The Impact of Antitrust Policy on Collusion with Imperfect Monitoring*

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Preliminary version

Abstract

We investigate the impact of antitrust enforcement by the means of fines in combination with different information policies and a leniency program on the sustainability of collusion. Using a model along the lines of Green and Porter (1984), we show that fines increase the sustainability of collusion in industries with relatively low probability of demand shocks. Even in situations where collusion is sustainable without antitrust enforcement, introducing a fine reduces welfare. Information spillovers from the antitrust authority to the colluding firms reinforce the effect of fines on collusion and enable industries facing a high probability of demand shocks to collude. The effect of leniency programs is ambiguous, since the program has a weakly positive effect in industries with low probability of demand shocks and an adverse effect if the probability of demand shocks is high.

JEL classification: L40; L41

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1 Introduction

Starting with Becker (1968) there has been a large debate in the literature on the impacts and the optimal adjustment of antitrust rules. In recent years the economic effects of leniency programs have been in the focus of the discussion (for a detailed overview see Spagnolo (2006)). Leniency programs are introduced to reduce the fines against colluding firms that report information on their cartel partners to the antitrust authority thus helping to punish other cartel members.

Brenner (2005)[1] reports that the number of decisions on cartels in the European Union has increased substantially after the introduction of leniency in 1996 from 15 cases in the period from 1990 to 1995, to 38 cases from 1996 to 2003. It is however not clear whether this increase is due to the effectiveness of the leniency program in encouraging whistle blowing or due to an increase in cartel activity.

While the literature on leniency programs has focused on the information spillovers from firms to the antitrust authority, we consider in this work an environment where the opposite information flow direction – from the antitrust authority to the firms – becomes relevant. The antitrust authority decides on the size of the fine, whether leniency is granted or not, and whether information during the antitrust procedure will be disclosed or not. Firms attempt to collude in a market with uncertain demand, while only observing their individual demand. Using a model along the lines of Green and Porter (1984), we show that the effect of leniency programs is ambiguous, since the program has a weakly positive effect in industries with low probability of demand shocks and an adverse effect if the probability of demand shocks is high. Leniency however unambiguously increase the number of prosecuted cartel cases. Similar to previous literature we find that fines can increase the sustainability of collusion, but only in industries with relatively low probability of demand shocks. Information spillovers from the antitrust authority to the colluding firms reinforce the effect of fines on collusion and enables industries to collude even when they face a high probability of demand shocks. In all cases, fines unambiguously reduce welfare, even in situations where collusion would have been sustainable without fines.

The intuition for these results is as follows: That fines can be used as a threat to sustain collusion has already been shown in Cyrenne (1999). Interestingly, this only works if the probability of demand shocks is low. For a high probability of demand shocks, even the introduction of a fine does not make collusion more stable, as then the fine would have to paid too often

[1] Based on data of Arlman (2005).

and collusion would never be profitable. In the standard Green Porter model temporary price wars in equilibrium are required to support collusion. By substituting the fine for these price wars, consumers are worse off, as the number of periods where collusion takes place increase. Thus welfare will be reduced. Informational disclosure by the antitrust authority allows the firm to learn about the behavior of their competitors. As this information is costly (the fine has to be paid), the model in this part works along the lines of the literature with costly private monitoring (see e.g. COMPTE (1998), KANDORI and MATSUSHIMA (2003), BEN-PORATH and KAHNEMAN (2003), and MARTIN (2006)). If discounting is not too strong, even in very uncertain industries this ability to monitor the competitors allows firms to collude. Finally, leniency in our model has the effect of reducing the expected fine. While for a small probability of demand shocks, a larger fine is useful in sustaining collusion, for a high probability of demand shocks it is the lower fine (which implies cheaper costly monitoring) which encourages collusion. Thus leniency works in both directions.

Our model contributes to the literature where information disclosure by the government influences market behavior. There exist several other examples where governmental institutions were helping industries to sustain collusive pricing. ALEXANDER (1994) shows that the National Industry Recovery Act (NIRA) between 1933 and 1935, which was introduced in the USA to stop price deflation and bankruptcies during the Depression, increased the concentration level of industries. LEVENSTEIN (1995) analyzed the price-enhancing effect of publishing firm specific transaction prices by the government in the American salt industry in the late nineteenth century. Similar effects were found by ALBEAK, MOLLGARD, and OVERGAARD (1997) which analyzed the price path of the Danish concrete industry and found an increase of prices during a period of price publishing by Danish antitrust authority.

The paper is organized as follows: Section 2 presents the model specifications and as a benchmark the standard model where no antitrust authority exits is discussed. In Section 3 we analyze the impact of fines on the possibility for firms to sustain collusive agreements. In section 4 the model it extended by an information disclosure policy and in section 5 by leniency programs. Section 6 concludes.

2 The Model

We follow the standard textbook analysis of infinitely repeated games under imperfect monitoring by TIROLE (1988), based on GREEN and PORTER

(1984).[2] We extend Tirole's model by an antitrust authority which can punish firms for collusive behavior.

2.1 Players

Firms

There are two firms in an industry, indexed by $i \in (1, 2)$. Firms compete in prices for an infinite number of periods $t \in \{0, 1, 2, .., \infty\}$ and produce a homogeneous product at constant marginal costs $c > 0$. In every period t, firm i sets the price p_i^t and observes its own demand D_i^t and profit Π_i^t, but neither the rival's price p_j^t nor demand D_j^t nor profit Π_j^t (with $j \neq i$).

Nature

The market demand D_k ($k \in \{l, h\}$) is stochastic and chosen by nature. Two states of demand are possible: With probability $1 - \alpha$, with $\alpha \in (0, 1)$ the market demand is strictly positive $D_h = D(p)$ (*high-demand state*). With probability α a demand shock accrues and market demand is zero $D_l = 0$, (*low-demand state*). The state of demand can not be observed by the firms directly.

To allow for correlated strategies later on, nature also chooses a random uniformly distributed signal $s \in [0, 1]$ which can be perfectly observed by the firms.

Antitrust authority

The antitrust authority implements a law enforcement policy, which consists of an exogenously given lump-sum fine $F \in [0, \infty)$, possibly a leniency program and rules for information allocation. The fines have to be paid by the firms being investigated and proven guilty with respect to collusive behavior. The success of an investigation depends on information about the collusion. We assume, this essential information can be revealed to the antitrust authority through *whistleblowing* by the firms only. Thus, if no firm does whistleblowing, the probability that the antitrust authority successfully proves firms guilty is equal to zero.[3] Otherwise, if at least one firm blows the

[2] For a concise description of the original model see TIROLE (1988), pp. 262-264.

[3] This assumption is made for simplicity. It can be justified by invoking a budget-constraint for the antitrust authority and sufficiently high investigation costs. As a result, the antitrust authority would never investigate the industry without information from at least one firm. The assumption of a budget-constrained antitrust authority has also been made by MOTTA and POLO (2003) and MARTIN (2006).

whistle, the antitrust authority will investigate the industry, convict firm i of collusion if it observes $p_i^t > c$ in the current investigation period.[4]

To analyze the impact of different strategies of the antitrust authority we will discuss the following policies:

Information Policy: There are two possible information policies. In the first case, antitrust authority uses the revealed information to convict firms, but does not reveal the price setting of a firm to its rival. In the second case, the antitrust authority discloses the price setting in period t and informs each firm i about the price p_j^t of its rival. Denote by $\{nd, d\}$ the antitrust authority's set of options, where $\{nd\}$ stands for a *non-disclosing* and $\{d\}$ for a *disclosing* antitrust authority.

Leniency Policy: If no leniency program is in place, colluding firms have to pay a fine F, independent of whether a firm was helping the antitrust authority by blowing the whistle or not. If a leniency program is installed, the whistleblowing firm has to pay a reduced fine $R = (1-r)F$ (with $r > 0$). Denote by $\{nl, l\}$ the antitrust authority's set of options, where $\{nl\}$ stands for *no leniency* and $\{l\}$ for *leniency*.

The fine F, the set of policies $\{d, nd\}$ and $\{l, nl\}$ are fixed before the firms start interacting.

The existence of a fine (full or reduced) and the policy of disclosing information, extend the strategy space of the firms compared with the firms in Tirole's model by two important aspects: First, by the possibility to use a new punishment tool (fine) provided by the antitrust authority.[5] And second, the possibility to obtain (formerly) private information by blowing the whistle. This second aspect changes the collusive game from collusion under imperfect monitoring to a collusive game where monitoring is possible. We describe the resulting changes in the structure of the game and in the firms' strategies in the following subsections.

2.2 Timing of the Game

In period $t = 0$ the legal environment is defined:

[4] As in AUBERT, REY, and KOVACIC (2006) it is assumed that the antitrust authority only considers current period prices.

[5] This aspect has also been discussed by CYRENNE (1999), who added a lump sum fine into GREEN and PORTER (1984).

Period t=0 : The antitrust authority sets the law enforcement policy parameters: It chooses a lump-sum fine F, commits to disclose the firms after investigation $\{d\}$ or not $\{nd\}$ and introduces leniency programs $\{l\}$ or not $\{nl\}$.

The pricing game proceeds in period $t = 1, 2, ...$ and every period has the following structure:

Stage 1 : Firms choose prices $p_i^t \in [c, p^M(c)]$.

Stage 2 : Nature chooses the market demand D^t and the signal s^t.

Stage 3 : Each firm i observes its own demand D_i^t with $i \in (1, 2)$ and the signal s^t, and obtains its profit Π_i^t. After that, each firm decides whether to blow the whistle or not. If no firm has chosen whistleblowing the game restarts at *Stage 1* in the next period $t + 1$. If at least one firm has blown the whistle, the game enters *Stage* 4.

Stage 4 : The industry will be investigated by the antitrust authority. The authority observes the price setting of each firm i. If the prices p_i^t $i \in \{1, 2\}$ have exceeded c, firm i is convicted of collusion and has to pay the fine F (or the reduced fine R). Depending on the information policy commitment in $t = 0$, prices p_i^t $i \in \{1, 2\}$ become public if $\{d\}$ was chosen or stay private knowledge for each firm $\{nd\}$. After that the game restarts at *Stage* 1 in the next period $t + 1$.

2.3 Firms' Strategies

In order to sustain the collusive agreement while rival's price setting can not be observed directly, firms have to use a punishment mechanism which is independent of direct observation. In Tirole's model the only way of punishment is a price war of finite duration for T periods. In our model firms are able to choose between (or combine) punishment by a price war for T periods and the fine punishment. Thus two different collusive strategies are analyzed, where in the line with the literature on modeling collusion in a dynamic framework, we concentrate on *Markov strategies*.[6]

GPP (Collusion and Green Porter Punishment) This is the standard strategy firms play in Tirole's model without an antitrust authority. Firms collude from $t = 1$ on. If in period t neither deviation from $p_i^t = p^M$ nor a demand shock occurs, each firm realizes a profit of

[6] For details see FUDENBERG and TIROLE (1991) pp. 501 et sqq.

$\Pi_i^t = \frac{1}{2}\Pi^M$ at the end of the period. If in period t the demand of at least one firm is zero, firms start in $t+1$ a price war of T periods. In $t+1+T$, they revert to collusion. If a deviation from the equilibrium GPP strategy occurs, they play "grim trigger" [Friedman (1971)], a price war with $p_i^t = c$ and profits $\Pi_i^t = 0$ forever.

GPFP (Collusion and Green Porter and Fine Punishment) This is a combination of punishment by price war and fine punishment provided by the antitrust authority. Again, firms collude from $t = 1$ on. If in period t no deviation from $p_i^t = p^M$ or a demand shock occurs, each firm realizes a profit of $\Pi_i^t = \frac{1}{2}\Pi^M$ at the end of the period. If in period t the demand of at least one firm is zero, firms blow the whistle with probability γ and reveal information to the antitrust authority. Furthermore, firms start in $t+1$ a price war for T^γ periods. In $t+1+T^\gamma$, they go back to collusion. With probability $1-\gamma$ no firm does whistleblowing, but a price war of T' periods is started in the next period. In $t+1+T'$ firms revert to collusion. Again, if at least one firm deviates from the collusive strategy, firms will compete with price equal to marginal costs in every following period.

2.4 Benchmark

We first study the benchmark case as described by Tirole (1988) where no antitrust authority exists. Firms choose a price equal to the monopoly price p^M in period $t = 1$. In doing so, each firm receives half of the monopoly profit in a *high-demand state*, $\Pi_i^{h,t} = \frac{1}{2}\Pi^M$ and no profit in *low-demand states*, $\Pi_i^{l,t} = 0$. A firm that unilaterally defects from $p^t = p^M$ attracts in a *high-demand state* the whole market and gets the monopoly profit Π^M. In the punishment phase which occurs after a *low-demand states* or when a firm has deviated, firms set $p^t = c$ for T-periods and hence obtain in each period $\Pi_i^{k,t} = 0$. Let V^+ denote the firm value in period t when the game is in a collusive phase. Let δ be the discount factor which is the same for each firm, with $0 \leq \delta < 1$. Then it holds:

$$V^+ = (1-\alpha)\left(\frac{1}{2}\Pi^M + \delta V^+\right) + \alpha(0 + \delta^{T+1}V^+) \tag{1}$$

The first term of equation (1) reflects that in each *high-demand state* firms get the collusive profit in every period of the game. The second term shows that in a *low-demand state*, profits are equal to zero and a phase of a T-period price war will be started.

If one firm unilaterally defects from the collusion, its firm value is:

$$V^D = (1-\alpha)\left(\Pi^M + \delta^{T+1}V^*\right) + \alpha(0 + \delta^{T+1}V^+). \tag{2}$$

It is obvious that firms have an incentive to collude if the firm value of a colluding firm V^+ is weakly larger than the firm value of a defecting firm, V^D. From $V^+ \geq V^D$ we get the following incentive compatibility constraint (IC):

$$(\delta - \delta^{T+1})V^+ \geq \frac{1}{2}\Pi^M. \tag{3}$$

The increase of the firm value by sticking to the collusion in period t has to be weakly larger than the additional profit $\frac{1}{2}\Pi^M$ from defecting in a *high-demand state*.

From equation (1) the firm value resulting from collusion can be determined:

$$V^+ = \frac{1}{2}\Pi^M\left(\frac{(1-\alpha)}{1-\delta+\alpha(\delta-\delta^{T+1})}\right). \tag{4}$$

Thus, the IC amounts to

$$\frac{1}{2}\Pi^M\left(\frac{(\delta-\delta^{T+1})(1-\alpha)}{1-\delta+\alpha(\delta-\delta^{T+1})}\right) \geq \frac{1}{2}\Pi^M \tag{5}$$

which can be reduced to

$$(1-2\alpha)(\delta-\delta^{T+1}) - (1-\delta) \geq 0. \tag{6}$$

From inequality (6) its obvious that firms have an incentive to collude if, for a given α, δ is not too small: $\delta \in [\delta(\alpha), 1)$, or if, for a given δ, α is not too large: $\alpha \in [0, \alpha(\delta)]$. The resulting critical parameters are described by Tirole (1988). In order to make the results comparable to the results in the following sections, we will present and prove them again.

Lemma 1
In absence of an antitrust authority, a GPP perfect Bayesian equilibrium exists in which firms collude from $t = 1$ on, and use a price war for T periods as punishment if

(i) $\alpha(\delta) \leq 1 - \frac{1}{2\delta} \leq \frac{1}{2}$

(ii) $\delta(\alpha) \geq \frac{1}{2(1-\alpha)} \geq \frac{1}{2}$

Proof From inequality (6) it follows directly that the IC can not be satisfied if $\alpha > \frac{1}{2}$ holds. So it is sufficient to consider the case $\alpha \leq \frac{1}{2}$. As $\frac{\partial IC}{\partial \alpha} \leq 0$,

$\frac{\partial IC}{\partial \delta} \geq 0$, and $\frac{\partial IC}{\partial T} = -(1-2\alpha)\delta^{T+1}\ln(\delta) \geq 0$ for $\alpha \leq \frac{1}{2}$, to calculate the minimal $\delta(\alpha)$ (maximal $\alpha(\delta)$) we set $T \to \infty$. Thus $IC \geq 0$ changes to $IC^{T\to\infty} = 2\delta(1-\alpha)-1 \geq 0$ which holds if $\delta(\alpha) \geq \frac{1}{2(1-\alpha)}$ and $\alpha(\delta) \leq 1-\frac{1}{2}\delta$.■

We define a specific industry as two firms producing the same product under the same cost structure and the same market conditions. These conditions are reflected by the industry-specific α and δ. The curve in *Figure 1* displays the boundary of industries where collusion is sustainable. Industries which are located in the hatched area left to the curve are the candidates for collusive activities using GPP.

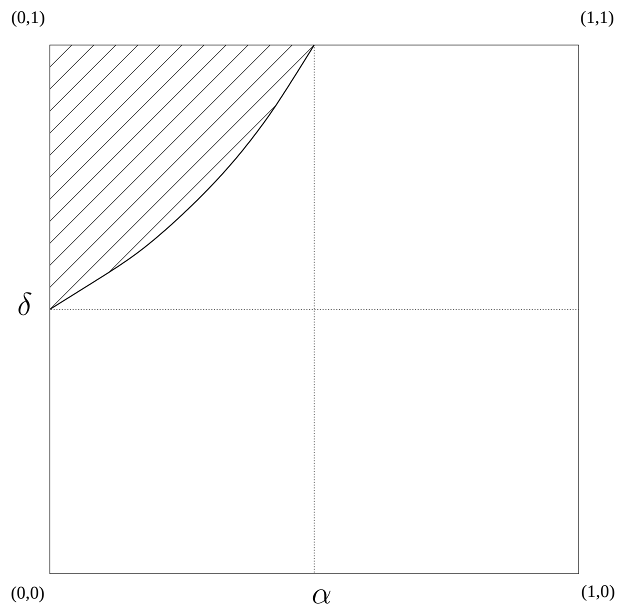

Figure 1: Sustainable Collusion by the use of the GPP strategy

The optimal strategy of firms using GPP is easy to see. From inequality (3) we get, that collusion is more likely to be stable if T is large. On the other hand, from equation (4) we see that $\frac{\partial V^+}{\partial T} \leq 0$. Thus, to maximize the collusive firm value, firms have to coordinate on a minimal T which is high enough to satisfy the IC. Thus, the optimization problem becomes:

$$\min T \equiv \arg\max V^+$$
$$\text{s.t.}$$
$$(1-2\alpha)(\delta-\delta^{T+1})-(1-\delta) \geq 0$$

3 *Non-disclosing* antitrust authority (law enforcement by fines only)

Now we add the antitrust authority to the benchmark model. The antitrust authority commits to a lump sum fine $F \in [0, \infty)$ in period $t = 0$, no leniency program exists, $\{nl\}$, and the antitrust authority chooses a *non-disclosing* policy, $\{nd\}$. Thus, firms do not obtain any information about the price setting of its rival if they are investigated. As a result, they are again not informed about the reason when observing zero demand, independent of whether a firm blows the whistle or not.

If firms use the GPP strategy no firm does whistleblowing in equilibrium and the outcome of the analysis is the same as in the benchmark. Thus, we only have to analyze the conditions for the GPFP strategy: If at least one firm faces no profit, blow the whistle with probability γ. To coordinate on a certain frequency of whistleblowing, firms use the signal s^t provided in every period t. Only if, $s^t \leq \gamma$ firms will (jointly) blow the whistle.

Recalling that the GPFP strategy specifies that firms, given they observe zero demand, undertake a price war of T^γ (T') periods if they blow (do not blow) the whistle, the values of the firms under collusion and deviation[7] can be calculated:

$$V^+ = (1-\alpha)\left(\frac{1}{2}\Pi^M + \delta V^+\right) + \alpha\left(\gamma\left[-F + \delta^{T^\gamma+1}V^+\right] + (1-\gamma)\delta^{T'+1}V^+\right) \quad (7)$$

and

$$\begin{aligned} V^D &= (1-\alpha)\left(\Pi^M + \gamma\left[-F + \delta^{T^\gamma+1}V^+\right] + (1-\gamma)\delta^{T'+1}V^+\right) + \\ &+ \alpha \left(\gamma\left[-F + \delta^{T^\gamma+1}V^+\right] + (1-\gamma)\delta^{T'+1}V^+\right). \end{aligned} \quad (8)$$

To sustain collusion, the IC, $V^+ \geq V^D$, has to hold again. Consequently, we get

$$\left(\delta - \left[\gamma\delta^{T^\gamma+1} + (1-\gamma)\delta^{T'+1}\right]\right)V^+ \geq \frac{1}{2}\Pi^M - \gamma F. \quad (9)$$

The term in the angled brackets $\gamma\delta^{T^\gamma+1} + (1-\gamma)\delta^{T'+1}$ can be interpreted as the *effective* reduction of firm value due to deviation caused periods of price war. From now on let denote this reduction as

[7] Note that if firm i deviates from the collusive strategy, it is indifferent in blowing the whistle with probability γ or not since firm j would do whistleblowing anyway.

$$\delta_{nd}^{eff} \equiv \gamma\delta^{T^\gamma+1} + (1-\gamma)\delta^{T'+1}. \tag{10}$$

Thus, inequality (9) changes to

$$\left(\delta - \delta_{nd}^{eff}\right) V^+ \geq \frac{1}{2}\Pi^M - \gamma F. \tag{11}$$

Compared with the corresponding inequality in the benchmark (3), the left hand side of inequality (11) represents again the difference of firm values in a *high-demand state* between staying in the collusion and after a deviation induced price war. While the right hand side is again the additional profit from defecting in a *high-demand state*, but reduced by the expected fine, a deviating firm has to pay. To determine the range of parameters where collusion is stable equation (7) is rearranged to give

$$V^+ = \frac{(1-\alpha)\Pi^M - 2\alpha\gamma F}{2[1-\delta+\alpha(\delta-\delta_{nd}^{eff})]} \tag{12}$$

and insert this into condition (11). Thus, $V^+ \geq V^D$ if:

$$(1-2\alpha)\left(\delta - \delta_{nd}^{eff}\right) + \left(\gamma\frac{2F}{\Pi^M} - 1\right)(1-\delta) \geq 0 \tag{13}$$

Whether the IC holds or not depends on the exogenous parameters α and δ but, compared with the benchmark, additionally on the term $\frac{2F}{\Pi^M}$. This parameter is the ratio of the fine F and half of the monopoly profit $\frac{1}{2}\Pi^M$, the additional profit from defecting in a *high-demand state*. Let ϕ be the fine/profit-ratio. Thus the IC reduces to:

$$(1-2\alpha)\left(\delta - \delta_{nd}^{eff}\right) + (\gamma\phi - 1)(1-\delta) \geq 0 \tag{14}$$

By choosing the length of the punishment phases $T^\gamma, T' \in \{0, 1, 2, ...\}$, firms can choose again the effective reduction of the firm value after a price war, δ_{nd}^{eff}. Additionally they can choose the expected payment to the antitrust authority, γF, by blowing the whistle with a frequency of $\gamma \in [0, 1]$. From inequality (14) it follows that for $\gamma\phi \geq 1$, firms do not need a reduction of firm value by choosing a δ_{nd}^{eff} to sustain collusion, as the IC holds anyway. However, from equation (12), using that $2F = \phi\Pi^M$, it can be seen that for a large expected fine/profit-ratio $\gamma\phi$ and a high probability of demand shocks α, V^+ may become negative. Therefore, an additional participation constrain (PC), $V^+ \geq 0$ has to be added. Since the denominator of inequality (12) never turns negative we can write the PC as:

$$(1-\alpha) - \alpha\gamma\phi \geq 0. \quad (15)$$

We state the condition for an equilibrium in the following Lemma 2.

Lemma 2
For $F > 0, \{nd\}$, and $\{nl\}$ a GPFP perfect Bayesian equilibrium exists if:

(i)

$$\alpha(\delta,\phi) \leq \begin{cases} 1 - \frac{1-(1-\delta)\phi}{2\delta} & \textit{if } \phi < 1 \\ \frac{1}{2} & \textit{if } \phi \geq 1 \end{cases}$$

(ii)

$$\delta(\alpha,\phi) \geq \begin{cases} \frac{1-\phi}{2(1-\alpha)-\phi} & \textit{if } \phi < 1 \\ 0 & \textit{if } \phi \geq 1 \end{cases}$$

Proof The PC (15) is satisfied if and only if $\gamma \leq \min\left[1, \frac{1-\alpha}{\alpha\phi}\right]$.
From inequality (14) it follows that $\frac{\partial IC}{\partial \delta_{nd}^{eff}} = -(1-2\alpha)$. To determine the border cases, for $\alpha \leq \frac{1}{2}$ we set $\delta_{nd}^{eff} = 0$ and for $\alpha > \frac{1}{2}$ we set δ_{nd}^{eff} to its maximal value, $\delta_{nd}^{eff} = \delta$. In both cases γ is also set to its maximum value.
Consider first the case $\alpha \leq \frac{1}{2}$: The IC then changes to $(1-2\alpha)\delta + \min\left[\phi, \frac{1-\alpha}{\alpha}\right] - 1)(1-\delta) \geq 0$. If $\phi \geq \frac{1-\alpha}{\alpha}(\geq 1)$ the IC holds. If $\phi < \frac{1-\alpha}{\alpha}$, the IC holds if $\alpha \leq 1 - \frac{1-(1-\delta)\phi}{2\delta}$ or $\delta \geq \frac{1-\phi}{2(1-\alpha)-\phi}$.
Next consider the case $\alpha > \frac{1}{2}$: As the PC requires that $\gamma\phi < 1$ and the IC now reads $(\gamma\phi - 1)(1-\delta) \geq 0$, one can see that both conditions can never hold simultaneously. ■

Compared with the benchmark case, the number of industries which are able to sustain collusion is increasing in the fines provided by a *non-disclosing* antitrust authority, since $\frac{\partial\alpha}{\partial\phi} > 0$ and $\frac{\partial\delta}{\partial\phi} < 0$. *Figure 2* displays the boundaries for industries where collusion can be sustained with respect to a given $\phi \geq 0$. All industries which are located in the area left to the curves are able to use the GPFP strategy in equilibrium.

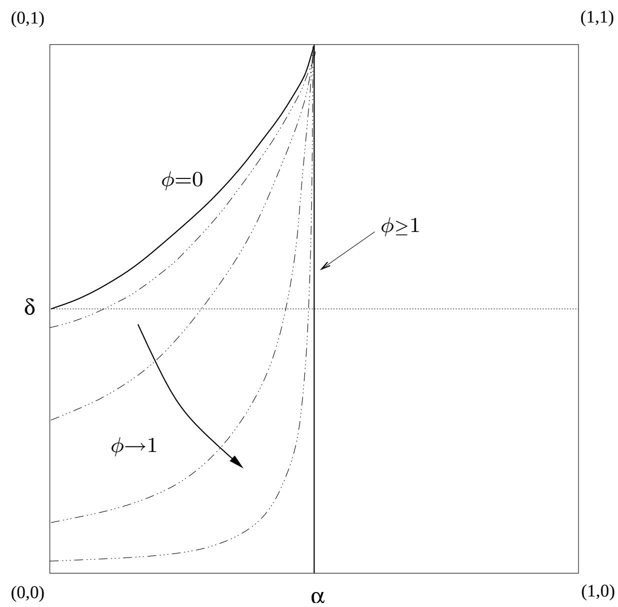

Figure 2: Sustainable Collusion under a regime of a *non-disclosing* antitrust authority

From comparing Lemma 1 and Lemma 2 it follows, if there is a *non-disclosing* antitrust authority, even firms with a relatively low discount factor ($\delta < \frac{1}{2}$) can sustain collusion. On the other hand, the result from the benchmark still holds: Firms which face a demand shock with a relative low probability only ($\alpha \leq \frac{1}{2}$) can sustain collusion. We summarize our results in the following Proposition:

Proposition 1 *Compared to a situation without an antitrust authority, introducing a non-disclosing antitrust authority with policy* $F, \{nd\}$, *and* $\{nl\}$

(i) leads to more collusive industries with $\alpha \leq \frac{1}{2}$

(ii) has no effect on industries with $\alpha > \frac{1}{2}$

Proof The proof follows immediately from Lemma 1 and Lemma 2. ■

Next the welfare consequences of an antitrust authority are analyzed. First, allowing for $\phi > 0$ makes it possible for more industries to collude, leading to welfare losses since prices are (in some periods) above marginal costs. Additionally, there is a second effect: Firms which were even able to sustain collusion without a fine might now use the fine punishment instead of the price war punishment. As the price war punishment brings with it a welfare gain due to marginal cost pricing for T periods instead of monopoly prices, reverting to a fine punishment would lead to a loss of welfare. However, for this argument to hold through, it first needs to show that firms indeed use the fine punishment if they have the choice between the two instruments.

As it turns out, if collusion is sustainable, both instruments are perfectly substitutable. This can be seen by the following argument: To keep the IC constant, a decrease in the frequency of whistleblowing (decrease in γ) has to be compensated by a decrease of δ_{nd}^{eff}, i.e. $\frac{d\delta_{nd}^{eff}}{d\gamma} = \frac{2F(1-\delta)}{\Pi^M(1-2\alpha)} \geq 0$. Now, as long as the IC binds, such a change in the mix of punishment instruments does not change the value of the firm, as the total change in V^+ is given by: $\frac{dV^+}{d\gamma} = \frac{\partial V^+}{\partial \delta_{nd}^{eff}} \frac{d\delta_{nd}^{eff}}{d\gamma} + \frac{\partial V^+}{\partial \gamma} = -\frac{\alpha^2 F \Pi^M \left[(1-2\alpha)(\delta-\delta_{nd}^{eff})+(\gamma\phi-1)(1-\delta)\right]}{(1-\delta+\alpha(\delta-\delta_{nd}^{eff}))\Pi^M(1-2\alpha)}$. This expression is zero as the term in brackets in the numerator is just the IC, which is assumed to bind. Thus, all relevant parameters can be freely chosen by the firms or can be adapted to any exogenous requirement.[8]

The results on the welfare consequences of an antitrust authority with a fine only are summarized in the next Proposition:

Proposition 2 *A fine reduces welfare through increasing the number of colluding industries. Even if collusion is sustainable without a fine, introducing a fine will lead to a reduction of welfare if firms blow the whistle with positive probability in equilibrium.*

Proof The first result immediately follows from Lemma 1 and Lemma 2. For the second result, it still needs to show that the new combination of fine and price wars (i.e. T^γ, T' instead of T) indeed leads to a reduction in welfare. Denote by Δ the welfare gain per period of price war. The expected

[8] An example for such a requirement could be, that firms have to make detailed reports about their activities for some periods after proven guilty for collusion ($T^\gamma \geq \underline{T}$), e.g. Motta and Polo (2003) introduced such a requirement in their model. They assume that firms have to interrupt the collusion for one period after the investigation of the antitrust authority.

welfare gain through price wars is then given by

$$\begin{aligned}
E[\Delta] &= \gamma \sum_{i=1}^{T^{\gamma}} \delta^i \Delta + (1-\gamma) \sum_{i=1}^{T'} \delta^i \Delta \\
&= \gamma \frac{(\delta - \delta^{T^{\gamma}+1})}{1-\delta} \Delta + (1-\gamma) \frac{(\delta - \delta^{T'+1})}{1-\delta} \Delta \\
&= \frac{\Delta}{1-\delta} \left[\delta - \left(\gamma \delta^{T^{\gamma}+1} + (1-\gamma) \delta^{T'+1} \right) \right] \\
&= \frac{\Delta}{1-\delta} \left[\delta - \delta_{nd}^{eff} \right]
\end{aligned}$$

The benchmark is represented by $\gamma = 0$. From Lemma 2 we know $\frac{\partial \delta_{nd}^{eff}}{\partial \gamma} = \frac{\phi(1-\delta)}{1-2\alpha} \geq 0$. Since $\frac{\partial E[\Delta]}{\partial \delta_{nd}^{eff}} < 0$, any $\gamma > 0$ reduces welfare. ■

4 ***Disclosing*** antitrust authority

Now the model is extended to analyze the effects of information spillovers between the antitrust authority and the colluding firms. If the antitrust authority informs each firm about the price of its rival in the current period t (commits to $\{d\}$ and $F > 0$ in $t = 0$), firms are able to monitor each other trough whistleblowing. If colluding firms blow the whistle and observe that no firm has deviated they can immediately go back to collusion. There is no need to punish the other by starting a price war. On the other hand, if it is observed that one firm has deviated this will trigger the breakdown of collusion, thus price equal marginal costs would be set in every period thereafter.

The firm value from collusion is therefore given by

$$V^+ = (1-\alpha) \left(\frac{1}{2} \Pi^M + \delta V^+ \right) + \alpha \left(\gamma \left[-F + \delta V^+ \right] + (1-\gamma) \delta^{T'+1} V^+ \right) \quad (16)$$

and the value of a firm which deviates once is

$$\begin{aligned}
V^D &= (1-\alpha) \left(\Pi^M + \gamma [-F] + (1-\gamma) \left[\delta^{T'+1} V^+ \right] \right) + \\
&+ \alpha \quad \left(\gamma [-F] + (1-\gamma) \delta^{T'+1} V^+ \right). \qquad (17)
\end{aligned}$$

Compared to equations (7) and (8) under a *non-disclosing* antitrust authority, there are two relevant modifications in the corresponding equations

(16) and (17). First, the firm value from collusion, V^+, is increased by $\alpha\gamma(\delta - \delta^{T^\gamma+1})V^+$: If firms blow the whistle, they are assured that the absence of demand was induced by nature. Thus, they are able to revert to collusion immediately if the antitrust authority informs them that deviation did not take place. Second, in the expression for the firm value from deviation, V^D, the term $\gamma\delta^{T^\gamma+1}V^+$ is missing. After a deviation is detected (with probability γ) there is no return to the collusive outcome (in effect $T^\gamma = \infty$). Both changes lead to the new IC:

$$(\delta - (1-\gamma)\delta^{T'+1})V^+ + \frac{\alpha}{(1-\alpha)}\gamma\delta V^+ \geq \frac{1}{2}\Pi^M - \gamma F \qquad (18)$$

The first term of the left hand side of inequality (18) represents again the difference of firm value between staying in the collusion and after a deviation induced price war conditional on being in a *high-demand state*. In analogy with the analysis of a *non-disclosing* antitrust authority we define a effective reduction of firm value due to deviation caused periods of price war by using that $T^\gamma \rightarrow \infty$:

$$\delta_d^{eff} \equiv \gamma\delta^\infty + (1-\gamma)\delta^{T'+1} = (1-\gamma)\delta^{T'+1}. \qquad (19)$$

This part of the IC represents the second effect discussed above.

The first effect finds itself in the second term of the left hand side of inequality (18).

The right hand side of inequality (18) is again the additional profit from defecting in a *high-demand state* and its reduction by the expected fine a deviating firm has to pay.

Proceeding as before and using definition (19) we can simplify equation (16) to

$$V^+ = \frac{(1-\alpha)\Pi^M - 2\alpha\gamma F}{2\left[1 - \delta + \alpha\left(\delta - \left[\gamma\delta + \delta_d^{eff}\right]\right)\right]}. \qquad (20)$$

Plugging (20) into (18) and using $\phi = \frac{2F}{\Pi^M}$, the IC changes to

$$(1-2\alpha)\left(\delta - \delta_d^{eff}\right) + (\gamma\phi - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\gamma\delta\left(2(1-\alpha) - \gamma\phi\right) \geq 0. \qquad (21)$$

As before, the PC, $V^+ \geq 0$, has to be considered as well. Again, the denominator of inequality (20) never turns negative. Thus, we can write the PC as before:

$$(1-\alpha) - \alpha\gamma\phi \geq 0. \qquad (22)$$

We state the condition for a GPFP perfect Bayesian equilibrium under a regime of a *disclosing* antitrust authority in the following Lemma 3.

Lemma 3
For $F > 0, \{d\}$, and $\{nl\}$ a perfect Bayesian equilibrium exists if

(i)

$$\alpha\left(\delta,\phi<1\right) \leq \begin{cases} 1-\frac{1-(1-\delta)\phi-(1-\phi)}{2\delta-(1-\phi)} & \text{if } \frac{\phi-1}{\phi-2}\leq\delta\leq\frac{\phi-2}{\phi-3} \\ \frac{((1-\delta)\phi+\delta)^2}{((1-\delta)\phi+\delta)^2+4\delta(1-\delta)\phi} & \text{if } \delta>\frac{\phi-2}{\phi-3} \end{cases}$$

$$\alpha\left(\delta,\phi\geq1\right) \leq \begin{cases} \frac{1}{2} & \text{if } \delta\leq\frac{\phi}{1+\phi} \\ \frac{((1-\delta)\phi+\delta)^2}{((1-\delta)\phi+\delta)^2+4\delta(1-\delta)\phi} & \text{if } \delta>\frac{\phi}{1+\phi} \end{cases}$$

(ii)

$$\delta\left(\alpha,\phi<1\right) \geq \begin{cases} \frac{(1-\alpha)(1-\phi)}{2(1-\alpha)-\phi} & \text{if } \alpha<\frac{1}{1+2\phi-\phi^2} \\ \frac{\left((3\alpha-1)+(1-\alpha)\phi+2\sqrt{2\alpha^2-\alpha}\right)\phi}{[2(3\alpha-1)+(1-\alpha)\phi]\phi+(1-\alpha)} & \text{if } \alpha\geq\frac{1}{1+2\phi-\phi^2} \end{cases}$$

$$\delta\left(\alpha,\phi\geq1\right) \geq \begin{cases} 0 & \text{if } \alpha\leq\frac{1}{2} \\ \frac{\left((3\alpha-1)+(1-\alpha)\phi+2\sqrt{2\alpha^2-\alpha}\right)\phi}{[2(3\alpha-1)+(1-\alpha)\phi]\phi+(1-\alpha)} & \text{if } \alpha>\frac{1}{2} \end{cases}$$

Proof We delegate the proof to the appendix because several cases has to be analyzed. ■

Lemma 3 shows that even for $\alpha > \frac{1}{2}$ collusion might be possible if the antitrust authority reveals information. The intuition for this can be most easily seen by assuming that the fine is zero, i.e. $(\phi = 0)$.[9] Then whistleblowing is costless and the situation is as in an environment with perfect monitoring. Thus, the standard result for collusion of two firms is obtained: for all $\alpha \leq 1$ collusion can be sustained as long as $\delta \geq \frac{1}{2}$.

Moreover, Lemma 3 shows that if the probability of demand shocks is relatively low ($\alpha \leq \frac{1}{2}$), the results are similar to the case of a regime of a *non-disclosing* antitrust authority: The number of industries which can sustain

[9]A zero expected fine might even be a realistic assumption to be made if proposals of a reward for whistleblowing go through. See the next section for a discussion of this.

collusion is increasing in ϕ. However, as we will see below, the overall range of parameters where collusion is possible is enlarged.

As before, if $\phi \geq 1$ all industries, with $\alpha \leq \frac{1}{2}$ and $\delta \geq 0$ can sustain collusion. If, in contrast, the probability of demand shocks is relatively high, ($\alpha > \frac{1}{2}$), the number of industries which can sustain collusion is decreasing in ϕ and sustainable collusion requires a larger δ if the fine/profit-ratio is increasing. For this reason, $\phi \to \infty$ is equal to an environment of an *non-disclosing* antitrust authority where no industry with $\alpha > \frac{1}{2}$ is able to sustain collusion.[10] *Figure 3* displays the boundaries for sustainable collusion for any given $\phi \geq 0$. All industries in the areas left (and above) the curves are able to sustain collusion with the GPFP strategy.

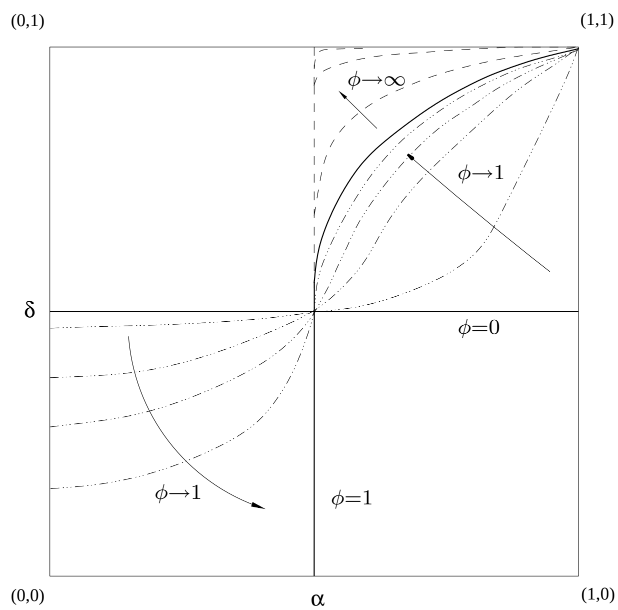

Figure 3: Sustainable Collusion under a regime of a *disclosing* antitrust authority

To compare between the outcomes of Lemma 2 and Lemma 3, we discuss the two cases $\alpha > \frac{1}{2}$ and $\alpha \leq \frac{1}{2}$ in turn.
If the probability of a demand shock is relatively high, $\alpha > \frac{1}{2}$, industries are able to sustain collusion only if the antitrust authority commits to $\{d\}$ in $t = 0$. If the probability of a demand shock is relatively low, $\alpha \leq \frac{1}{2}$,

[10] These results are in the line with Ben-Porath and Kahneman (2003) who show that if perfect monitoring is possible, and even when the costs of monitoring are high, every payoff vector which is an interior point in the set of feasible and individually rational payoffs can be implemented in a repeated game if discount factor is high enough.

for any $\phi < 1$, then the critical discount rate where collusion can barley be sustained for a given ϕ is weakly lower for a *disclosing* than for a *non-disclosing* antitrust authority.[11] *Figure 4* gives an example comparing the critical discount rates in the two scenarios for a given α. In case $\phi \geq 1$ (and still $\alpha \leq \frac{1}{2}$) collusion can be sustained for any $\delta \geq 0$, independent of the information policy.

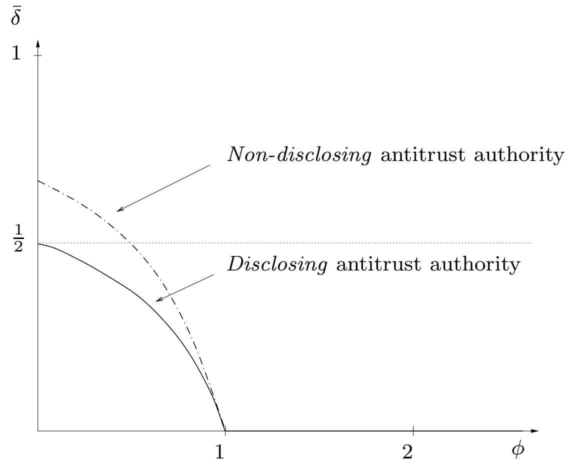

Figure 4: Comparison: Critical discount rates under regime of a *non-disclosing* and a *disclosing* antitrust authority, for $\alpha = \frac{1}{4}$.

These results are summarized in the following Proposition:

Proposition 3 *Compared to a non-disclosing antitrust authority, a disclosing antitrust authority, which commits to $F \geq 0$, $\{d\}$ and $\{nl\}$ in $t = 0$, increases the number of colluding industries.*

Proof The proof immediately follows from the discussion above. ■

Before turning to the welfare analysis, we first have to analyze whether firms will indeed use the fine as a punishment if they have the choice between different instruments. However, while under a *non-disclosing* policy the firms were indifferent between the two instrument, in the case of a *disclosing* antitrust authority firms always prefer to blow the whistle and price wars will no longer be observed. This yields the following proposition:

[11] From Lemma 2 we get that a *non-disclosing* antitrust authority requires a critical discount rate of $\delta \geq \frac{1-\phi}{2(1-\alpha)-\phi}$. While Lemma 3 shows that under a *disclosing* antitrust authority collusion can barley be sustained if $\delta \geq \frac{(1-\alpha)(1-\phi)}{2(1-\alpha)-\phi}$.

Proposition 4 *Under a non-disclosing policy firms will never use price wars to sustain collusion.*

Proof We delegate the proof to the appendix, since it requires tedious calculations. ■

Since it is never optimal to choose $T' > 0$ and thus $\delta_d^{eff} = (1-\gamma)\delta$, the optimization problem is:

$$\min \gamma \equiv \arg \max_{\gamma \in (0,1]} V^+$$
$$\text{s.t.}$$
$$(1-2\alpha)(\delta - (1-\gamma)\delta) + (\gamma\phi - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\gamma\delta\,(2(1-\alpha) - \gamma\phi) \geq 0$$
$$(1-\alpha) - \alpha\gamma\phi \geq 0$$

Now we can analyze the consequences of a *disclosing* antitrust authority on welfare. There are three different effects.

First, as discussed above, both for $\alpha \leq \frac{1}{2}$ and for $\alpha > \frac{1}{2}$ there will be more parameter values for which collusion is stable, if the antitrust authority commits to disclose information.

Second, even if industries could have colluded anyway, there will be less price war periods. As was shown above, under a *disclosing* antitrust authority profit maximizing colluding firms will never resort to price wars, while with a *non-disclosing* antitrust authority price wars might either be necessary or firms are at least not worse off by using a price war than by using the fine punishment. As price wars lead to marginal cost pricing and thus to a welfare gain compared to monopoly prices, using fines only reduces welfare.

If paying fines is positive for welfare (e.g. due to welfare losses in raising taxes which might be avoided by obtaining the fine), then there is a third welfare reducing effect: With a *disclosing* antitrust authority, firms pay less fines on average. To see this, assume that parameter values are such that firms are able to sustain collusion under a regime of a *non-disclosing* antitrust authority without price war.[12] For such industries, the number of price war periods are unaffected by the disclosing of information. However, since $T^\gamma = T' = 0$, inequality $\delta_{nd}^{eff} \leq \delta_d^{eff}$ implies that for a given ϕ the frequency of whistleblowing under a regime of a *disclosing* antitrust authority is lower than the frequency of whistleblowing under a regime of a *non-disclosing* antitrust authority.

[12]For example if $\phi > 1$.

Proposition 5 *Compared to a regime of a non-disclosing antitrust authority, introducing a disclosing antitrust authority is always welfare reducing.*

Proof The proof follows immediately from the discussion above. ■

5 Leniency Policy

In this section the model is extended to analyze an antitrust authority which commits to a leniency policy $\{l\}$ in $t = 0$. In doing so, a firm that has blown the whistle will get a reduced fine $R = (1 - r)F$ with leniency parameter $r > 0$. In the lines with the current antitrust policy of the European Commission and the US DoJ we will not allow for rewards for whistleblowing firms.[13] Thus, the leniency parameter is limited to $r \leq 1$.[14] The antitrust authority commits to reduce the fine only for the first firm which blows the whistle.[15] If both firms blow the whistle simultaneously, one of them is randomly chosen as the first whistleblower. For our analysis, where firms either do not blow the whistle at all or do it simultaneously, a whistle blowing firm thus expects a fine of

$$E[F] = \left(1 - \frac{1}{2}r\right) F < F$$

From $r \in (0, 1]$ it is obvious that $E[F] < F$.

Again, we have to analyze two cases: The first, where a *non-disclosing* antitrust authority commits to leniency $\{l\}$ for whistleblowing firms. And second, the case of a *disclosing* antitrust authority commits to $\{l\}$.

5.1 *Non-disclosing* antitrust authority

If the antitrust authority commits to $F > 0$, $\{nd\}$, and $\{l\}$ with $r > 0$ in $t = 0$, the expected fine/profit-ratio is $E[\phi_l] = \frac{2E[F]}{\Pi^M}$ which is lower than $\phi_{nl} = \frac{2F}{\Pi^M}$. From Lemma 2 it follows that sustainability of collusion requires for any given $\alpha \leq \frac{1}{2}$ and $\phi < 1$ a discount rate of

$$\delta \geq \frac{1 - \phi}{2(1 - \alpha) - \phi}.$$

[13] An overview of the similarities and varieties of the leniency policy in the EU and in the US is given in Section 3 of Spagnolo (2006).

[14] We will relax this assumption in Section 5.3.

[15] This assumption will be relaxed in Section 5.3, too.

Since $\frac{\partial \delta}{\partial \phi} < 0$, introducing a leniency programs with $r > 0$ always decreases sustainability of collusion under a regime of a *non-disclosing* antitrust authority if $E[\phi_l] < 1$. This is shown in *Figure 5*.

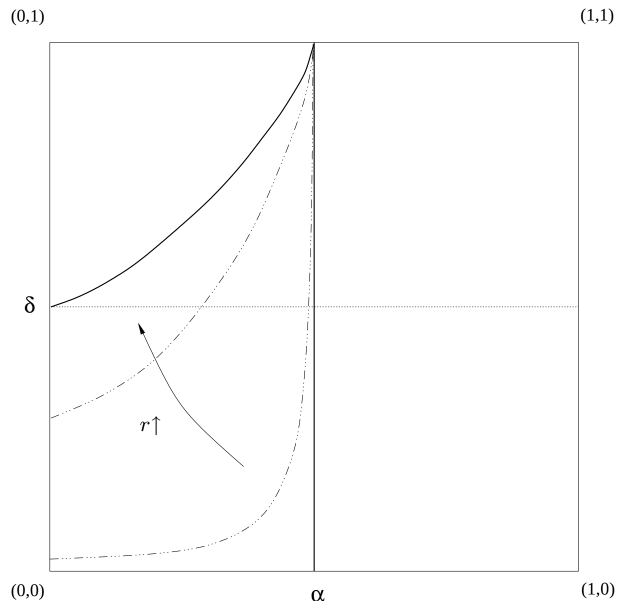

Figure 5: Effect of leniency programs on sustainability of collusion under a regime of a *non-disclosing* antitrust authority with $E[\phi_l] < 1$.

On the other hand, following Lemma 2, if $\phi \geq 1$, all industries with $\delta \geq 0$ and $\alpha \leq \frac{1}{2}$ are able to sustain collusion. For $r \leq 1$, the fine/profit-ratio a firm expects when blowing the whistle, $E[\phi_l]$, is equal or larger than $\frac{1}{2}\phi_{nl}$. Consequently, the number of colluding industries is not affected by leniency if $\phi_{nl} > 2$. Under such an environment, leniency only reduces the fine/profit-ratio firms expect to pay, $\frac{\partial E[\phi_l]}{\partial r} < 0$, and thus ceteris paribus[16] increases the frequency of whistleblowing which is necessary to sustain collusion, $\frac{\partial \gamma}{\partial \phi} < 0$.

We summarize our results in the following Proposition.

Proposition 6 *Introducing a leniency program under a regime of a non-disclosing antitrust authority*

(i) leads to less collusion if the expected fine is not to large ($E[\phi_l] < 1$),

(ii) has no effect if the expected fine is large ($E[\phi_l] \geq 1$),

[16] Holding δ_{nd}^{eff} constant.

(iii) increases the frequency of whistleblowing γ for a given duration of price wars.

Proof The proof follows immediately from the discussion above. ■

5.2 *Disclosing* antitrust authority

A *disclosing* antitrust authority which commits to $\{l\}$ with $r > 0$ in $t = 0$ has the same effect on reduction of fines as discussed above, $\frac{1}{2}\phi_{nl} \leq E[\phi_l] < \phi_{nl}$. From Lemma 3 it follows, for a given $\alpha \leq \frac{1}{2}$ and $\phi < 1$, sustainable collusion requires a discount rate of

$$\delta \geq \frac{(1-\alpha)(1-\phi)}{2(1-\alpha)-\phi}.$$

Again, since $\frac{\partial \delta}{\partial \phi} < 0$ if and only if $\alpha \leq \frac{1}{2}$, leniency leads to less collusion as long as the fine/profit-ratio without leniency was relatively low, $\phi_{nl} < 2$, and thus $\phi = E[\phi_l] < 1$.

On the other hand, for a given $\alpha > \frac{1}{2}$ and $\phi < 1$, sustainable collusion requires a discount rate of

$$\delta \geq \begin{cases} \frac{(1-\alpha)(1-\phi)}{2(1-\alpha)-\phi} & \text{if} \quad \alpha < \frac{1}{1+2\phi-\phi^2} \\ \frac{\left((3\alpha-1)+(1-\alpha)\phi+2\sqrt{2\alpha^2-\alpha}\right)\phi}{[2(3\alpha-1)+(1-\alpha)\phi]\phi+(1-\alpha)} & \text{if} \quad \alpha \geq \frac{1}{1+2\phi-\phi^2}. \end{cases}$$

Thus $\frac{\partial \delta}{\partial \phi} > 0$. So as a consequence, if the probability of demand shock is relatively high, $\alpha > \frac{1}{2}$, introducing a leniency program leads to more collusion.

Figure 6 shows the trade-off a *disclosing* antitrust authority faces when introducing a leniency program starting from a relative low fine/profit-ratio $\phi_{nl} < 2$.

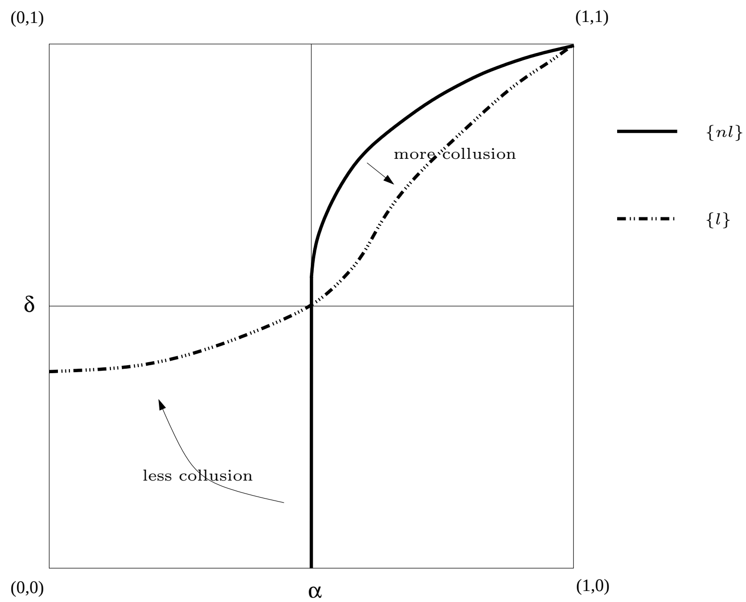

Figure 6: Effect of a leniency program on sustainability of collusion under a regime of a *disclosing* antitrust authority with $\phi_{nl} = 1$, $r = 1$ and $E[\phi_l] = \frac{1}{2}$.

If $\phi \geq 1$ the same results as for a *non-disclosing* antitrust authority holds: If $\alpha \leq \frac{1}{2}$, all industries with $\delta \geq 0$ are able to sustain collusion. Thus, the number of industries which face a relative low probability of a demand shock, $\alpha \leq \frac{1}{2}$, is not affected by leniency if $\phi_{nl} \geq 2$ and thus $E[\phi_l] \geq 1$. On the other hand, from Lemma 3 and from the discussion above we know, reducing the fine/profit-ratio for industries which face $\alpha > \frac{1}{2}$, always increases the number of industries which can sustain collusion. Consequently, if $\phi_{nl} \geq 1$ introducing a leniency program always increases the number of industries which are able to sustain collusion. An example for this result is given in *Figure 7*.

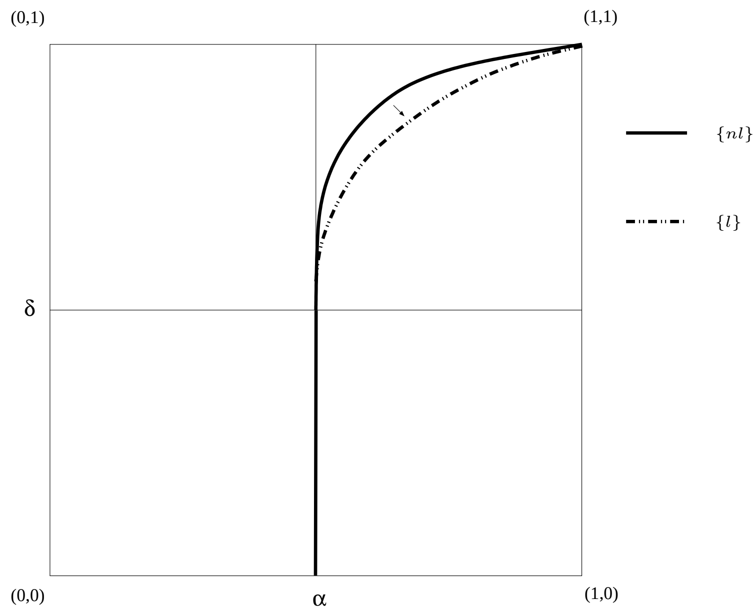

Figure 7: Effect of a leniency program on sustainability of collusion under a regime of a *disclosing* antitrust authority with $\phi_{nl} = 2$, $r = 1$ and $E[\phi_l] = 1$.

In the proof of Lemma 3 it was also shown that $\frac{\partial\gamma}{\partial\phi} < 0$, i.e. the frequency of whistleblowing is increased if the expected fine/profit-ratio is reduced. One can show[17], that the total effect of a reduced fine/profit-ratio and an increased frequency of whistleblowing γ is weakly negative: $\gamma^l E[\phi^l] \leq \gamma^{nl}\phi^{nl}$. Thus leniency programs increases the expected firm value of collusive firms.

We summarize our results in the following Proposition.

Proposition 7 *Introducing a leniency program under a regime of a disclosing antitrust authority*

(i) leads to less collusion in industries with a relative low probability of demand shocks ($\alpha \leq \frac{1}{2}$), if the expected fine is not to large ($E[\phi] < 1$),

(ii) has no effect in industries with a relative low probability of demand shocks ($\alpha \leq \frac{1}{2}$), if the expected fine is large ($E[\phi] \geq 1$),

(iii) leads to more collusion in industries with a relative high probability of demand shocks ($\alpha > \frac{1}{2}$),

(iv) increases the frequency of whistleblowing,

[17] $dV^+ = \frac{\partial V^+}{\partial F}\frac{dF}{d\gamma} + \frac{\partial V^+}{\partial\gamma} \geq 0$ if $T' = 0$ and $IC = 0$.

(v) increases the firm value of collusive firms.

Proof The proof follows immediately from the discussion above. ■

5.3 Extension

Following the discussion around leniency programs at the moment we consider two extensions to the leniency program. First, as e.g. argued in Aubert, Rey, and Kovacic (2006) rewards ($r > 1$) for whistleblowers are introduced.[18] Second, as practised in the European leniency program and being discussed in Motchenkova and van der Laan (2005), leniency will not only be granted to the first firm which blows the whistle, but, possibly with a lower reduction in the fine, also for later firms.

In our framework, both changes have the same effect in that they reduce the expected fine even further. Consider first the reward. As $E[F] = (1 - \frac{1}{2}r)F$, allowing for larger r reduces the fine.

Granting leniency not just to the first firm (with leniency parameter r_1) but also to the second firm (with leniency parameter r_2) reduces the expected fine in case of simultaneous whistleblowing to

$$E[F] = (1 - \frac{1}{2}r_1 - \frac{1}{2}r_2)F,$$

Both changes have the effect of reducing the expected fine. In the extreme case (full rewards for whistleblowing, $r = 2$) or full leniency for the second whistleblower ($r_1 = r_2 = 1$) the expected fine is reduced to zero: $E[F] = 0$. In any case, these changes strengthen the effects of a leniency program as discussed in the previous section.

6 Conclusions

The model developed above identifies the effects of antitrust policy on the collusive strategies of firms, which can not directly observe the market demand, and the resulting market structure.

We find that an antitrust authority charging fines for collusive behavior allow firms with a low discount factor to collude and always leads to a loss in welfare, even if in industries which could have colluded even without a fine. An antitrust authority which discloses information to the collusive industries increases the and allows industries to collude even if the probability

[18]We restrict r to be smaller than 2, since otherwise firms would have the incentive to launching cartels over and over again with the aim to be rewarded for whistleblowing.

of demand shocks is high. Introducing a leniency program reduces the fines and has ambiguous consequences for sustainability of collusion in general: If fines are not too high, leniency programs may reduce the number of industries which collude in an environment with low probability of demand shocks. In contrast, the number of industries which collude in an environment with a high probability of demand shocks is increasing. Finally, leniency always increases the frequency of whistleblowing and leads to a higher collusive firm value if information is disclosed.

Appendix

Proof of Lemma 3

Proof We start with proving condition (ii). We define condition $IC \geq 0$ as

$$IC: \;\; (1-2\alpha)(\delta-\delta_d^{eff})+(\gamma\phi-1)(1-\delta)+\frac{\alpha}{1-\alpha}\gamma\delta\,(2(1-\alpha)-\gamma\phi)\geq 0$$

and $PC \geq 0$ as

$$PC: \;\; 1-\alpha-\alpha\gamma\phi\geq 0.$$

As $\frac{\partial IC}{\partial \delta_d^{eff}} = -(1-2\alpha) \leq 0$ for $\alpha \leq \frac{1}{2}$ and $\frac{\partial IC}{\partial \delta_d^{eff}} > 0$ if $\alpha > \frac{1}{2}$, we will set $\delta_d^{eff} = 0$ if $\alpha \leq \frac{1}{2}$ and $\delta_d^{eff} = (1-\gamma)\delta$ if $\alpha > \frac{1}{2}$ to calculate the minimal $\delta(\alpha,\phi)$ where $IC \geq 0$ holds.

- $\delta(\alpha,\phi=0)$:

If $\phi=0$, firms can always choose $\gamma=1$, thus $IC\geq 0$ and $PC \geq 0$ change to $IC_{\gamma=1}^{\phi=0} = 2\delta-1\geq 0$ and $PC_{\phi=0}=(1-\alpha)\geq 0$. This leads to $\delta(\alpha,\phi=0)\geq \frac{1}{2}$ for all $\alpha\leq 1$.

- $\delta\left(\alpha\leq\frac{1}{2},\phi<1\right)$:

If $\alpha \leq \frac{1}{2}$, $PC \geq 0$ always holds if $\phi < 1$ and thus $\gamma^{max}=1$. Furthermore $IC\geq 0$ changes to $IC^{\delta_d^{eff}=0} = (1-2\alpha)\delta+(\gamma\phi-1)(1-\delta)+\frac{\alpha}{1-\alpha}\gamma\delta\,(2(1-\alpha)-\gamma\phi)\geq 0$. From $\frac{\partial IC^{\delta_d^{eff}=0}}{\partial\gamma} = \phi(1-\delta)+\frac{\alpha\delta(2-2\alpha-\gamma\phi)}{1-\alpha}-\frac{\alpha\delta\gamma\phi}{1-\alpha}$, we get $\gamma^* = \frac{(1-\alpha)[(1-\delta)\phi+2\alpha\delta]}{2\alpha\delta\phi}$. Since $\frac{\partial IC^{\delta_d^{eff}=0}}{\partial\delta} = \left(1+\frac{\alpha\gamma}{1-\alpha}\right)(2(1-\alpha)-\gamma\phi)\geq 0$ if $\alpha\leq\frac{1}{2}$, $\phi<1$, and $\gamma^{max}=1$, the minimal $\delta(\alpha\leq\frac{1}{2},\phi<1)$ where $IC\geq 0$ holds is determined trough $\gamma = min[1,\gamma^*]$. Since $\frac{\partial\gamma^*}{\partial\alpha} = -\frac{(1-\delta)\phi+2\alpha^2\delta}{2\alpha^2\delta\phi} < 0$, $\frac{\partial\gamma^*}{\partial\delta} = -\frac{1-\alpha}{2\alpha\delta^2} < 0$, and $\frac{\partial\gamma^*}{\partial\phi} = -\frac{1-\alpha}{\phi^2} < 0$, it is easy to show that for $\alpha\leq\frac{1}{2}$, $\delta\to\frac{1}{2}$, and $\phi\to 1 \Rightarrow \gamma^*>1$. Thus we set $\gamma=1$ and $IC^{\delta_d^{eff}=0}$ changes to

$IC_{\gamma=1}^{\delta_d^{eff}=0} = (1-2\alpha)\delta + (\phi - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\delta(2(1-\alpha) - \phi) \geq 0$ which holds if $\delta(\alpha \leq \frac{1}{2}, \phi < 1) \geq \frac{(1-\alpha)(1-\phi)}{2(1-\alpha)-\phi}$.

- $\delta\left(\alpha > \frac{1}{2}, \phi < 1\right)$:

If $\alpha > \frac{1}{2}$, $PC \geq 0$ only holds if $\gamma \leq \frac{1-\alpha}{\alpha\phi} \equiv \gamma^{max}$ and $IC \geq 0$ changes to $IC^{\delta_d^{eff}=(1-\gamma)\delta} = (1-2\alpha)(\delta-(1-\gamma)\delta)(\gamma\phi - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\gamma\delta(2(1-\alpha) - \gamma\phi) \geq 0$. From $\frac{\partial IC^{\delta_d^{eff}=(1-\gamma)\delta}}{\partial\gamma} = (1-2\alpha)\delta + \phi(1-\delta) + \frac{\alpha\delta(2-2\alpha-\gamma\phi)}{1-\alpha} - \frac{\alpha\delta\gamma\phi}{1-\alpha}$, we get $\gamma^* = \frac{1-\alpha}{\alpha\phi}\frac{(1-\delta)\phi+\delta}{2\delta}$ with $\gamma^* \gtreqless \gamma^{max}$.

Plugging γ^* into $IC^{\delta_d^{eff}=(1-\gamma)\delta}$ we get $IC_{\gamma=\gamma^*}^{\delta_d^{eff}=(1-\gamma)\delta} = \frac{1}{4\alpha\delta\phi}[((1-\alpha)(1-\delta)^2)\phi^2 + (2\delta(1-\delta)(1-3\alpha))\phi + (1-\alpha)\delta^2] \geq 0$. This condition holds if $\delta \geq \frac{\left((3\alpha-1)+(1-\alpha)\phi+2\sqrt{2\alpha^2-\alpha}\right)\phi}{[2(3\alpha-1)+(1-\alpha)\phi]\phi+(1-\alpha)} \equiv \delta_1$ and if $\delta \leq \frac{\left((3\alpha-1)+(1-\alpha)\phi-2\sqrt{2\alpha^2-\alpha}\right)\phi}{[2(3\alpha-1)+(1-\alpha)\phi]\phi+(1-\alpha)} \equiv \delta_2$.

Plugging δ_1 into γ^* we get $\gamma^*_{\delta=\delta_1} = \frac{1-\alpha}{\alpha\phi}\frac{(1+\phi)\alpha+(1-\phi)\sqrt{2\alpha^2-\alpha}}{(3\alpha-1)+(1-\alpha)\phi+2\sqrt{2\alpha^2-\alpha}} \gtreqless 1$ with $\frac{\partial\gamma^*_{\delta=\delta_1}}{\partial\alpha} < 0$, $\frac{\partial\gamma^*_{\delta=\delta_1}}{\partial\phi} < 0$, and $\gamma^*_{\delta=\delta_1} \leq \gamma^{max}$. It can be shown that $\gamma^* \geq 1$ if $\alpha \leq \frac{1}{1+2\phi-\phi^2}$.

Thus, if $\alpha < \frac{1}{1+2\phi-\phi^2}$, we set $\gamma = 1$ and $IC^{\delta_d^{eff}=(1-\gamma)\delta}$ changes to $IC_{\gamma=1}^{\delta_d^{eff}=(1-\gamma)\delta} = (1-2\alpha)\delta + (\phi-1)(1-\delta) + \frac{\alpha}{1-\alpha}\delta(2(1-\alpha)-\phi) \geq 0$ which holds if $\delta(\alpha < \frac{1}{1+2\phi-\phi^2}, \phi < 1) \geq \frac{(1-\alpha)(1-\phi)}{2(1-\alpha)-\phi}$.

If $\alpha \geq \frac{1}{1+2\phi-\phi^2}$, we set $\gamma = \gamma^*$, $IC^{\delta_d^{eff}=(1-\gamma)\delta}$ changes to $IC_{\gamma=\gamma^*}^{\delta_d^{eff}=(1-\gamma)\delta} \geq 0$ which holds if $\delta(\alpha \geq \frac{1}{1+2\phi-\phi^2}, \phi < 1) \geq \delta_1$.

We ignore δ_2 as $\gamma^*_{\delta=\delta_2} = \frac{1-\alpha}{\alpha\phi}\frac{(1+\phi)\alpha-(1-\phi)\sqrt{2\alpha^2-\alpha}}{(3\alpha-1)+(1-\alpha)\phi-2\sqrt{2\alpha^2-\alpha}}$ with $\frac{\partial\gamma^*_{\delta=\delta_2}}{\partial\alpha} > 0$ and $\frac{\partial\gamma^*_{\delta=\delta_2}}{\partial\phi} < 0$ because if we go to the limits with $\alpha > \frac{1}{2}$ and $\phi \to 1$, we get $\gamma^*_{\delta=\delta_2} > 1$.

- $\delta\left(\alpha \leq \frac{1}{2}, \phi \geq 1\right)$:

If $\alpha \leq \frac{1}{2}$, it is easy to see that $PC \geq 0$ with $\gamma^{max} = \frac{1-\alpha}{\alpha\phi}$ and $IC \geq 0$ always holds if $\alpha \leq \frac{1}{2}$ and $\gamma = \frac{1}{\phi} \leq \gamma^{max} \leq 1$. If $\gamma = \frac{1}{\phi}$, $IC^{\delta_d^{eff}=0}$ changes to $IC_{\gamma=\frac{1}{\phi}}^{\delta_d^{eff}=0} = \frac{(1-2\alpha)\delta}{(1-\alpha)\phi} \geq 0$ and $PC \geq 0$ to $PC_{\gamma=\frac{1}{\phi}} = 1-2\alpha \geq 0$. Both conditions hold if $\delta(\alpha \leq \frac{1}{2}, \phi \geq 1) \geq 0$.

- $\delta\left(\alpha > \frac{1}{2}, \phi \geq 1\right)$:

Again, $PC \geq 0$ holds if $\gamma \leq \frac{1-\alpha}{\alpha\phi} \equiv \gamma^{max}$. Since $\phi > 1 \Rightarrow \gamma^{max} < 1$ for all $\alpha > \frac{1}{2}$. Furthermore $IC \geq 0$ again changes to $IC^{\delta_d^{eff}=(1-\gamma)\delta} \geq 0$, thus the following proof is similar to the proof above for $\delta\left(\alpha > \frac{1}{2}, \phi < 1\right)$. We get $\gamma^* = \frac{1-\alpha}{\alpha\phi}\frac{(1-\delta)\phi+\delta}{2\delta}$ with $\gamma^* \gtreqless \gamma^{max}$ and $IC_{\gamma=\gamma^*}^{\delta_d^{eff}=(1-\gamma)\delta} \geq 0$ holds if $\delta \geq \delta_1$ and if $\delta \leq \delta_2$.

Plugging δ_1 into γ^* we get $\gamma^*_{\delta=\delta_1} = \frac{1-\alpha}{\alpha\phi}\frac{(1+\phi)\alpha+(1-\phi)\sqrt{2\alpha^2-\alpha}}{(3\alpha-1)+(1-\alpha)\phi+2\sqrt{2\alpha^2-\alpha}}$ with $\frac{\partial\gamma^*_{\delta=\delta_1}}{\partial\alpha} < 0$, $\frac{\partial\gamma^*_{\delta=\delta_1}}{\partial\phi} < 0$, and $\gamma^*_{\delta=\delta_1} \leq \gamma^{max} < 1$. Thus we set $\gamma = \gamma^*$ for all $\alpha > \frac{1}{2}$ and $IC^{\delta_d^{eff}=(1-\gamma)\delta}$ changes to $IC^{\delta_d^{eff}=(1-\gamma)\delta}_{\gamma=\gamma^*} \geq 0$ which holds if $\delta(\alpha > \frac{1}{2}, \phi \geq 1) \geq \delta_1$.
We ignore δ_2 again as we get $\gamma^*_{\delta=\delta_2} = \frac{1-\alpha}{\alpha\phi}\frac{(1+\phi)\alpha-(1-\phi)\sqrt{2\alpha^2-\alpha}}{(3\alpha-1)+(1-\alpha)\phi-2\sqrt{2\alpha^2-\alpha}}$ with $\frac{\partial\gamma^*_{\delta=\delta_2}}{\partial\alpha} > 0$. If we go to the limit where $\alpha \to \frac{1}{2}$, $\gamma^*_{\delta=\delta_2} \to \frac{1}{\phi} > \gamma^{max}$ if $\alpha > \frac{1}{2}$.

The proof of condition (i) can be done in a similar way as to the proof of condition (ii). ∎

Proof of Proposition 4

Proof The proof requires to analyze two cases, the case $\alpha > \frac{1}{2}$ and the case $\alpha \leq \frac{1}{2}$.
• For $\alpha > \frac{1}{2}$ it follows $\frac{\partial V^+}{\partial\delta_d^{eff}} \geq 0$ and $\frac{\partial IC}{\partial\delta_d^{eff}} > 0$. Thus, firms choose δ_d^{eff} as large as possible, $\delta_d^{eff} = (1-\gamma)\delta$ and thus $T' = 0$, to maximize its profit. Since $\frac{\partial V^{+\delta_d^{eff}=(1-\gamma)\delta}}{\partial\gamma} = -\frac{\alpha F}{1-\delta} < 0$ firms choose the frequency of whistleblowing γ as large as necessary to sustain collusion and as low as possible to maximize V^+.

• To prove that firms choose $T' = 0$ if $\alpha \leq \frac{1}{2}$, we show that it is always a firm-value-maximizing strategy to choose $T' = 0$. Thus, we show that

$$\frac{dV^+}{d\gamma} = \frac{\partial V^+}{\partial\delta^{T'+1}}\frac{d\delta^{T'+1}}{d\gamma} + \frac{\partial V^+}{\partial\gamma} > 0$$

if it is assumed that the IC just binds when firms choose no price wars, $IC_{T'=0} = 0$. If this holds, firms will never have an incentive the sustain collusion by a positive number of price war periods.

The collusive firm value is given by

$$V^+ = \frac{(1-\alpha)\Pi^M - 2\alpha\gamma F}{2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]}$$

and the IC by:

$$IC : (1-2\alpha)(\delta-(1-\gamma)\delta^{T'+1}) + (\gamma\frac{2F}{\Pi^M} - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\gamma\delta(2(1-\alpha) - \gamma\frac{2F}{\Pi^M}) \geq 0.$$

From these two expressions we get:

$$\begin{aligned}
\frac{\partial IC}{\partial \gamma} &= (1-2\alpha)\delta^{T'+1} + (\frac{2F}{\Pi^M} - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\delta(2(1-\alpha) - 2\gamma\frac{2F}{\Pi^M}) \\
\frac{\partial IC}{\partial \delta^{T'+1}} &= -(1-2\alpha)(1-\gamma) \\
\frac{\partial V^+}{\partial \gamma} &= \frac{2\alpha(\delta - \delta^{T'+1})[(1-\alpha)\Pi^M - 2\alpha\gamma F]}{\big(2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]\big)^2} \\
&\quad - \frac{2\alpha F}{2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]} \\
\frac{\partial V^+}{\partial \delta^{T'+1}} &= \frac{2\alpha(1-\gamma)[(1-\alpha)\Pi^M - 2\alpha\gamma F]}{\big(2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]\big)^2}
\end{aligned}$$

From $\frac{d\delta^{T'+1}}{d\gamma} = -\frac{\partial IC/\partial\gamma}{\partial IC/\partial\delta^{T'+1}}$, it follows:

$$\frac{d\delta^{T'+1}}{d\gamma} = \frac{(1-2\alpha)\delta^{T'+1} + (\frac{2F}{\Pi^M} - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\delta(2(1-\alpha) - 2\gamma\frac{2F}{\Pi^M})}{(1-2\alpha)(1-\gamma)},$$

and thus the total differential is given by

$$\begin{aligned}
&\frac{dV^+}{d\gamma} = \\
&\frac{2\alpha[(1-\alpha)\Pi^M - 2\alpha\gamma F]\left[(1-2\alpha)\delta^{T'+1} + (\frac{2F}{\Pi^M} - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\delta(2(1-\alpha) - 2\gamma\frac{2F}{\Pi^M})\right]}{\big(2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]\big)^2(1-2\alpha)} + \\
&+\frac{2\alpha(\delta - \delta^{T'+1})[(1-\alpha)\Pi^M - 2\alpha\gamma F]}{\big(2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]\big)^2} - \frac{2\alpha F}{2[1-\delta+\alpha(\delta-[\gamma\delta+(1-\gamma)\delta^{T'+1}])]}.
\end{aligned}$$

We first turn to the conditions were $IC_{T'=0}$ just binds: By setting $T' = 0$ we get

$$IC_{T'=0} : (1-2\alpha)(\delta-(1-\gamma)\delta) + (\gamma\frac{2F}{\Pi^M} - 1)(1-\delta) + \frac{\alpha}{1-\alpha}\gamma\delta(2(1-\alpha) - \gamma\frac{2F}{\Pi^M}) \geq 0.$$

Setting this expression equal to zero and solving for γ reveals that the $IC_{T'=0}$ just binds if

$$\gamma = \frac{1}{8\alpha\delta F}\Big[2[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)] \pm \\ \pm 2\sqrt{[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F]}\Big].$$

To maximize the firm value from collusion, V^+, we only need the lowest possible γ. Thus,

$$\gamma_{IC}^0 = \frac{1}{8\alpha\delta F}\Big[2[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)] - \\ -2\sqrt{[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F]}\Big]$$

From this expression we observe:

1. *$\gamma_{IC}^0 \in \mathbb{R}$ if $\alpha \leq \frac{1}{2}$.*

   Term under the root of γ_{IC}^0 is given by:

   $$r = [(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F]$$

   It follows, $r = 0$ if and only if:

   $$\alpha = \begin{cases} 1 \\ \frac{(\delta\Pi+2F(1-\delta))^2}{(\delta\Pi+2F(1-\delta))^2+8\delta\Pi F(1-\delta)} \end{cases}$$

   The second null, $\frac{(\delta\Pi+2F(1-\delta))^2}{(\delta\Pi+2F(1-\delta))^2+8\delta\Pi F(1-\delta)}$ is always equal or larger than $\frac{1}{2}$, as $\frac{(\delta\Pi+2F(1-\delta))^2}{(\delta\Pi+2F(1-\delta))^2+8\delta\Pi F(1-\delta)} = \frac{1}{x}$ requires $\delta = \frac{2F\left[(3-x)\Pi-2F(1-x)\pm 2\sqrt{(2-x)\Pi^2}\right]}{(x-1)(\Pi-2F)^2+8\Pi F}$ which is not defined for $x > 2$. For $\alpha \leq \frac{1}{2}$, $r \geq 0$ since for $\alpha = 0$, $r = (\delta\Pi + 2F(1-\delta))^2 > 0$.

2. *From γ_{IC}^0 it is easy to see that γ_{IC}^0 never turns negative.*

3. *For $\gamma = 0$, the $IC_{e=d}$ is reduced to $-1+\delta < 0$.*

Given these observation it follows: If $\gamma \geq \gamma_{IC}^0$, $IC_{T'=0} \geq 0$.

Now we turn the analysis wether the total differential, $\frac{dV^+}{d\gamma}$, is positive if it is assumed that $T' = 0$ is chosen by the firms. By setting $T' = 0$ the total differential reduces to:

$$\frac{dV^+}{d\gamma} = \frac{2\alpha[(1-\alpha)\Pi^M - 2\alpha\gamma F]\left[(1-2\alpha)\delta + (\frac{2F}{\Pi^M}-1)(1-\delta) + \frac{\alpha}{1-\alpha}\delta(2(1-\alpha) - 2\gamma\frac{2F}{\Pi^M})\right]}{(2[1-\delta])^2(1-2\alpha)} - \\ -\frac{\alpha F}{1-\delta}$$

Again, setting this expression equal to zero and solving for γ reveals that $\frac{dV^+}{d\gamma} = 0$ if and only if:

$$\gamma = \frac{1}{8\alpha\delta F}\Bigg[3(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta) \pm \\ \pm\sqrt{[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F] + + 2(1-2\alpha)[4\delta(1-\alpha)(1-\delta)\Pi F]}\Bigg]$$

As done before, we only need the lowest possible γ. Thus we get

$$\gamma^0_{dV^+} = \frac{1}{8\alpha\delta F}\Bigg[3(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta) - \\ -\sqrt{[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F] + + 2(1-2\alpha)[4\delta(1-\alpha)(1-\delta)\Pi F]}\Bigg]$$

From $\gamma^0_{dV^+}$ we observe:

4. *The term under the root of $\gamma^0_{dV^+}$ is larger than the term under the root of γ^0_{IC} for all $\alpha \leq \frac{1}{2}$.*

5. *From observation (1.) it follows that $\gamma^0_{dV^+}$ never turns negative for $\alpha \leq \frac{1}{2}$.*

6. *For $\gamma = 0$, the total differential, $\frac{dV^+}{d\gamma}$, reduces to $\frac{\alpha[(1-\alpha)\delta\Pi + 2F(1-\delta)\alpha]}{2(1-\alpha)(1-\delta)^2} > 0$.*

Consequently it follows, if $\gamma \leq \gamma^0_{dV^+}$, $\frac{dV^+}{d\gamma} > 0$. With the result above, that $IC_{T'=0} \geq 0$ if $\gamma \geq \gamma^0_{IC}$ we get that, $\frac{dV^+}{d\gamma} > 0$ if $\gamma^0_{dV^+} \geq \gamma^0_{IC}$.

We define the difference of the two nulls as $Z = \gamma^0_{dV^+} - \gamma^0_{IC}$. We get that

$Z \geq 0$ if

$$Z = (1-\alpha)\delta\Pi - 2F(1-\alpha)(1-\delta) + \Big[- \sqrt{[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F] + 2(1-2\alpha)[4\delta(1-\alpha)(1-\delta)\Pi F]} + 2\sqrt{[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F]} \Big] \geq 0.$$

From $Z \geq 0$ we observe:

7. The term in the squared bracket of Z never turns negative if $\alpha \leq \frac{1}{2}$.

To prove this observation we proceed as in the proof of observation *(1.)*. To clarify our approach note the following simplification:

$$Z' = X + \left[-\sqrt{W+y} + 2\sqrt{W} \right]$$

The term in the squared bracket is larger or equal than zero if and only if $3W \geq y$. Using this simplification to solve our problem we get:

$$3\big([(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2\alpha[4\delta(1-\alpha)(1-\delta)\Pi F]\big) \geq 2(1-2\alpha)[4\delta(1-\alpha)(1-\delta)\Pi F]$$

which is equal to:

$$r = 3[(1-\alpha)\delta\Pi + 2F(1-\alpha)(1-\delta)]^2 - 2(1+\alpha)[4\delta(1-\alpha)(1-\delta)\Pi F] \geq 0$$

It follows, $r = 0$ if and only if:

$$\alpha = \begin{cases} 1 \\ \frac{3\delta^2\Pi^2 + 4\delta(1-\delta)\Pi F + 12(1-d)^2F^2}{3\delta^2\Pi^2 + 20\delta(1-\delta)\Pi F + 12(1-d)^2F^2}. \end{cases}$$

The second null, $\frac{3\delta^2\Pi^2 + 4\delta(1-\delta)\Pi F + 12(1-d)^2F^2}{3\delta^2\Pi^2 + 20\delta(1-\delta)\Pi F + 12(1-d)^2F^2}$ is always larger or equal than $\frac{1}{2}$, since $\frac{3\delta^2\Pi^2 + 4\delta(1-\delta)\Pi F + 12(1-d)^2F^2}{3\delta^2\Pi^2 + 20\delta(1-\delta)\Pi F + 12(1-d)^2F^2} = \frac{1}{x}$ requires $\delta = \frac{2F\left[(5-x)\Pi - 6(1-x)F \pm 2\sqrt{2(2-x)(1+x)\Pi^2}\right]}{3(x-1)\Pi^2 + 4(5-x)\Pi F + 12(x-1)F^2}$ which is not defined for $x > 2$. For $\alpha \leq \frac{1}{2}$, $r \geq 0$ since for $\alpha = 0$, $r = 3\delta^2\Pi^2 + 4\delta(1-\delta)\Pi F + 12F^2(1-\delta)^2 > 0$.

8. If $\alpha = 0$, $Z \geq 0$.

From setting $\alpha = 0$ it follows,

$$Z_{\alpha=0} = \delta\Pi - 2F(1-\delta) + \left[-\sqrt{(\delta\Pi + 2F(1-\delta))^2 + 8\delta(1-\delta)\Pi F} + 2\sqrt{(\delta\Pi + 2F(1-\delta))^2}\right]$$

Setting $Z_{\alpha=0}$ equal to zero and solving for δ reveals that there is a null for $\delta = 0$. The second null at $\delta = 1$ is not relevant as $\Pi > 0$. As for $\delta = 1$, $Z = 2\Pi > 0$, it follows that $Z_{\alpha=0} \geq 0$.

9. *If* $(1-\alpha)\delta\Pi - 2F(1-\alpha)(1-\delta) > 0$, $Z = 0$, *if and only if* $\alpha = 1$.

   Setting Z equal to zero and solving for α reveals that there is a null at $\alpha = 1$, if $(1-\alpha)\delta\Pi - 2F(1-\alpha)(1-\delta) > 0$.

10. *If* $(1-\alpha)\delta\Pi - 2F(1-\alpha)(1-\delta) \leq 0$, $Z = 0$, *if* $\alpha = \frac{1}{2}$ *and* $\alpha = 1$.

    Setting Z equal to zero and solving for α reveals that there is a null at $\alpha = 1$ and additionally a second null at $\alpha = \frac{1}{2}$, if $(1-\alpha)\delta\Pi - 2F(1-\alpha)(1-\delta) \leq 0$.

Given these observations it follows that $\gamma^0_{dV^+} \geq \gamma^0_{IC}$ if $\alpha \leq \frac{1}{2}$. Consequently, the total differential is always positive, $\frac{dV^+}{d\gamma} \geq 0$, if collusion can just be sustained with $T' = 0$, $IC_{T'=0} = 0$. Thus, in order to maximize the collusive firm value, firms will never choose price wars to sustain collusion. ■

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