Reshuffling the cards: Regulation and competition in a capacity accumulation game

Bertrand Villeneuve (U. Tours and CREST)
Yanhua Zhang (U. Toulouse)

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Strategic Firm-Authority Interaction in Antitrust, Merger Control and Regulation

Motivating examples

The electricity market in China

- Regional monopolies with (to some extent) region specific technologies
- Inter-connection growing
- Restructuring the industry?

The electricity market in France

- Historic monopoly: EDF
- Static restructuring: divestiture
- Dynamic restructuring: authorization/laissez-faire

Differential game approach

- A game of capacity accumulation
- Open-loop strategies
  - Nash: one's strategy does not depend on the other's "reaction"
  - Equilibrium not necessarily subgame perfect

Formal literature

- Besanko and Doraszelski (2004), Hanig (1986), Reynolds (1987), Cellini and Lambertini (2003)

Overview of the results

1. A simple theory of site allocation with impact on investment costs
2. Effect in the long-run
3. Effect of initial condition on the transition
4. Optimum: symmetric initial conditions and symmetric investment opportunities
5. Intuitive (and strong) results: if not possible, compensate smaller firm with better opportunities
6. Problem: commitment

Main assumptions

- Infinite time $t \in [0, +\infty)$
- Duopoly : 1 and 2 with i for a generic firm (j for the generic competitor)
- Inverse demand function at date t: $P(t) = A - q_1(t) - q_2(t)$
- Capacity accumulation $i = 1, 2$

$$\dot{k}_i(t) = I_i(t) - \delta_i k_i(t)$$

Main assumptions (continued)

Investment

- Quadratic instantaneous cost of investment

$$C_i(I_i) = \frac{c_i}{2} I_i^2$$

- (c_1, c_2) belongs to convex set $\Omega \subset \mathbb{R}_+^2$

Production

- No production cost (for simplicity)
- Full capacity utilization (relaxed in paper)

Sites and costs: a simple example for Ω

- A continuum of available sites parameterized by $\theta \in [\underline{\theta}, \overline{\theta}]$
- Site specific investment represented by function $z(\theta)$
- θ site specific investment cost $c(\theta) = \frac{\theta}{2} z(\theta)^2$
- Firm i described by sites it owns (indicator $\omega_i(\theta)$)
- Let each firm optimize investment with its sites
- We find a global constraint

$$\frac{1}{c_1} + \frac{1}{c_2} = \text{Constant}$$

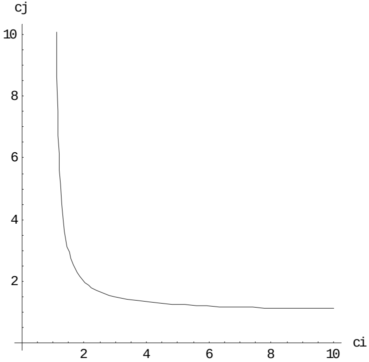

Figure: Cost frontier (c_1, c_2)

The open-loop Cournot-Nash equilibrium

- Firm i maximizes the present value of the profit flows

$$\max_{I_i(\cdot)} \int_0^{+\infty} \pi_i(t) e^{-\rho t} dt$$
$$\text{where } \pi_i(t) = P(t) q_i(t) - \frac{c_i}{2} I_i(t)^2$$

- Control variables: $I_i(t)$ and $I_j(t)$
- State variables: $k_i(t)$ and $k_j(t)$
- Equilibrium when one's path is best response to the other's path
- Open-loop not an inferior concept
  - Information
  - Investment programming
  - Commitment
  - ... tractable!

- Dynamic equation

$$(\rho+\delta_i)c_iI_i - c_i\dot{I}_i = [A - 2k_i - k_j]$$

- With accumulation equations

$$\ddot{k}_i + \delta_i\dot{k}_i - \left[\frac{2}{c_i} + (\rho+\delta_i)\delta_i\right]k_i + \frac{A-k_j}{c_i} = 0$$

- Define functions of time $h_1 = \dot{k}_1$ and $h_2 = \dot{k}_2$
- 2nd-order system of equations solved as a 4-dimensional 1st-order system:

$$\dot{H} = MH - N,$$

where $H = (h_1, k_1, h_2, k_2)^T$, $N = (\frac{A}{c_1}, 0, \frac{A}{c_2}, 0)^T$ and

$$M = \begin{pmatrix} -\delta_1 & \frac{2}{c_1} + (\rho+\delta_1)\delta_1 & 0 & \frac{1}{c_1} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{c_2} & -\delta_2 & \frac{2}{c_2} + (\rho+\delta_2)\delta_2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- Eigenvalues of M, λ_s with $s = 1, 2, 3, 4$
- At least one is negative (Tr$[M] < 0$)
- Even number of negative eigenvalues (Det$M >$)
- Eigenvalues can't be all negative (Coeff. of 2nd order term in characteristic polynomial is negative)

Proposition

There are two positive eigenvalues and two negative ones

- Weights given to diverging exponentials must be null (otherwise capacity diverges to $\pm\infty$).
- So capacities, as a function of time, have the form

$$k_i(t) = c_i^0 + c_i^1 e^{\lambda_1 t} + c_i^3 e^{\lambda_3 t}$$

6 parameters identified with

- Initial conditions (2 equations)
- Particular solution of system $=$ steady state (2 equations)
- Eigenvectors (2 equations—1 per vector)

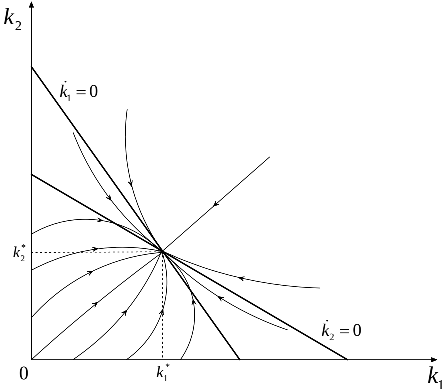

Figure: The phase diagram

Comparative statics of steady state

Cost indicators $C_1 = c_1(\rho + \delta_1)\delta_1$ and $C_2 = c_2(\rho + \delta_2)\delta_2$

- Investment

$$I_i^* = \frac{(1 + C_j)A\delta_i}{(2 + C_i)(2 + C_j) - 1}$$

- Capacity

$$k_i^* = \frac{(1 + C_j)A}{(2 + C_i)(2 + C_j) - 1}$$

Comparative statics of steady state

A definite sign for each derivative

Proposition (Steady state profit)

We have

$$\frac{\partial \pi_i^*}{\partial c_i} < 0, \frac{\partial \pi_i^*}{\partial c_j} > 0, \frac{\partial \pi_i^*}{\partial \delta_i} < 0, \frac{\partial \pi_i^*}{\partial \delta_j} > 0$$

Explains the ambiguity in the symmetric case

Remark (In the symmetric case)

Changing cost affect the whole industry in parallel, bringing no clear advantage.

Proposition (Symmetry optimal in long run)

If Ω is symmetric, an allocation of sites equalizing costs maximizes long run total capacity and minimizes long run total profits.

More on the dynamics

- Where does the economy go? **OK**
- Where does it start from?
- How does it make the transition?

Constraint on the allocation of capacity

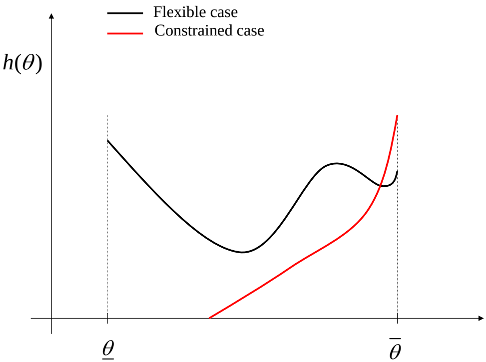

Figure: Distribution of initial capacity over sites

A useful case

- Focus on asymmetry in c_1 and c_2 while keeping symmetric depreciation rates ($\delta_1 = \delta_2 = \delta$)
- We can then calculate the negative eigenvalues of M:

$$\lambda_1 = -\frac{\delta}{2} - \frac{\sqrt{c_1 c_2 (4c_2 - 4\sqrt{c_1^2 - c_1 c_2 + c_2^2} + c_1(4 + c_2\delta(5\delta + 4\rho)))}}{2c_1 c_2},$$

$$\lambda_3 = -\frac{\delta}{2} - \frac{\sqrt{c_1 c_2 (4c_2 + 4\sqrt{c_1^2 - c_1 c_2 + c_2^2} + c_1(4 + c_2\delta(5\delta + 4\rho)))}}{2c_1 c_2}$$

- Natural angle is total **capacity over time = total consumption**

Proposition

Fix total initial capacity K_0 and costs c_1 and c_2 (wlog $c_1 < c_2$).
βK_0 goes to firm 1 and $(1-\beta)K_0$ goes to firm 2.

1. *Total capacity at date 0 and in the long run independent of β*
2. *Total capacity increases more slowly (or decreases faster) at date 0 as β increases*
3. *Total capacity at any date $t > 0$ is smaller for larger β*

If investment cost cannot be changed, if no fine tuning done (regulator plays once), **give at initial date as much as possible to less-favored firm**

A summary

- Long run objective: symmetry always preferred
- Short run objective: asymmetry may be a second-best
- Optimum is a trade-off

Asymmetric costs and capacities: an example

- $\delta = 0.1$, $\rho = 0.1$, $A = 1$, $C = 1$ and initial total capacity $K_0 = 1/2$
- Two cases
  - $c_1 = c_2 = 2$
  - $c_1 = 1.33$ and $c_2 = 4.33$

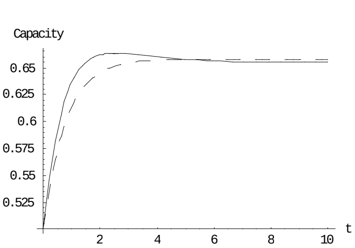

Figure: Total capacity (sym. and asym.). Firm specific and total capacity

Policy implication

- When priority is on long-run objective, symmetry dominates
- Asymmetric may be optimal for transition given discounting of future
- Regulatory (in)consistency: incentives to symmetrize every so often
- Closed-loop: on-going research