Cartel behavior is designed to alter outcomes that would prevail in the absence of coordination among competitors. Discerning differences in prices, output, levels of investment and promotion and potential changes in technologies that would occur among firms is made even more difficult in cases where markets are populated by a small number of firms whose strategic interactions leave them far from acting as simple price takers. U.S. antitrust precedents recognize the inherent challenges in this process and make allowance for the precision with which damage measures must be made.\textsuperscript{1}

In his survey of cartel overcharges, John Connor found that the average overcharges based on a broad review of cartel studies of all types of private cartels are three times as high as the presumed levels and lead to penalties that rarely recover more than half of the price-fixing profits. Such results reflect that the need for accurate and robust damage estimation techniques is considerable and growing.\textsuperscript{2} Combined with the steps currently taken by the European Commission (Commission) to promote the “private enforcement” of antitrust law and based on the recent publication of its Green Paper, the quantification of damages will become more common in Europe. The Commission expects that damages claims will play a significant role in that effort recognizing that price-fixing cartels and other violations of EC antitrust law cause

\textsuperscript{1} Authors are Vice Chairman and Senior Vice President of CapAnalysis LLC, respectively.

\textsuperscript{1} U.S. antitrust law recognizes that the standard of proof for establishing the amount of damages is less stringent than the standard for establishing the fact of damage. See Dolphin Tours, 773 F. 2d. Once a plaintiff establishes the fact of damages with reasonable certainty, the jury will be permitted to “make a just and reasonable estimate of damage based on the relevant data.” Bigelow, 327 U.S. at 264

damage to consumers and companies. This paper is intended to review how U.S. antitrust precedents recognize the inherent challenges in this process of estimating damages and make allowance for the precision with which damage measures must be made.

This analysis starts with the recognition that empirical analysis of economic issues is an essential element of antitrust analysis when used correctly and conscientiously. It has proven to be a valuable tool in resolving legal disputes. Despite the enduring question of whether “some number is better than no number” it appears to be the case, particularly in damages estimation in litigation, that quantitative analysis is a key tool for the finder of fact and, as such, can control the debate and focus the arguments. Econometric models provide direct statistical connections between facts and allegations. Even though it may be a regrettable necessity, the need to value impact on the market is present in most antitrust inquiries.

Principles of U.S. antitrust law as they pertain to damages in private actions have coalesced around two considerations – damages must flow from or establish causation with the alleged conduct (applicable whether the conduct is alleged to have injured competitors or consumers) and that the damages model or methodology address any measure within the context of how competitors/consumers would have fared in the absence of the conduct at issue. In what has become a recognizable metric, damage models must provide the difference between the actual and “but-for” market. As discussed in more detail below, these principles require that a damage methodology, regardless of the intended quantitative rigor, produce supportable predictions of how a market has been affected by the offending conduct rather than a model that suitably explains how the market actually performed. This distinction is often thought of as the difference between models that fit or explain actual prices, but do not produce

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supportable predictions or forecasts of prices that would have prevailed "but-for" the conduct in question.

Multiple regression analysis has been misused, misapplied and misunderstood in cases that have not validated this most relevant element, that they provide reliable measures of the increment of prices attributable to the offending conduct. Regression estimates of antitrust damages become most reliable when the types of analyses and diagnostics discussed below are applied to estimates before they are presented to fact finders who typically have a limited basis to distinguish between questions of statistical significance (are the underlying parameters from which damages are estimated different from zero) and the weight to be assigned to competing assumptions used in rival damage models (do small adjustments to assumptions underlying the damages model eliminate the estimated overcharges).

Multiple regression analysis is a common method relied on when it is well accepted that numerous factors contribute in setting prices where the contribution of various potential forms of anticompetitive conduct must be isolated from a range of likely unaffected market dynamics to determine whether in fact prices have been elevated and who may have incurred any alleged overcharges.\(^4\) Multiple regression is recognized as meeting three of the tests concerning the admissibility of scientific evidence in U.S. courts, namely that as a methodology it is well accepted, testable and reproducible. U.S. courts have increasingly accepted the application of multiple regression models for purposes of identifying competitors or consumers that have been injured as the result of anticompetitive conduct and for purposes of valuing such injuries.\(^5\) Multiple regression analysis is capable of determining whether a particular

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\(^4\) Multiple regression is a well accepted method to isolate the conduct or practices at issue from other factors that influence market outcomes. See Askew v. City of Rome, 127 F. 3d 1355, 1365 n. 2 (11th Cir.1997), Petruzzi's 998 F.2d at 1238.

\(^5\) This trend began in earnest in the mid-1970's and has increased steadily over the past 30 years. Multiple regression models have been used as the basis for valuing overcharges, foregone sales and market share and lost profits in cases alleging cartel behavior, bid rigging, and monopolization. See for example Rea v. Ford Motor Co. 355 F. Supp 842 (W.D. PA 1973), Coleman Motor Co. v. Chrysler Corp.
effect is present, measuring the magnitude of the effect and forecasting what the effect would be but for an intervening event. Beyond the most limited set of market dynamics where changes in price-cost margins may adequately distinguish between competition and coordination, most markets have multiple independent influences affecting near and long terms prices at any point in time. Fact finders have increasingly been presented with the results of multiple regressions of varying forms with the expectation that they will accept measurement of the influence of offending conduct, while not recognizing that a conduct "variable" will may well be correlated with other unrelated determinants of prices and output.

**Regression Models**

Determining antitrust damages through the use of a multiple regression model can be considered a four-stage process:

- Determine the structure of the model. Assess the proper mathematical form to explain how relevant economic factors (independent variables) relate to the appropriate measure of price (dependent variable). As described below, a truncated process referred to as a reduced form model is often selected. This may be an iterative process in that preliminary results based on an assumed linear model may not be confirmed when a log-linear model of the same form is estimated. Reliability concerns over varying outcomes based on functional forms may be solved by using more flexible estimation techniques or the econometric equivalence of allowing the data to identify the best approximation.

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• Collect the requisite data and assess econometric techniques to be used in estimating the model. In many cases this involves combining widely available measures of broad economic activity with detailed measures of industry and firm specific operating measures that become available through the production of information by the parties to the dispute.

• Evaluate statistical tests measuring the significance of the results.

• Fourth and perhaps most importantly, an economic interpretation of the results. This element involves analysis of whether the results allow for reasonable and reliable measure of any damages. For example, are the rates of return implied by the predicted prices credible? Was the coordination designed to protect select inefficient competitors or to avoid uncertain investments such that returns during the alleged affected period vary substantially across participants? Are technological changes introduced during select affected or unaffected periods accounted for in a way that allows for their comparison with other periods incorporated in the damages study?

Reduced Form Models and the Problem of Omitted Variables

Regression models used to estimate antitrust damages typically take the form of linear relationships where product prices are determined as a function of a vector of variables that affect costs, a vector of variables affecting demand, in some cases a vector of variables that account for structural conditions for the market under study and depending on data availability a set of indicators that allow the intercept \( a \) to vary among relevant groups of observations. This latter element is often used to distinguish
periods or transactions subject to coordination from so-called “clean” observations. Econometric measures of this form are referred to as reduced form models.\(^6\)

\[
P_{it} = a + \beta W_{it} + ? Y_{it} + d S_{it} + ? D_{it} + e_{it}
\]

For purposes of convenience and ease of explanation, rather than identifying the relevant structural elements that equate supply and demand in equilibrium, reduced form models allow for only one endogenous variable, price, while treating elements of supply and demand as exogenous variables. Using actual values for \(W_t, Y_t, S_t\) and \(D_t\) and the estimated values of the regression parameters \((\beta, ?, d, ? \text{ and } e)\) the predicted value for prices can be estimated for any consumer in any period.\(^7\) The predicted prices are then subtracted from actual prices in arriving at the measure of damages.

While perhaps convenient and efficient in terms of the availability of the relevant information and in ease of explanation to unacquainted fact finders, reliance on reduced form models may suffer from uncertainty as to the interpretation of certain variables and the potential for introducing bias in the results. Despite circumstances where elements of a reduced form model demonstrate statistically significant relationships with the chosen measure of prices, the parameters used to calculate any overcharge may be less tied with economic theory than elements that would comprise the structural equations used to separately explain the elements of supply and demand in the same market. Stated simply, reduced form models are found to be attractive because they can serve to isolate transactions thought to be subject to overcharges and identify the extent to which such prices exceed unaffected prices while holding other factors constant. Such outcomes may be accepted regardless of whether the analysis that would be necessary to

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\(^6\) Elements of supply and demand can be reduced as a single expression of prices. If a demand function is given by \(Q = a - bP + cI\), where \(P\) is price, \(Q\) is quantity and \(I\) income, and a supply function given by \(Q = d + eP - fC\), where \(C\) is cost, the reduced form price equation for \(P\) is \(P = (a-d)/(b+e) = [c/(b+e)]I + [f/(b+e)]C\).

\(^7\) The random error term, \(e_{it}\), is assumed to be independent of and uncorrelated with the other explanatory variables.
separately identify and establish the supply and demand can be determined with the appropriate degree of reliability.

Among the criticisms often levied against the use of reduced form models is that they are prone to omitted variable bias. The issue here is the omission of variables that change underlying supply and/or demand conditions at points that coincide with periods (or for transactions thought to be) affected by anticompetitive conduct. For example, if changes in costs among factors not incorporated in the reduced form regression coincide with the identification of affected transactions, some or all the difference in prices may erroneously be attributed to the alleged anticompetitive conduct. Unaccounted for changes in the nature of demand can result in the same effects.

**Robustness**

Measures of the goodness of fit ($R^2$) and the statistical significance of the independent variables included in a multiple regression model do not ensure that the results of the model will serve as reliable estimates of damages. Robustness connotes the extent to which regression results are sensitive to small misspecifications in assumptions. If the assumptions of the regression model are valid, standard statistical tests can be applied. However, when the assumptions of the model are violated, standard tests can overstate or understate the significance of the results. Feedback effects, correlation among the explanatory variables themselves and the sensitivity of

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8 Omitted variable bias is particularly problematic to the extent that it has been recognized as an error subject to criticism and adjustment as to the weight that such evidence provides fact finders in assessing damages, not as to whether such measures are admissible. In a non-antitrust context, the U.S. Supreme Court found that, “While the omission of variables from a regression analysis may render the analysis less probative than it otherwise might be, it can hardly be said that, absent some other infirmity, that an analysis which accounts for the major factors must be considered unacceptable as evidence ... Normally, failure to include variables will affect the analysis' probative value, not its admissibility.” Brazemore v. Friday, 478 U.S. 385, 400, 92 L. Ed 2d 315, 106 S. Ct. 3000 (1986).

results to select data points, among other effects, can undermine the reliability of damage estimates.

In determining the regression equation, the analyst assumes that changes in the explanatory variables contribute to changes in the dependent variable, generally the measure of prices in a damages model. This assumption does not account for circumstances where changes in prices themselves contribute to changes in the explanatory variables. In cases where prices work back through elements of the supply and demand structures, unless accounted for outside the reduced form model, feedback effects can bias the results of the model, resulting in spurious correlation between prices and any of the other explanatory variables.\(^\text{10}\)

This issue is potentially found in price fixing claims where it becomes likely that while changes in demand contribute to changes in price, it may well be the case that demand reflects price changes as well. A feedback effect between demand and price can be expected in cases where close substitutes exist between the product at issue and competing products that may not be subject to the conduct in question or where the alleged cartel period may be of a sufficient period that consumers cultivated such alternatives. Feedback effects may be present in cost measures as well. In cases where a cartel acts to raise prices, cartel members may fail to control their own costs or become willing to accept supracompetitive prices for its inputs. Judgments about the likelihood of feedback effects and dealing externally with the potential importance of such effects are recognized as the remedy in assessing the reliability of regression models. Testing the sensitivity of changes in the measure of damages where a variable thought to be affected by the dependent variable is omitted is an option as is externally modeling the relationship between the dependent variable and the explanatory variable at issue.

\(^{10}\) Spurious correlation may be the outcome when two or more variables move in the same direction, appearing to be related to one another, when in fact they bear no causal relationship with one another.
Robustness also pertains to the influence of select observations on the outcome of the regression model. Particularly in cases with limited data sets, it is not unusual for the coefficients in a multiple regression analysis to change substantially if one or more observations are excluded. The issue of influential observations is not raised to suggest that the exclusion of select data points inherently makes regression estimates more reliable, but rather as a caution in recognizing that further investigation is warranted in cases where the exclusion of select observations materially influences the conclusions and estimates drawn from the model. Often influential observations are the most valuable information in the data set as they accurately account for changes in economic activity, while in other cases additional analysis should be conducted to determine the credibility of attaching vastly different outcomes to minimal amounts of information about the operation of a market.

A recently litigated case involving the distribution and retail sale of moist snuff tobacco throughout the U.S. highlighted the pitfalls of reliance on a regression where a limited amount of information, in this case a single observation, determined the outcome of a damage model that served as the basis for a $350 million damage award. In 1998, Conwood Company L.P. filed a complaint alleging that the leading manufacturer of moist snuff tobacco (MST), United States Smokeless Tobacco Company, Inc., (USSTC) monopolized the moist snuff market in the U.S. by excluding competitors’ display racks, advertising and products. The underpinnings of the damages theory advanced by Conwood was a regression model where its expert attempted to establish a “foothold” theory where, relying on shares of sales at the state level, the change in Conwood’s share of moist snuff sales over time was a function of its share at the outset of the period alleged to have been affected by USSTC’s conduct. The testimony of Conwood personnel

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12 CapAnalysis served as consultants to USSTC in follow-on litigations to the Conwood case.
Rather than investigate other factors that may have contributed to the change in Conwood's share of moist snuff sales over two periods in question, 1984-1990 and 1990-1997, Conwood's expert posited a simple model where aggregate changes in the firms' share of sales was specified as a function of share in the initial period (1984 and 1990). The estimated equation was:

\[
\text{Change in Market Share} = a + \beta \text{MS}_{1990}
\]

An OLS model was run for two periods - 1984-1990 and 1990-1997. The results for the 1984-1990 period showed a minimal negative relationship between initial share and change in share that was not statistically significant. Conwood had alleged that USSTC's anticompetitive actions had increased in the period since 1990 and that Conwood's performance was particularly hampered in states where its share was lowest. Conwood's expert determined that the same regression run over the 1990-1997 period supported the premise that the change in Conwood's share of sales at the state level was a function of its share in 1990. The model showed a statistically significant positive relationship between initial market share and subsequent growth in share. Plaintiffs' expert extended this finding to compute damages under the assumption that in the absence of USSTC's anticompetitive conduct, growth in Conwood's share in "low share" states would have been commensurate with growth in other, so-called "high" share states.\(^\text{14}\)

\(^{13}\) "The hypothesis was that USSTC's anticompetitive behavior had a greater impact on Conwood's market performance in cases where Conwood had a relevantly [sic] low market share in 1990". Leftwich Trial Transcript at 11.

\(^{14}\) A range of damages were presented using 15 and 20% cut-offs to distinguish low share and high share states.
As noted above, one principle associated with testing the robustness of any regression finding is the extent to which the model is sensitive to select observations. In the case of the Conwood damage model, the premise thought to support the entire measure of damages was shown to turn on the treatment of a single observation.

Plaintiffs regression model designed to test the relationship between Conwood’s initial share and growth in share over time consisted of state-based shares for 48 states and the District of Columbia (49 total data points). During the periods in question, U.S. consumption of moist snuff tobacco ranged from 36.5 to 53.8 million pounds a year. Consumption of moist snuff tobacco varies substantially on a regional basis throughout the U.S., suggesting that some weighting process may be appropriate to account for how differences in regional performance affects overall brand sales. For example, plaintiff’s treatment of Conwood’s sales gave equal accounting to the performance of Conwood’s share of sales in the District of Columbia, Rhode Island and Nevada, which respectively account for 0.03 percent, 0.02 percent and 0.3 percent of total U.S. consumption as it did for sales in Texas, Pennsylvania and North Carolina, which account for 9.6 percent, 5.1 percent and 5.0 percent of sales during the relevant period. As one may expect, small changes in Conwood’s sales in areas where consumption is limited resulted in substantial changes in its share of moist snuff sales in those areas. A 144 percent increase in Conwood’s share of sales in the District of Columbia and 985 percent increase in its share in Rhode Island between 1990 and 1997 validated this presumption. This variability was even more pronounced in terms of testing plaintiff’s theory of causation, namely that the low share “foothold” effect coincided with the allegations that USSTC’s anticompetitive conduct limited Conwood beginning in 1990, whereas growth in its shares over the preceding period (1984 to 1990) was not related to its initial share during that period. Over the three relevant time considerations, 1984, 1990 and 1997, Conwood’s share of sales in the District of Columbia declined from 21.0 percent to 7.2 percent before increasing to 17.6 percent in 1997.
Given the limited amount of information comprising the Conwood regression model (49 data points) and the substantial variance associated with select observations, it was not unlikely to expect that the results of the model would be sensitive to the presence or exclusion of a few data points. In fact, one data point, Conwood’s share of sales in the District of Columbia, allowed the outcome of the regression to support plaintiff’s theory of causation. If Conwood’s share of sales is omitted from the 1984 to 1990 regression, rather than a negative relationship between initial share and growth in share (-0.13), the coefficient changes to 0.26. Excluding the DC observation from the 1990 to 1997 period, the alleged affected period, increased the coefficient marginally (0.22 to 0.24), leaving the effect of initial share on share growth largely indistinguishable between the hypothetical “clean” period and the alleged affected period.

Leaving aside the issue whether factors other than initial share should have been tested to understand whether the “before” and “after” periods differed, a visual plot of
changes in the state-based shares, with the recognition of the geographic variability in consumption of moist snuff, should have conveyed concern that the regression may produce results that were highly sensitive to the treatment and reliability of select observations. This is not to suggest that regression is an inappropriate technique where the results include outliers. Rather, accepted tests exist to identify the extent to which regression results are influenced by select observations and other techniques to potentially limit or scale the effects associated with transactions or other market measures that represent marginal contributions to the market as a whole.  

**Joint Fit, Forecast Models and Stability Tests**

As noted at the outset, quantifying the damages resulting from coordination among competitors involves measuring the increment of prices attributable to the alleged wrong doing. This signifies that the task is largely a forecasting process rather than an effort to “model” those elements of supply and demand that suitably explain the prices subject to the alleged overcharges. Regression models can be fit to explain prices that prevail during conspiracy equally well as those present during competitive interaction between the same firms in the same market(s) at different times. This is the distinction between modeling price variation over time, be it during periods of competition or coordination, and modeling differences in price levels. Multiple regression is thought to be better suited to explain price variation than the extent to which price levels differ.

Depending on the adequacy of data made available for the market(s) in question, multiple regression damage models typically take one of two forms. Dummy variable or

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15 There are three standard test statistics that may be used to identify the effects of outliers. DFFITS is the normalized change in the OLS estimate of the ith value of the dependent variable resulting from omitting the ith observation when calculating the OLS coefficient estimates. DFBETA is the normalized change in an OLS coefficient estimate resulting from omitting the ith observation. Cook’s Distance (D) is the sum of the squared differences between the estimated y values using all the observations and the estimated y values deleting the ith observation. See H. Takeuchi “Assessment of Influence of Individual Observations on Prediction Mean Square Errors In Variable Selection Problems” J. Japan Statistics Soc., Vol. 32, No. 1 2002 43-55.
“joint fit” models incorporate observations from both competitive and alleged uncompetitive periods. A binary variable distinguishes the observations from both periods, measuring the extent to which average prices during the alleged effected period exceed average prices during the competitive period. After controlling for all other relevant factors, the dummy variable measures all remaining differences in the mean price between the two periods. In this sense, the dummy variable may act to test the hypothesis that the alleged coordination successfully raised prices above what they otherwise would be. A positive and statistically significant coefficient on the dummy variable indicates that the mean of prices during the alleged conspiracy period in fact exceeds average prices during the unaffected period.

A second approach, the forecasting or residuals method, relies on the assumptions that supply and demand behavior is unaffected by the conduct at issue and is able to be modeled the same way in both periods. If these assumptions hold, the forecasting method makes use of the coefficients derived in one period, typically the competitive period, as the basis to predict or forecast prices that would prevail during the conspiracy period. Given the difference in these two approaches, they are unlikely to lead to the same measure of any overcharge, raising the issue of which measure offers the prospect of producing the most reliable measure of any overcharge.

Providing sufficient data exists to fit a model that adequately explains the variation in prices during a period thought to be free of the influences of the alleged coordinated behavior, the experts’ task in using the forecast approach to produce a reliable damage estimate shifts to verifying that the “clean” period is sufficiently stable to serve as a “benchmark”. If a damage model were reliable, two outcomes should be expected (1) unaffected sub-periods should serve as strong predictors of prices during other unaffected sub-periods and (2) the model, if estimated over the entire affected and unaffected period should not fit well during the affected period. In this context instability refers to whether the period deemed to be unaffected contains one or more structural breaks such that developing coefficients over a portion of the unaffected
period produces forecasts materially different than a forecast based on the period as a whole. This process can be referred to inside sampling. A second test in this regard would be to run a single model over both periods to identify the extent to which the fit during the alleged coordinated period differs from the unaffected period.

Using the forecast approach to assess the stability of the unaffected period can be particularly important in circumstances where evidence as to when the alleged collusive period began or ended is deemed questionable. It is common in markets where cartel behavior is alleged for members to test the resolve of their competitors by “buying” market share for some period before complying with the designs of an agreement and similarly to misperceive the actions of a competitor and break ranks in advance of a formal end to coordinating conduct. Similarly, the dissolution of a cartel may have an imperceptible effect on prices, as despite the fact that the sum of individual outputs will generally exceed the joint output resulting from collusion, firm responses gleaned from periods of coordination may serve to temper price competition even where formal agreements have lapsed. Furthermore, the forecast method may help serve to reconcile evidence that a market structure conducive to “cheating” may result in an agreement among competitors that resulted in price levels equivalent to those that would have prevailed in the absence of coordination.

**Conclusions**

Multiple regression is a well accepted and flexible tool that in some cases may offer the best prospect for isolating the effects of anticompetitive behavior from other factors that contribute to prices and other market outcomes. Where shortcomings have

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16 An F statistic can be computed to determine the presence of unique sub-periods within a longer period thought to be suitable for purposes of forecasting overcharges. A “Chow” test is recognized for the purpose of measuring the stability of regression coefficients estimated over two or more periods.

17 Findings of a regression model put forward by plaintiff’s expert in In Re Industrial Silicon Antitrust Litigation seemed to be rebutted when the model run over the entire affected and unaffected periods found no indication of a departure from market conditions that prevailed during the unaffected period.
been identified in the use of multiple regression and other econometric techniques in U.S. antitrust litigation, the problems may largely lie with legal process associated with discovery and evidentiary disclosure as opposed to the inherent difficulty in expecting lay juries or fact finders to decipher meaningful distinctions in competing models. The size of the confidence intervals that bound multiple regression damage estimates are rarely the true indicators of the sensitivity of the estimates. Rather, the underlying assumptions behind the selection of the appropriate model to describe competitor interaction in the absence of the alleged conduct or the validity of the data used in the measurement are typically more critical in assessing the credibility of the estimates.

It is generally the case that experts presenting the outcome of any empirical analysis are obligated to reveal only the information they rely on in presenting the estimate they intend fact finders to use in arriving at a proposed award or penalty. The process of sorting out alternative models or tests that bear on the sensitivity of any estimates may never be revealed in a way that provides fact finders with a better sense of the “practical” probability that the damage estimates presented closely approximate market outcomes in the absence of the alleged conduct. An important consideration in this regard is whether, by way of the adversarial process, the findings of an opposing expert are a suitable alternative to more complete disclosure of the sensitivity of any results by the expert sponsoring the estimates. Too often it appears that fact finders consider rebuttal analysis of this type as a substitute for plaintiff’s estimate of damages, effectively creating a binary choice between competing models. A more equitable approach may be to apply and allow for the degree of disclosure of the complete economic analysis comprising plaintiff’s damage estimate, a process typically associated with peer reviewed academic research.
References On The Use and Abuse of Multiple Regression Analysis in Antitrust


Daniel McFadden, Kenneth Wise, Susan Guthrie and Paul Liu, The Misuse of Econometrics in Estimating Damages, Econometrics Legal, Practical and Technical Issues, ABA, Section of Antitrust Law (Year?).