire all shares, its resulting payoff is again $V_{BI}^D + V_{BI}^U - V_0^U$, so the acquisition is once again profitable if and only if $G \geq L$. Hence, the situation under an MBR is exactly as in the full integration case.

Under forward integration, U_1 does not need to acquire control in D_1, so the MBR may not apply. To see what happens if it does, recall from Section 4.3 that if D_1 has an initial controller, then U_1 needs to pay him $b^D = \alpha V_{FI}^D(\alpha) - \alpha_C\left(V_{FI}^D(\alpha) - V_0^D\right)$ for a stake α. Offering the same price to D_1's passive shareholders, implies a price per share of $\frac{b^D}{\alpha}$. Now suppose by way of negation that the passive shareholders accept the offer. Then U_1 becomes the sole owner of D_1, so there is no tunneling. By (8), $V_{FI}^D(\alpha) \geq V_0^D$, so $\frac{b^D}{\alpha} = V_{FI}^D(\alpha) - \frac{\alpha_C}{\alpha}\left(V_{FI}^D(\alpha) - V_0^D\right) \leq V_{FI}^D(\alpha)$. Hence, D_1's passive shareholders are better-off rejecting the offer, a contradiction. Hence an MBR is irrelevant. When D_1's ownership is dispersed, U_1 needs to offer them the post-acquisition value of shares, otherwise they will reject the offer. But then U_1 breaks even on the acquisition, and since its upstream profit weakly falls, it has no incentive to purse an acquisition.[36]

In sum, an MBR has no effect on foreclosure under forward integration, but under backward integration, it implies that a foreclosure equilibrium arises if and only if it also arises under full integration. Since the latter occurs for a more limited set of parameters than under backward integration without an MBR, it follows that an MBR reduces the set of parameters for which backward integration leads to foreclosure. Hence, while the corporate finance literature has already

[36] In Section 4.3, U_1 may still wish to buy a controlling stake in D_1 by making a restricted offer for a stake $\underline{\alpha}$ and then exploit the remaining passive shareholders by engaging in tunneling. Here by contrast, U_1 cannot make a restricted offer and hence it cannot gain from tunneling.

pointed out that an MBR can protect minority shareholders against inefficient transfers of control (see e.g., Bebchuk (1994)), our results show that an MBR can also alleviate the concern for input foreclosure under backward integration.

7 Conclusion

We have merged ideas from IO and from corporate finance in order to develop a general framework that allows us to study the interaction between the takeover process and the antitrust implications of partial vertical integration. In particular, we studied the incentive to acquire partial stakes in vertically related firms and then foreclose a downstream rival. Such foreclosure generates a downstream gain by weakening the downstream rival, but entails an upstream loss due to the forgone sales to the downstream rival. Although we focused on the foreclosure of a downstream rival (input foreclosure), our theory can also apply, with some modifications, to the foreclosure of upstream rivals (customer foreclosure).

A main insight from our analysis is that under backward integration, the passive shareholders of U_1 bear part of the upstream loss from input foreclosure, so partial backward integration is more profitable when their post-acquisition stake is large. We showed that when U_1 is initially held by dispersed shareholders, D_1 can acquire just the minimal stake that ensures control and hence there are relatively many passive shareholders who subsidize the foreclosure. When control is acquired from an initial controller, whose controlling stake is above the minimum needed for control, the stake of passive shareholders is smaller, so partial backward integration is less profitable. Acquisition of control from an initial controller is even less profitable when D_1 holds a toehold in U_1, because the toehold reduces further the stake of passive shareholders in U_1 who can be exploited. By contrast, a toehold is irrelevant when U_1's ownership is initially dispersed, because the toehold allows D_1 to acquire a smaller stake in order to gain control, and hence it does not affect the ultimate stake of passive shareholders in U_1. We also showed that input foreclosure is particularly profitable when D_1 has an initial controller who can acquire a controlling stake in U_1 through some other firm which he controls. In that case, some of the upstream loss from foreclosure is borne by the passive shareholders of the acquiring firm.

Under forward integration, input foreclosure can arise regardless of whether U_1 gains control over D_1 because the decision to foreclose is taken by U_1 rather than by D_1. Nevertheless, the foreclosure of a downstream rival boosts the value of D_1, so when D_1 is initially held by dispersed shareholders, the shareholders agree to sell their shares only if U_1 offers them the post-acquisition value of their shares. This renders the acquisition unprofitable because U_1 breaks even on the acquired shares and cannot cover the associated upstream loss. When D_1 has an initial controller, the stake of passive shareholders, who capture part of the downstream gain from foreclosure, is smaller, so now the acquisition might be profitable, provided that the downstream gain is sufficiently larger than the upstream loss.

From an antitrust perspective, our analysis suggests that antitrust authorities should pay special attention to the post-acquisition stake of passive shareholders in the target firm and to whether these shareholders stand to gain or lose from foreclosure. Our theory also implies that strong corporate governance is another important factor that should be taken into account, because it affects the acquire's ability to exploit the passive shareholders of the target.

Although we considered several scenarios and extensions, there are still many open questions which are left for future research. We now briefly mention three questions. First, we only considered the possibility that one upstream firm and one downstream firm integrate. But as Ordover, Saloner, and Salop (1990) show, foreclosure may induce another pair of upstream and downstream firms to merge; this possibility constrains the ability of the merged entity to foreclose the downstream rival. It would be interesting to consider how the possibility of a countermerger affects the incentive to integrate in our model in the first place. Second, we did not consider competition for acquiring the target. If the two downstream firms, say, compete to acquire a stake in U_1, then each would have a stronger incentive to control U_1 because it can both foreclosure the rival and prevent the rival from foreclosing it. The question then is how this extra incentive affects matters. Third, firms in our model are symmetric. The question is whether asymmetry in either the downstream profits or upstream costs makes vertical foreclosure more or less likely, relative to the symmetric case, and whether the more or less efficient firms are more likely to be the first to integrate.

A Proofs

Following are the proofs of Lemmas 1-2, Corollary 1, and Propositions 4-9, and 11.

Proof of Lemma 1: By Assumption A2, in equilibrium each supplier sells to at least one downstream firm. Now suppose by way of negation that there exists an equilibrium in which k_1 suppliers sell exclusively to D_1, k_2 suppliers sell exclusively to D_2, and $N - k_1 - k_2 \geq 0$ suppliers sell to both downstream firms. In this equilibrium, D_1 buys $N - k_2$ inputs and D_2 buys $N - k_1$ inputs. Hence, the marginal willingness of D_1 to pay for inputs is $\Delta_1 (N - k_2, N - k_1)$, while the marginal willingness of D_2 to pay for inputs is $\Delta_1 (N - k_1, N - k_2)$. Since the upstream suppliers make take-it-or-leave-it offers to the two downstream firms, in equilibrium, each downstream firm pays a price equal to its marginal willingness to pay. Consequently, the profit of each supplier that sells exclusively to D_1 is

$$\Delta_1 (N - k_2, N - k_1) - c.$$

If the supplier also sells to D_2, its profit becomes:

$$\underbrace{\Delta_1 (N - k_2, N - k_1 + 1)}_{\text{The price that } \mathrm{D}_1 \text{ pays}} + \underbrace{\Delta_1 (N - k_1 + 1, N - k_2)}_{\text{The price that } \mathrm{D}_2 \text{ pays}} - 2c.$$

Selling to both D_1 and D_2 is more profitable since

$$\begin{aligned} &[\Delta_1 (N - k_2, N - k_1 + 1) + \Delta_1 (N - k_1 + 1, N - k_2) - 2c] - [\Delta_1 (N - k_2, N - k_1) - c] \\ &= \Delta_1 (N - k_1 + 1, N - k_2) - c - \underbrace{[\Delta_1 (N - k_2, N - k_1) - \Delta_1 (N - k_2, N - k_1 + 1)]}_{-\Delta_{12}(N-k_2, N-k_1+1)} > 0, \end{aligned}$$

where the inequality follows from Assumption A4. A similar argument applies when suppliers sell exclusively to D_2. Hence, in equilibrium, suppliers $2, \ldots, N$ sell to both D_1 and D_2.

The last part of the lemma follows because D_1 and D_2 pay input prices that reflect their marginal willingness to pay. ■

Proof of Corollary 1: Assumption A1 implies that $\Delta_{11} (\cdot, \cdot) < 0$, so $\Delta_1 (k, N) > \Delta_1 (N, N)$ for all $k < N$. Hence,

$$V_0^D = \pi (0, N) + \sum_{k=1}^{N} \Delta_1 (k, N) - N\Delta_1 (N, N) = \pi (0, N) + \sum_{k=1}^{N} (\Delta_1 (k, N) - \Delta_1 (N, N)) > 0.$$

By Assumption A2, $V_0^U \equiv 2 (\Delta_1 (N, N) - c) > 0$. ■

Proof of Lemma 2: First, notice that if $V \leq V_{BI}^{U}(\alpha)$ (the price that D_1 offers is below the post-acquisition value of U_1), it is a dominant strategy for each shareholder not to tender. And, given the assumption in the lemma, shareholders also do not tender if $V_{BI}^{U}(\alpha) \leq V < V_0^U$. Hence, the tender offer fails for sure if $V < V_0^U$. By contrast, if $V \geq V_0^U$, then it is a weakly dominant strategy for each shareholder to fully tender his shares: if the offer succeeds, the shareholder gets V_0^U on the sold shares, but gets only $V_{BI}^{U}(\alpha) \leq V_0^U$ on retained shares; if the offer fails, the value of the shares is V_0^U regardless of whether they are tendered. Since the tender offer surely succeeds, it is optimal for D_1 to set $V = V_0^U$, which is the lowest price that ensures success. ■

Proof of Proposition 4: Recall that when $\alpha < \frac{L}{G}$, U_1 needs to pay the atomistic shareholders of D_1 the pre-acquisition value of their shares, V_0^D, and when $\alpha \geq \frac{L}{G}$, U_1 needs to set V equal to the post-acquisition value of D_1, which is either V_1^D absent tunneling, or $V_1^D - t$ with tunneling. Recalling in addition that $V_1^U \equiv V_0^U - L$, and rearranging terms, the post-acquisition payoff of U_1, as a function of the size of the acquired stake, α, is given by

$$Y^U(\alpha) = \begin{cases} V_0^U + \alpha V_0^D - \alpha V_0^D = V_0^U, & \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\}, \\ V_1^U + \alpha V_1^D - \alpha V_1^D = V_0^U - L, & \frac{L}{G} < \alpha < \underline{\alpha}, \\ V_0^U + t + \alpha\left(V_0^D - t\right) - \alpha V_0^D = V_0^U + (1-\alpha)\,t, & \underline{\alpha} \leq \alpha < \frac{L}{G}, \\ V_1^U + t + \alpha\left(V_1^D - t\right) - \alpha\left(V_1^D - t\right) = V_0^U - L + t, & \alpha \geq \max\left\{\underline{\alpha}, \frac{L}{G}\right\}. \end{cases} \tag{15}$$

Given Assumption A5, $V_0^U + (1-\alpha)\,t \geq V_0^U > V_0^U - L + t \geq V_0^U - L$, where the first and third inequalities are strict when $t > 0$. Since $V_0^U + (1-\alpha)\,t$ is decreasing with α, $Y^U(\alpha)$ is maximized at $\alpha = \underline{\alpha}$ if $\underline{\alpha} < \frac{L}{G}$. If $\underline{\alpha} > \frac{L}{G}$, the third line of $Y^U(\alpha)$ is irrelevant, so now $Y^U(\alpha)$ is maximized by choosing some $\alpha < \frac{L}{G}$ (the first line of (15)). At this α, $Y^U(\alpha) = V_0^U$, so U_1 has no incentive to pursue an acquisition. ■

Proof of Proposition 5: Given b^D, U_1's payoff if it acquires a stake α is $V_{FI}^{U}(\alpha) + \alpha V_{FI}^{D}(\alpha) - b^D$. Using (8), (9) and (11) and rearranging terms, U_1's payoff as a function of α can be written as follows:

$$Y^U(\alpha) = \begin{cases} V_0^U, & \alpha < \min\left\{\underline{\alpha}, \frac{L}{G}\right\}, \\ V_0^U + \alpha_C G - L, & \frac{L}{G} < \alpha < \underline{\alpha}, \\ V_0^U + (1-\alpha_C)\,t, & \underline{\alpha} \leq \alpha < \frac{L}{G}, \\ V_0^U + \alpha_C G - L + (1-\alpha_C)\,t, & \alpha \geq \max\left\{\underline{\alpha}, \frac{L}{G}\right\}. \end{cases} \tag{16}$$

There are now two possibilities: (i) If $\alpha_C G \geq L$, then $Y^U(\alpha)$ is maximized at $\alpha \geq \max\left\{\underline{\alpha}, \frac{L}{G}\right\}$ (the last line in 16)). In equilibrium, U_1 will foreclose D_2. (ii) If $\alpha_C G < L$, then $Y^U(\alpha)$ is maximized at $\alpha \in [\underline{\alpha}, \alpha_C]$ (the third line of (16)). In equilibrium, U_1 will not foreclose D_2. ■

Proof of Proposition 6: Suppose that $\alpha_C^U \geq \frac{L}{G}$ and $\alpha_C^U < \frac{G}{L}$, so we get a foreclosure equilibrium regardless of whom controls the partially integrated firm. Given (1), (2), (8), and (9), the difference between the joint payoffs of the two controllers under backward and forward integration is given by

$$\begin{aligned} \Delta &= \left[\alpha_C^D \left(V_1^D + t + \alpha_C^U \left(V_1^U - t\right) - b^U\right) + b^U\right] - \left[\alpha_C^U \left(V_1^U + t + \alpha_C^D \left(V_1^D - t\right) - b^D\right) + b^D\right] \\ &= \left(1 - \alpha_C^U\right)\left(\alpha_C^D \left(V_1^D + t\right) - b^D\right) - \left(1 - \alpha_C^D\right)\left(\alpha_C^U \left(V_1^U + t\right) - b^U\right). \end{aligned}$$

To determine the sign of Δ, notice that $\Delta = 0$ when $\alpha_C^D = \alpha_C^U = 1$ (but since $\alpha_C^U \geq \frac{L}{G}$ and $\alpha_C^U < \frac{G}{L}$, this can occur only when $G = L$). Otherwise, U_1's controller will agree that U_1 will acquire the stake of D_1's controller only if this forward integration boosts the value of his controlling stake in U_1, i.e., only if $\alpha_C^U \left(V_1^U + t + \alpha_C^D \left(V_1^D - t\right) - b^D\right) \geq \alpha_C^U V_0^U$ or $b^D \leq \alpha_C^D \left(V_1^D - t\right) + V_1^U - V_0^U + t$. Moreover, to induce U_1's controller to sell his stake to D_1, b^U must be at least equal to the controller's payoff absent a deal, i.e., $b^U \geq \alpha_C^U V_0^U$. Hence,

$$\begin{aligned} \Delta &\geq \left(1 - \alpha_C^U\right)\left(\alpha_C^D \left(V_1^D + t\right) - \alpha_C^D \left(V_1^D - t\right) - V_1^U + V_0^U - t\right) - \left(1 - \alpha_C^D\right)\left(\alpha_C^U \left(V_1^U + t\right) - \alpha_C^U V_0^U\right) \\ &\equiv \left(1 - \alpha_C^U\right)\left(2\alpha_C^D t + L - t\right) + \left(1 - \alpha_C^D\right)\alpha_C^U (L - t) > 0, \end{aligned}$$

where the last inequality follows from Assumption A5. Moreover, the joint payoff of the two controllers under backward integration exceeds their joint payoff absent integration:

$$\begin{aligned} \alpha_C^D \left(V_1^D + t + \alpha_C^U \left(V_1^U - t\right) - b^U\right) + b^U &\geq \alpha_C^D \left(V_1^D + t + \alpha_C^U \left(V_1^U - t\right)\right) + \alpha_C^U \left(1 - \alpha_C^D\right) V_0^U \\ &\equiv \alpha_C^D \left(V_0^D + G + t + \alpha_C^U \left(V_0^U - L - t\right)\right) + \alpha_C^U \left(1 - \alpha_C^D\right) V_0^U \\ &= \alpha_C^D V_0^D + \alpha_C^U V_0^U + \alpha_C^D \left(G - \alpha_C^U L + \left(1 - \alpha_C^U\right) t\right) \\ &> \alpha_C^D V_0^D + \alpha_C^U V_0^U, \end{aligned}$$

where the inequality follows since by assumption, $G \geq \alpha_C^U L$, and since $t > 0$. ■

Proof of Proposition 7: Suppose that shareholder i accepted D_1's offer, shareholder j rejected it, and D_1 made a tender offer to the dispersed shareholders of U_1 for a stake $\underline{\alpha} - \frac{\alpha_C}{2}$ that ensures control. Shareholder j can outbid D_1 and prevent foreclosure by increasing his stake in U_1 from $\frac{\alpha_C}{2}$ to $\underline{\alpha}$ and thereby ensuring that D_1 cannot gain control. The highest bid, $b_{\max}$, that shareholder j will make to gain control over U_1 is such that

$$\underline{\alpha} V_0^U - b_{\max} = \frac{\alpha_C}{2}\left(V_1^U - t\right) \qquad \Longrightarrow \qquad b_{\max} = \underline{\alpha} V_0^U - \frac{\alpha_C}{2}\left(V_1^U - t\right).$$

Therefore, if D_1 wants to gain control over U_1, it must offer U_1's dispersed shareholders at least $b_{\max}$. Such an offer is profitable for D_1 if and only if

$$\underbrace{V_1^D + t + \underline{\alpha}\left(V_1^U - t\right)}_{\text{Post-acquisition payoff}} - \underbrace{\left(\underline{\alpha}V_0^U - \frac{\alpha_C}{2}\left(V_1^U - t\right)\right)}_{b_{\max}} \geq \underbrace{V_0^D + \frac{\alpha_C}{2}V_0^U}_{\text{Payoff absent an offer}}, \quad \Longrightarrow \quad G+t \geq \left(\underline{\alpha} + \frac{\alpha_C}{2}\right)(L+t)\,. \tag{17}$$

Suppose that (17) holds. Then shareholder 2 realizes that once shareholder 1 accepted D_1's offer, he cannot prevent D_1 from gaining control over U_1. Hence, shareholder 2 will accept any offer b that gives him the post-acquisition value of his shares, i.e., $b \geq \frac{\alpha_C}{2}\left(V_1^U - t\right)$. Consequently, D_1 acquires a controlling stake α_C in U_1 without having to make a tender offer to the dispersed shareholders of U_1.

By contrast, if shareholder 1 rejected D_1's offer, then shareholder 2 is pivotal, so D_1 must offer him the pre-acquisition value of his shares, $b_2 = \frac{\alpha_C}{2}V_0^U$, to induce him to tender his shares. D_1 can then buy the rest of the shares needed for control from U_1's dispersed shareholders at $b_{\max}$. Condition (17) ensures that D_1's post-acquisition payoff exceeds its pre-acquisition payoff:

$$\underbrace{V_1^D + t + \underline{\alpha}\left(V_1^U - t\right)}_{\text{Post-acquisition payoff}} - \underbrace{\frac{\alpha_C}{2}V_0^U}_{b_2} - \underbrace{\left(\underline{\alpha}V_0^U - \frac{\alpha_C}{2}\left(V_1^U - t\right)\right)}_{b_{\max}} = V_0^D + G + t - \left(\underline{\alpha} + \frac{\alpha_C}{2}\right)(L+t) \geq V_0^D.$$

Since D_1 gains control over U_1 regardless of whether shareholder 1 accepts or rejects its offer, shareholder 1 will agree to sell D_1 his stake in the first place at its post-acquisition value $\frac{\alpha_C}{2}\left(V_1^U - t\right)$. But since D_1 fully acquires shareholder 1's stake, shareholder 2 will also agree to sell his stake to D_1 for $\frac{\alpha_C}{2}\left(V_1^U - t\right)$. Altogether then, when (17) holds, D_1 acquires the stakes of shareholders 1 and 2 by paying them $2\left(\frac{\alpha_C}{2}\right)\left(V_1^U - t\right) = \alpha_C\left(V_1^U - t\right)$. The post-acquisition payoff of D_1 is

$$Y_2^D = V_1^D + t + \alpha_C\left(V_1^U - t\right) - \alpha_C\left(V_1^U - t\right) \equiv V_0^D + G + t.$$

The acquisition is profitable since $Y_2^D > V_0^D$, where V_0^D is D_1's pre-acquisition payoff.

Next, suppose that (17) fails. Then, D_1 can gain control over U_1 only if it acquires the stakes of both shareholders 1 and 2. Suppose moreover that shareholder 1 already sold his stake to D_1 (otherwise D_1 cannot gain control over U_1 and the game ends). Since shareholder 2 is pivotal, D_1 must offer him a price b_2, which is at least as high as the shareholder's payoff if he rejects D_1's offer. To compute this payoff, note that if shareholder 2 rejects D_1's offer, D_1 can make a tender offer to the dispersed shareholders of U_1 for a stake $\underline{\alpha} - \frac{\alpha_C}{2}$. If shareholder 2 does not make a counter tender offer, D_1 gains control over U_1 and his total stake is $\frac{\alpha_C}{2} + \left(\underline{\alpha} - \frac{\alpha_C}{2}\right) = \underline{\alpha}$. Since $G > \underline{\alpha}L$ by Assumption A6, D_1 will use its control to foreclose D_2.

To compute b_2, note that the highest bid, $b_{\max}$, that D_1 will make the dispersed shareholders of U_1 for a stake $\underline{\alpha} - \frac{\alpha_C}{2}$ if shareholder 2 rejects his offer is

$$\underbrace{V_1^D + t + \underline{\alpha}\left(V_1^U - t\right)}_{\text{Post-acquisition payoff}} - b_{\max} = \underbrace{V_0^D + \frac{\alpha_C}{2} V_0^U}_{\text{Payoff absent an offer}} \quad \Longrightarrow \quad b_{\max} = \underbrace{V_1^D - V_0^D}_{G} + t + \underline{\alpha}\left(V_1^U - t\right) - \frac{\alpha_C}{2} V_0^U.$$

Shareholder 2's payoff if he outbids $b_{\max}$ and blocks D_1's acquisition is

$$\underline{\alpha} V_0^U - \underbrace{\left(G + t + \underline{\alpha}\left(V_1^U - t\right) - \frac{\alpha_C}{2} V_0^U\right)}_{b_{\max}} = \frac{\alpha_C}{2} V_0^U - \left(G + t - \underline{\alpha}\left(L + t\right)\right).$$

Given this payoff, D_1 can buy shareholder 2's stake by offering him

$$b_2 = \frac{\alpha_C}{2} V_0^U - \left(G + t - \underline{\alpha}\left(L + t\right)\right).$$

As for shareholder 1, D_1 must offer him $b_1 = \frac{\alpha_C}{2} V_0^U$ for his stake, otherwise shareholder 1 will reject D_1's offer, in which case D_1 will be unable to gain control over U_1 and would obtain a payoff of V_0^D. The overall payoff of D_1 from acquiring the stakes of shareholders 1 and 2 is therefore

$$\begin{aligned} Y_2^D &= \underbrace{V_1^D + t + \alpha_C\left(V_1^U - t\right)}_{\text{Post-acquisition payoff}} - \underbrace{\frac{\alpha_C}{2} V_0^U}_{b_1} - \underbrace{\left(\frac{\alpha_C}{2} V_0^U - \left(G + t - \underline{\alpha}\left(L + t\right)\right)\right)}_{b_2} \\ &\equiv V_0^D + 2\left(G + t\right) - \left(\alpha_C + \underline{\alpha}\right)\left(L + t\right). \end{aligned}$$

Since $G \geq \alpha_C L \geq \underline{\alpha} L$, then $Y_2^D \geq V_0^D$, so the acquisition is profitable.

In sum, D_1 acquires a total stake α_C in U_1 and since $G \geq \alpha_C L$ it uses it to foreclose D_2.

■

Proof of Proposition 8: First, suppose that U_1 is initially held by a continuum of atomistic shareholders. As in Section 4.2, given D_1's eventual stake α, foreclosure would arise if and only if $G \geq \alpha L$. By Lemma 2, D_1 would offer the dispersed shareholders a price that reflects a value of V_0^U for the entire firm and would therefore pay a total of $\left(\alpha - \alpha_1\right) V_0^U$ for the acquired stake. Hence, the post-acquisition value of D_1 is

$$Y^D\left(\alpha\right) = \begin{cases} V_0^D + \alpha V_0^U - \left(\alpha - \alpha_1\right) V_0^U, & \alpha < \underline{\alpha}, \\ V_1^D + t + \alpha\left(V_1^U - t\right) - \left(\alpha - \alpha_1\right) V_0^U, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^D + t + \alpha\left(V_0^U - t\right) - \left(\alpha - \alpha_1\right) V_0^U, & \alpha > \frac{G}{L}. \end{cases}$$

Using the definitions of G and L, and rearranging, we get

$$Y^D\left(\alpha\right) = \begin{cases} V_0^D + \alpha_1 V_0^U, & \alpha < \underline{\alpha}, \\ V_0^D + \alpha_1 V_0^U + G - \alpha L + \left(1 - \alpha\right) t, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^D + \alpha_1 V_0^U + \left(1 - \alpha\right) t, & \alpha > \frac{G}{L}. \end{cases}$$

By Assumption A6, $Y^D(\alpha)$ is maximized at $\alpha = \underline{\alpha}$. Hence, D_1 would acquire $\underline{\alpha} - \alpha_1$ shares and would foreclose D_2 after the acquisition. Since the condition that ensure foreclosure, $G \geq \underline{\alpha} L$, is identical to that in Proposition 3, the toehold does not affect the equilibrium.

Next, suppose that U_1 is initially controlled by a single shareholder, whose initial stake is α_C. The minimal offer that D_1 needs to make to induce U_1's initial controller to sell is given by (3), except that now, $\alpha - \alpha_1$ replaces than α. Hence, the post-acquisition payoff of D_1 is given by:

$$V_{BI}^D(\alpha)+\alpha V_{BI}^U(\alpha)-\underbrace{\left((\alpha-\alpha_1)V_{BI}^U(\alpha)+\alpha_C\left(V_0^U - V_{BI}^U(\alpha)\right)\right)}_{b^U} = V_{BI}^D(\alpha)+\alpha_1 V_{BI}^U(\alpha)-\alpha_C\left(V_0^U - V_{BI}^U(\alpha)\right).$$

Using (1) and (2) and rearranging terms, D_1's post-acquisition payoff becomes:

$$Y^D(\alpha) = \begin{cases} V_0^D + \alpha_1 V_0^U, & \alpha < \underline{\alpha}, \\ V_0^D + \alpha_1 V_0^U + G - (\alpha_C + \alpha_1)L + (1-\alpha_C-\alpha_1)t, & \underline{\alpha} \leq \alpha \leq \frac{G}{L}, \\ V_0^D + \alpha_1 V_0^U + (1-\alpha_C-\alpha_1)t, & \alpha > \frac{G}{L}. \end{cases}$$

$Y^D(\alpha)$ is maximized at $\alpha \in \left[\underline{\alpha}, \frac{G}{L}\right]$ when $G \geq (\alpha_C + \alpha_1)L$, and at $\alpha > \frac{G}{L}$ when $G < (\alpha_C + \alpha_1)L$. In the former case we get foreclosure. ∎

Proof of Proposition 9: Conditional on acquiring a controlling stake $\alpha \geq \underline{\alpha}$ in U_1 through firm $i = 0, 1, \ldots, m$ ("firm 0" means that the controller acquires the stake directly, so naturally, $\beta_0 = 1$), the controller will use his control over U_1 to lower D_1's payment for U_1's input by t if his stake in D_1 is at least as large as his stake in U_1, i.e., if $\beta_1 \geq \beta_i \alpha$, or $\underline{\alpha} \leq \alpha \leq \frac{\beta_1}{\beta_i}$. If $\alpha > \frac{\beta_1}{\beta_i}$, the controller will raise D_1's payment for U_1's input by t (the controller can do it even if he has no control over U_1). In this case, $t < 0$ (i.e., funds flow from D_1 to U_1).

Moreover, the controller would use his control over U_1 to foreclose D_2 if this increases his post-acquisition payoff:

$$\beta_1\left(V_1^D + t\right)+\beta_i\alpha\left(V_1^U - t\right) \geq \beta_1\left(V_0^D + t\right)+\beta_i\alpha\left(V_0^U - t\right), \qquad \Longrightarrow \qquad \beta_1\underbrace{\left(V_1^D - V_0^D\right)}_{G} \geq \beta_i\alpha\underbrace{\left(V_0^U - V_1^U\right)}_{L}.$$

That is, foreclosure arises if and only if $\alpha \leq \frac{\beta_1}{\beta_i}\frac{G}{L}$. Note that the decision to foreclose D_2 is independent of t. The payoffs of D_1 and U_1 under (partial) backward integration are given by:

$$V_{BI}^D(\alpha) = \begin{cases} V_0^D, & \alpha < \underline{\alpha}, \\ V_1^D + t \equiv V_0^D + G + t, & \underline{\alpha} \leq \alpha \leq \frac{\beta_1}{\beta_i}\frac{G}{L}, \\ V_0^D + t, & \alpha > \frac{\beta_1}{\beta_i}\frac{G}{L}, \end{cases} \tag{18}$$

and

$$V_{BI}^{U}(\alpha)=\begin{cases} V_0^U, & \alpha<\underline{\alpha}, \\ V_1^U-t\equiv V_0^U-L-t, & \underline{\alpha}\leq\alpha\leq\frac{\beta_1}{\beta_i}\frac{G}{L}, \\ V_0^U-t, & \alpha>\frac{\beta_1}{\beta_i}\frac{G}{L}, \end{cases} \tag{19}$$

where $t>0$ if $\alpha\leq\frac{\beta_1}{\beta_i}$ and $t<0$ if $\alpha>\frac{\beta_1}{\beta_i}$.

Given that firm i pays b for a stake α in U_1, the controller's payoff is given by

$$\beta_1 V_{BI}^{D}(\alpha)+\beta_i\left(\alpha V_{BI}^{U}(\alpha)-b\right)+\sum_{j=2,\ldots,m}\beta_j V_j, \tag{20}$$

where V_j is the value of firm $j=2,\ldots,m$.

Now, suppose that U_1 is initially controlled by a single shareholder, whose initial stake is α_C. The minimal offer that firm i needs to make to induce U_1's initial controller to sell his shares is given by (3). Substituting for $b=b^U$ in (20), the post-acquisition payoff of D_1's controller becomes:

$$\begin{aligned} & \beta_1 V_{BI}^{D}(\alpha)+\beta_i\left[\alpha V_{BI}^{U}(\alpha)-\underbrace{\left(\alpha V_{BI}^{U}(\alpha)+\alpha_C\left(V_0^U-V_{BI}^{U}(\alpha)\right)\right)}_{b^U}\right]+\sum_{j=2,\ldots,m}\beta_j V_j \\ = \quad & \beta_1 V_{BI}^{D}(\alpha)-\beta_i\alpha_C\left(V_0^U-V_{BI}^{U}(\alpha)\right)+\sum_{j=2,\ldots,m}\beta_j V_j. \end{aligned}$$

Using equations (18) and (19) and rearranging terms, the post-acquisition payoff of D_1's controller becomes

$$Y^C(\alpha)=\sum_{j=2,\ldots,m}\beta_j V_j+\begin{cases} \beta_1 V_0^D, & \alpha<\underline{\alpha}, \\ \beta_1 V_0^D+\beta_1 G-\beta_i\alpha_C L+(\beta_1-\beta_i\alpha_C)\,t, & \underline{\alpha}\leq\alpha\leq\frac{\beta_1}{\beta_i}\frac{G}{L}, \\ \beta_1 V_0^D+(\beta_1-\beta_i\alpha_C)\,t, & \alpha>\frac{\beta_1}{\beta_i}\frac{G}{L}. \end{cases} \tag{21}$$

Notice that $Y^C(\alpha)$ is decreasing with β_i, which is obvious when $t>0$, but is also true when $t<0$ since by Assumption A5, $L>t$. Hence, if D_1's controller acquires a controlling stake in U_1, he will do it via the firm in which he holds the lowest controlling stake. As a result, $\beta_i\leq\beta_1$, which implies in turn that $\alpha\leq\frac{\beta_1}{\beta_i}$, so $t>0$: after the acquisition, D_1's controller will use his control over U_1 to lower D_1's payment for U_1's input.

Since $t>0$, the maximum of $Y^C(\alpha)$ is attained at the second line of (21) if $\beta_1 G\geq\beta_i\alpha_C L$, and since $\alpha\leq\alpha_C$ (the initial controller's stake is at most α_C), the size of the acquired controlling stake is $\alpha\in[\underline{\alpha},\alpha_C]$. If $\beta_1 G<\beta_i\alpha_C L$, then $Y^C(\alpha)$ attains a maximum at the second line of (21), and since $\alpha\leq\alpha_C$, the size of the acquired controlling stake now is $\alpha\in\left(\frac{\beta_1}{\beta_i}\frac{G}{L},\alpha_C\right]$. Foreclosure arises only in the former case.

Next, suppose that U_1 is initially held by a continuum of atomistic shareholders. By Lemma 2, the acquirer i would offer the dispersed shareholders a price that reflects a value of V_0^U for the entire firm and would therefore pay $b = \alpha V_0^U$ for the acquired stake. Substituting in (20), using equations (19) and (18) and rearranging terms, the controller's post-acquisition payoff becomes:

$$Y^C(\alpha) = \sum_{j=2,\ldots,m} \beta_j V_j + \begin{cases} \beta_1 V_0^D, & \alpha < \underline{\alpha}, \\ \beta_1 V_0^D + \beta_1 G - \beta_i \alpha L + (\beta_1 - \beta_i \alpha)\, t, & \underline{\alpha} \leq \alpha \leq \frac{\beta_1}{\beta_i}\frac{G}{L}, \\ \beta_1 V_0^D + (\beta_1 - \beta_i \alpha)\, t, & \alpha > \frac{\beta_1}{\beta_i}\frac{G}{L}. \end{cases} \tag{22}$$

(22) differs from (21) only in that α replaces α_C. Hence, D_1's controller would acquire a controlling stake in U_1 through the firm in which he holds the lowest controlling stake. Moreover, $Y^C(\alpha)$ decreases with α and hence is maximized at $\alpha = \underline{\alpha}$. In equilibrium D_2 will be foreclosed if and only if $\beta_1 G \geq \beta_i \underline{\alpha} L$. ■

Proof of Proposition 11: Suppose that $G \geq \widehat{\delta} L$, otherwise foreclosure never arises, and suppose that initially, U_1 has an initial controller whose ownership stake is α_C. Suppose first that $G > L$; as Figure 1 shows, in this case, $\delta(\alpha^*) > \delta(\alpha)$ for all $\alpha \in (\alpha^*, 1]$.

If $\alpha_C \geq \alpha^*$, an acquisition of $\alpha \in [\alpha^*, \alpha_C]$ leads to foreclosure since $G = \delta(\alpha^*) L \geq \delta(\alpha) L$. Now note that $\omega(\alpha) \leq 1$ implies that $\delta(\alpha) \geq \alpha$ and recall that $G \equiv V_1^D - V_0^D$. Then, the acquisition is profitable for D_1 since after paying the initial controller $b^U = \alpha V_1^U + \alpha_C L$ (see (3)), D_1's payoff exceeds its pre-acquisition payoff, V_0^D:

$$\begin{aligned} V_1^D + \alpha V_1^U - \underbrace{\left(\alpha V_1^U + \alpha_C L\right)}_{b^U} &= V_0^D + G - \alpha_C L \\ &\geq V_0^D + G - \delta(\alpha_C) L \\ &\geq V_0^D + G - \delta(\alpha^*) L \\ &= V_0^D, \end{aligned}$$

where the second inequality follows because $\alpha_C \geq \alpha^*$ implies $\delta(\alpha_C) \leq \delta(\alpha^*)$. Since D_1's payoff is independent of α, D_1 will acquire in this case any $\alpha \in [\alpha^*, \alpha_C]$.

If $\alpha_C < \alpha^*$, then an acquisition of $\alpha \leq \alpha_C$ is insufficient to induce foreclosure because $G = \delta(\alpha^*) L < \delta(\alpha_C) L \leq \delta(\alpha) L$. However, D_1 can buy an additional stake $\alpha - \alpha_C$ from U_1's passive shareholders, such that after the acquisition its stake is $\alpha \geq \alpha^*$, in which case U_1 will foreclose D_2. Since D_1 needs to pay both the initial controller and the passive shareholders of

U_1 a price that reflects the pre-acquisition value of U_1, the resulting payoff of D_1 exceeds its pre-acquisition payoff:

$$V_1^D + \alpha V_1^U - \alpha_C V_0^U - (\alpha - \alpha_C)\, V_0^U = V_0^D + G - \alpha \underbrace{\left(V_0^U - V_1^U\right)}_{L}.$$

Since this expression decreases with α, D_1 will only acquire from the passive shareholders a stake of $\alpha^* - \alpha_C$, such that its final stake in U_1 is α^*. Given that $\delta(\alpha) \geq \alpha$, the acquisition is profitable since D_1's resulting payoff is

$$V_0^D + G - \alpha^* \underbrace{\left(V_0^U - V_1^U\right)}_{L} \geq V_0^D + G - \delta(\alpha^*)\, L = V_0^D. \tag{23}$$

In sum, when U_1 has an initial controller and $G > L$, we get a foreclosure equilibrium, and D_1's stake in U_1 is any $\alpha \in [\alpha^*, \alpha_C]$ if $\alpha^* < \alpha_C$ and α^* otherwise.

Next, suppose that $G \leq L$. Then as Figure 1 shows, $G > \delta(\alpha)\, L$ for all $\alpha \in (\alpha^*, \alpha^{**})$. The analysis is as before when $\alpha_C \leq \alpha^{**}$. Things are different however when $\alpha_C > \alpha^{**}$. To induce foreclosure, D_1 must acquire a stake $\alpha \leq \min\{\alpha^{**}, \alpha_C\}$. After paying the initial controller $b^U = \alpha V_1^U + \alpha_C L$ for this stake, D_1's post-acquisition payoff is

$$V_1^D + \alpha V_1^U - \underbrace{\left(\alpha V_1^U + \alpha_C L\right)}_{b^U} = V_0^D + G - \alpha_C L.$$

This payoff is (weakly) higher than the pre-acquisition payoff if and only if $G \geq \alpha_C L$. Again, D_1 is indifferent as to the actual stake it acquires provided that it induces foreclosure, i.e., is such that $\alpha \in [\alpha^*, \alpha^{**}]$.

Finally, assume that initially U_1's ownership is dispersed and assume that D_1 acquires a stake α which leads to the foreclosure of D_2 (otherwise the acquisition is not profitable). Since D_1 needs to pay the passive shareholders of U_1 a price that reflects the pre-acquisition value of U_1, the post-acquisition payoff of D_1 is

$$V_1^D + \alpha V_1^U - \alpha V_0^U = V_0^D + G - \alpha \underbrace{\left(V_0^U - V_1^U\right)}_{L}.$$

Since this payoff decreases with α, D_1 will acquire the minimal stake that ensures foreclosure, i.e., α^*. Hence, D_1's resulting payoff is as in (23), so the acquisition is profitable. ■

B The properties of the reduced firm profits

Following is an explicit model of downstream competition that is intended to motivate the assumptions made in Sections 2 and 3 on the downstream profit functions. Suppose that D_1 and D_2 are located at the two ends of a unit line and compete by setting prices. Consumers are uniformly distributed on the line and the utility of a consumer located at point x is

$$U_1(x) = v \log(n_1 + 1) - tx - p_1,$$

if he buys from D_1 and

$$U_2(x) = v \log(n_2 + 1) - t(1 - x) - p_2,$$

if he buys from D_2, where $v \log(n_1 + 1)$ and $v \log(n_2 + 1)$ are the "qualities" of D_1 and D_2 which increases with the number of inputs that D_1 and D_2 use, $t > 0$ is the transportation cost per unit of distance, and p_1 and p_2 are the prices that D_1 and D_2 charge. If the consumer does not buy at all, his utility is 0.[37]

Assuming that the market is fully covered, the location of the indifferent consumer between D_1 and D_2 is

$$x^*(p_1, p_2, n_1, n_2) = \frac{1}{2} + \frac{p_2 - p_1 + v \log\left(\frac{n_1+1}{n_2+1}\right)}{2t}. \tag{24}$$

Assuming in addition that D_1 and D_2 pay a fixed price for the inputs (the input prices are independent of actual sales) and normalizing their additional costs to 0, the gross profits of D_1 and D_2 are given by

$$\pi_1 = p_1 x^*(p_1, p_2, n_1, n_2), \qquad \pi_2 = p_2(1 - x^*(p_1, p_2, n_1, n_2)).$$

Solving for the Nash equilibrium prices, we obtain:

$$p_1^*(n_1, n_2) = t + \frac{v}{3}\log\left(\frac{n_1+1}{n_2+1}\right), \qquad p_2^*(n_1, n_2) = t - \frac{v}{3}\log\left(\frac{n_1+1}{n_2+1}\right).$$

To avoid uninteresting complications, we shall assume that t is large but not too large:

$$\frac{v}{3}\left|\log\left(\frac{n_1+1}{n_2+1}\right)\right| < t < \frac{v}{3}\log(n_1+1)(n_2+1).$$

[37] This formulation is similar to the simple model in Gavazza (2011), where the willingness of consumers to pay increase with the number of different products that each firm offers. Gavazza referes to this effect as "demand spillover." Unlike here, Gavazza considers a discrete choice model and $\log(n_i)$ is the intercept of the consumer's utility when he buys from firm i.

This assumption ensures that $p_1^*(n_1, n_2)$ and $p_2^*(n_1, n_2)$ are both nonnegative and the market is covered as we assumed.[38]

Substituting $p_1^*(n_1, n_2)$, $p_2^*(n_1, n_2)$ in the profit functions and using (24), the profit of a downstream firm when it uses k inputs and its rival uses l inputs (e.g., the profit of D_1 when $n_1 = k$ and $n_2 = l$) is

$$\pi(k, l) = \frac{\left[t + \frac{v}{3}\log\left(\frac{k+1}{l+1}\right)\right]^2}{2t}.$$

Our assumption that $t > \frac{v}{3}\left|\log\left(\frac{n_1+1}{n_2+1}\right)\right|$ ensures that $\pi(k, l)$ is increasing with k and decreasing with l as Assumption A1 states.

Now,

$$\Delta_1(k, l) \equiv \pi(k, l) - (k-1, l) = \frac{v}{6t}\log\left(\frac{k+1}{k}\right)\left[2t + \frac{v}{3}\log\left(\frac{k(k+1)}{(l+1)^2}\right)\right],$$

and

$$\Delta_2(k, l) \equiv \pi(k, l) - (k, l-1) = \frac{v}{6t}\log\left(\frac{l}{l+1}\right)\left[2t + \frac{v}{3}\log\left(\frac{(k+1)^2}{l(l+1)}\right)\right].$$

Assumption A3 holds since

$$\begin{aligned}\Delta_{12}(l, k) &\equiv \underbrace{(\pi(l, k) - (l-1, k))}_{\Delta_1(l,k)} - \underbrace{(\pi(l, k-1) - (l-1, k-1))}_{\Delta_1(l-1,k)} \\ &= \frac{v^2}{9t}\log\left(\frac{k+1}{k}\right)\log\left(\frac{l}{l+1}\right) < 0.\end{aligned}$$

Assumption A2 holds if

$$\frac{v}{6t}\log\left(\frac{N+1}{N}\right)\left[2t + \frac{v}{3}\log\left(\frac{N}{N+1}\right)\right] > c,$$

and Assumption A4 holds if

$$\frac{v}{6t}\log\left(\frac{k+1}{k}\right)\left[2t + \frac{v}{3}\log\left(\frac{k(k+1)l^2}{(l+1)^4}\right)\right] > c.$$

[38] Similar restrictions are also needed in the textbook version of the Hotelling model since when t is too high, the two firms become local monopolies, and when t is too small, the equilibrium is unstable (starting from a candidate interior equilibrium, firms may wish to cut prices drastically and corner the market).

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