Partial Commitments in Antitrust*

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Abstract

We analyze the impact of the introduction of a commitments procedure in terms of efficiency and deterrence in antitrust contexts. This procedure allows certain cases of abuse of dominant position to be handled effectively and quickly by offering an immunity of fine to the firm in return for some behavior modification. We first show that this procedure should only be proposed when the anticompetitive harm is large enough and may allow partial remedies to the infringement when incentives to cooperation ask for it. Under asymmetric information, informational rents associated to the firm's gain with the alleged practice diminish the CA's interest to negotiate and we provide some sufficient conditions under which this procedure is welfare enhancing. We show that coupled with a partial reduction of fine, the introduction of such a procedure always increases the consumers' surplus.

Keywords: Commitments in antitrust; Plea bargaining; Consumer Surplus.

JEL Codes: K21, K42, L41

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1 Introduction

Some tools have been recently introduced in the European antitrust arsenal in order to facilitate the work of the Competition Authorities (CA) by quickening the treatment of cases and by giving the right incentives to the firms to cooperate with the CA. Amongst them, one can cite leniency programs, commitments and "no-contest" procedures. These tools can be viewed as negotiation abilities in the sense that they can be discretionarily used by the CA, and that the cooperation of the firm in investigations and/or concerning their future behavior are obtained under the threat of the trial fine.

Leniency programs are designed for cases of collusion. They provide some incentives for a firm to reveal its participation to a cartel before or during an investigation: the whistleblowing firm obtains an immunity of fine if it facilitates the establishment of guiltiness of the other members of the cartel. The economic literature have paid much attention on these programs. For example, Motta and Polo (2003), Aubert, Kovacic and Rey (2006) and Harrington (2008) analyze incentives provided by reductions of the fine in order to make tacit collusion harder to sustain on a given market. Essentially, these programs act like an improvement of the audit technology of the CA that makes in expectation the trial sanction more probable.

The two other procedures have received less attention in antitrust contexts. Both of these are possible in cases of unilateral practices, and contain, in addition to the cooperation in investigations, the acceptance of some commitments concerning the future behavior of the firm on the market. The main difference between these last two is that the CA does not establish the guiltiness of the firm in the commitments procedure decision, while it does in the "no-contest" and thus has the ability to maintain a positive fine. In this article, we focus on the sole commitments procedure, but we also consider the case where the CA is provided with a possibility to partially reduce the fine in the negotiation of commitments.[1]

While the commitments procedure has been recently introduced in the European law, such a solution is more than established in front of the *US Department of Justice*, where *consent decrees* concern 70 to 80 % of antitrust cases.[2]

We find that the commitments procedure has typically two opposite effects. On the one hand, consumers benefit of the avoidance of the trial and the associated uncertainty. On the other hand, by closing a case with commitments, the

[1]Our objective is to analyze the negotiation of commitments, that is more uncertain for the firm in the no-contest procedure, and we present the possibility to introduce a positive fine in the commitments procedure. For more detailed juridical exposures, see Furse (2004), Cook (2006), Wils (2006) and Vialfont (2007). For an analysis of the interaction between leniency programs and the commitments procedure in tacit collusion cases, see Vialfont (2008). Here, we will only study unilateral practices. Examples of such cases are found in *German Football League* (COMP/37.214, decision of Jan., 19th of 2005), *Coca-Cola* (COMP/39.116, decision of Jun., 22nd of 2005), *Alrosa & De Beers* (COMP/E-2/38.381, decision of Feb., 22nd of 2006), *Repsol* (COMP/B-1/38.348, decision of Apr., 12th of 2006). However, there is no such limitation in the European dispositions. See for example: *Buma and Sabam* (COMP/37.749, decision of Aug., 17th of 2005), *Scandinavian Airline System and Australian Airlines* (COMP/39.152, decision of Sept., 22nd of 2005) and *DaimlerChrysler, Toyota, General Motors and Fiat* (COMP/39.140 to 143, communication IP/07/409 of Mar., 23rd 2007).

[2]In European law, it has been introduced under article 9 of Regulation No 1/2003 and transposed in the article L 464-2-I of the French code of commercial law under the ordinance of the 4th of November 2004.

competition authority gives up the fine, and inevitably looses some deterrent effect of its intervention.

The main purpose of this article is to analyze the tradeoff for the authority between these two effects when injunctions in trial are also expected. We show that the commitments procedure may enhance the efficiency of the enforcement of the competition law, through the enhancement of the consumers' surplus: the firm always has the ability to choose the trial procedure. However, in some situations, the introduction of the commitments procedure is harmful to consumers. These cases are presented in a framework where the alleged practice initially reduces the consumers' surplus. Finally, coupled with a partially reduced fine, this procedure is always introduced by the authority.

This paper is related to the literature on plea bargaining and settlement, where a defendant is given the option to plead guilty in exchange for a reduction of the penalty.[3] However, in the antitrust context, we must also take into account that the decision of the CA has an impact on the future equilibrium on the market[4]. Using injunctions in trial, the firm is imposed to take an end to the practice, and we will allow in this paper for a negotiation of commitments. These negotiations will not consist in price fixing by the authority but in behavioral remedies that in equilibrium may be less constraining than injunctions for incentive reasons.

We use a model in which the firm's gain with the practice is its private information. The timing makes it close to the Bebchuk (1984) model with a continuum of types. Indeed, the firm proposes the use of the commitments procedure, but we assume that the CA is able to announce its acceptance rule prior the begining of the game, for example with guidelines or reputation. Alternatively, we could assume that the authority detains all the bargaining power.[5]

Landes (1971), Gould (1973), Posner (1973), and Shavell (1982) are among the early papers on this topic. In this literature, avoiding the trial costs or quickening the treatment of cases, are the first justification for both parties to negotiate. However, we do not explicitly take these costs into account in order to focus on other interesting effects of the commitments procedure.

A second rational for plea bargaining is provided by Grossman and Katz (1983) who first identified their insurance effect against judicial errors. Here, we assume that a firm that did not adopt a potentially anticompetitive practice is never prosecuted. However, even with risk-neutral parties, we will find some

[3]See Daughety and Reinganum (2005, 2008) and Spier (2007) for excellent surveys on the settlement literature.

[4]In the European competition law, CA are generally not supposed to grant direct compensations to consumers or plaintiffs: the fine is paid back to the public revenue department. Trial injunctions and commitments are closer to compensation schemes, but only refer to the future behavior of the firm. Hence, only the fine affects directly the firm while commitments and injunctions gets in the future consumers surplus. In the settlement literature, Polinsky and Che (1991) first analyzed the decoupled liability providing some optimal differentiated transfers between trial parties, in terms of crime deterrence and suits reduction. Daughety and Reinganum (2003) first analyzed decoupled liability in asymmetrical information. See Chu and Chien (2007), for an article measuring the compliance effect of settlement, where however the probability of a damage occurrence does not depend on the degree of precaution taken by the offender.

[5]Reinganum and Wilde (1986) analyze a settlement game where the informed party moves first. This signaling model implies that all types have a positive chance of going to trial. In Bebchuk (1984), the uninformed party moves first, so that the offer acts like a screening device that split the informed one into those who will accept the offer and those who will choose the traditional suit procedure.

value to the insurance effect of commitments against "type-II errors" in trial.

Finally, the impact of negotiations in terms of deterrence has been addressed in the plea bargaining literature with a budget constrained authority. In asymmetric information models, the compliance incentives of settlements and their effects on welfare are analyzed by Reinganum (1993), Miceli (1996) and Franzoni (1999).[6] These articles mainly show that the reduction of sanctions needed in order to incite negotiations reduces deterrence. In our model, the anticompetitive qualification of a practice is only characterized in a formal decision. However, we also find that the introduction of the commitments procedure induces more types of firm to adopt a practice that harms consumers: a firm that could be deterred with the trial intervention is always part of those that are interested in a commitments proposal. As compared to this literature, the antitrust context pushes for a damage to consumers that vary with the gain of the firm. In addition, we have some economical basis to the value of the deterrence objective in the efficiency tradeoff.

We do consider a simple model where all these effects are at play in an antitrust context: a firm can engage in a practice that enhances its profit and harms consumers, but not necessarily in a one-to-one proportion.

We first show that, under symmetric information, the CA never uses commitments in smallest cases, but always introduces it in its abilities for larger cases. Under asymmetric information, the firm's informational rent may lead the authority to never introduce the commitments procedure, according to the distribution of the firm's gain. When a partial reduction of the fine is possible, and under any information structure, there exists an introduction of the commitments that is beneficial to consumers.

The paper continues as follows. In section 2, we present the model. In section 3, we present the case where the CA only uses trial. In section 4, we look for the equilibrium under different information regimes. Section 5 deals with some extensions concerning the main assumptions of the model on the fine reduction and the consumers' surplus. Concluding remarks follow in section 6.

2 The Model

2.1 Presentation of the Game

A firm considers engaging in a practice that increases its profits and reduces the consumers' surplus. We denote $\Delta \in [0, \overline{\Delta}]$ the incremental gain resulting from the practice, which follows a density $\phi(.)$ on the concerned sector.

The intervention of the authority is modelled as follows. We assume that the CA maximizes the consumers surplus, so that its first-best would be to deter any practice. In order to make an interest to implement a commitments procedure, we do assume that the fine is not high enough to achieve this result. We assume that the firm is never prosecuted when it does not adopt the practice.

Concerning the detection and sanction of the practice we make the following assumptions. If the firm engages in the practice, it might be detected in a preliminary investigation with a probability $\alpha \in (0, 1)$. While detected, the practice is found anticompetitive in trial with a probability $\beta \in (0, 1)$, in which

[6]Polinsky and Rubinfeld (1988) first tackled the deterring consequences of a fine reduction in negotiations in a symmetric information game.

case the firm pays a fine F and receives an injunction to end the practice. In addition, we assume that Δ cannot be credibly disclosed by the firm, and that, unless specified, it remains its private information.[7] Moreover, we assume that α and β are exogenous, knowing that we assume no prosecution of firm that has not adopt the practice, in the sense of Franzoni (1999): during any stage of the game the CA would like to set them as high as possible, and we assume an implicit budget constraint.

The extensive form of the game and the resulting firm's payoffs are presented in figure 1. Points correspond to decisions of the firm and circles random occurrences.

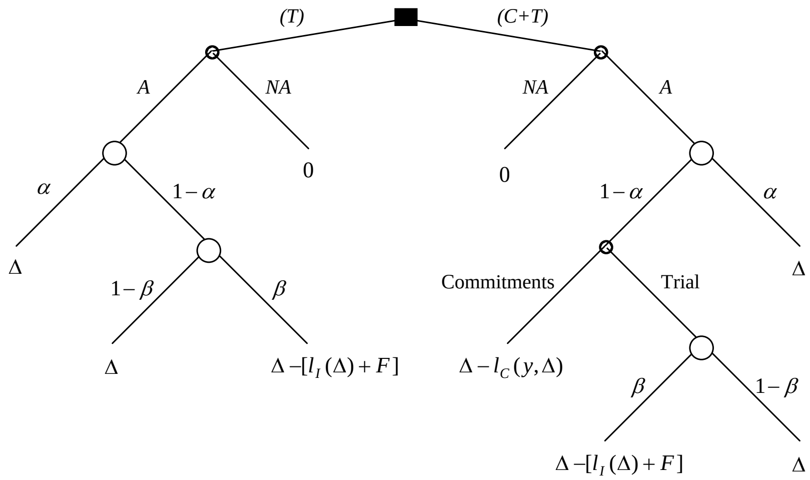

Figure 1: Extensive form of the game and firm's expected payoffs

At the first stage of the game, the CA announces if in prosecutions only trial will be used (T subgame) or if commitments may be proposed prior to trial ($C{+}T$ subgame). Under the latter option, we assume that the authority credibly announces a requested *level of effort* in commitments, denoted $y \in [0,1]$.[8]

At the second stage, given this announcement, the firm decides whether to abuse of its dominant position (A) or not (NA). If it adopts the practice, an audit takes place with probability α and the firm is found guilty in trial with probability β.

In the $(C+T)$ option, the firm also decides prior to the trial whether to propose commitments or to go to trial.[9]

We denote Π^d the profit of a firm that do not adopt the practice (NA), where d stands for deterrence, and normalize it to zero. Note that, for $\alpha = 0$ or $\beta = 0$, the firm always adopts the practice: it gains Δ and is never sanctioned.

[7] We discuss in section 4 the result of Shavell (1989), where private information can be credibly revealed, such that no trial would occur in equilibrium when it is costly.

[8] In a former version of this paper we analyzed the credibility of this effort announcement or alternatively a case where the firm has all the bargaining power. In such a case, we find that there is no room for an interest of the commitments procedure in efficiency terms.

[9] Results of a multi-period model are qualitatively identical when the firm proposes commitments in a first period, and that trial happens in the following one. Tradeoffs of the firm would be of the same kind, and also the determinants of commitments proposals. In addition, we would find that incentive compatible efforts increase with the actualization rate.

We denote Π^t the expected profits of a firm that adopts the practice and that will face trial once prosecuted: $\Pi^t = \Delta - \alpha\beta\,[l_I(\Delta) + F]$, where $l_I(\Delta)$ is the loss associated to injunctions. The case where F is a function of Δ is discussed in section 4.1.3. For simplicity of the exposure, we assume that $l_I(\Delta) = \Delta$, knowing that $l_I(\Delta)$ is legally constrained by Δ and that a strict increasing property with Δ would be sufficient.

Under the $(C + T)$ option, we denote Π^c the expected profits of a firm that decides to adopt the practice and that proposes commitments once prosecuted: $\Pi^c = \Delta - \alpha l_C(y, \Delta)$, where $y \in [0, 1]$ is the requested effort on which the firm commits to restore competition. In accordance to the jurisprudence of the Court of First Instance of the European Communities,[10] we assume that this effort is at most as much constraining as the one requested in injuctions. In this model, this can be interpreted as a condition on y that can not exceed unity, while we assume that $l_C(y, \Delta)$ is strictly increasing with y and that $l_C(1, \Delta) = l_I(\Delta)$. In addition, we assume that $l_C(y, \Delta)$ is strictly increasing with Δ and twice differentiable w.r.t. y and Δ.

Before solving the game, we present the assumed implicit link between the firm's loss and the requested effort. It will allow us to analyze the how consumers are affected by the imposed or negotiated ending of the practice.

2.2 Consumers' surplus and firm's profit on the market

First, we assume that the consumers' surplus is only a function of the equilibrium price, that will depend on the effort of the firm in restoring competition.

We denote $p(y, \Delta)$ the price associated to a type of firm Δ and an effort y. We make the following assumption concerning the type of cases we consider in this model.

Assumption 1. *We assume that the* equilibrium residual profit *of the firm and the* counterfactual price on the market *decrease with the effort $y \leqslant 1$ and Δ. In addition, we consider that the residual profit does not change of concavity for the possible prices without the practice.*

This assumption means that when the profit of the firm after an effort to restore competition decrease there is also a reduction of the equilibrium price. In addition, it means that the larger this effort, or, for a given effort, the larger the initial gain of the firm with the practice, the smaller will be the price after the intervention of the CA. The last part is discussed with examples and assumes no inflexion point between the residual profit and the equilibrium price.

In order to illustrate this assumption with examples, let us define the different prices on the market with respect to the intervention of the CA. For a given Δ-type of firm, we can define four different equilibrium prices. We denote $p_A > c$ the price observed by the CA in case where the firm adopts the practice, where c is its constant marginal cost. $p_{NA} \in [c, p_A]$ is the equilibrium price without any practice, observed under deterrence but unknown by the CA during investigations as Δ. After some injunctions, the complete effort implies a price $p_I \in [p_{NA}, p_A]$, while commitments imply a price $p_C \in [p_I, p_A]$.

We assume that injunctions perfectly restore competition: $l_I(\Delta) = \Delta$ and $p_I = p_{NA}$. In addition, with the function $p(y, \Delta)$, we have $p(y, \Delta) = p_C \geqslant p_I = p(1, \Delta)$, and $p_C = p_I$ for $y = 1$.

[10] Decision of the CFIEC of Jul., 11th of 2007 in the case T-170/06 Alrosa.

Finally, we denote $f(p)$ the equilibrium residual profit of the firm associated to a counterfactual price p. With $l_C(y,\Delta) \leqslant l_I(\Delta)$, this residual profit is implicitly defined according to y and Δ:

$$f(p_C) = D(p_A)(p_A - c) - l_C(y,\Delta)$$

where $D(p)$ is the demand of the firm with the practice. $D(.)$ is assumed to be log-concave, which includes linear and isoelastic demand function.

To illustrate assumption 1, we detail in appendix how the profit of the firm is linked to the equilibrium price under four situations. We first present a Bertrand competition with a dominant firm who owns an essential facility and uses it to raise its rivals' marginal cost. This situation is also considered with a Cournot competition. Then we consider a Cournot competition with a dominant firm who has the same marginal cost as the potential entrants but restricts the number of active competitors. Finally, this framework may also deal with collusion models on market sharing or no-competition clause with symmetric firms and out of the dynamic dimension.

In all these cases, if Δ is not a common knowledge, there is a single source of private information concerning the counterfactual situation during an investigation. For example, the dominant firm privately knows the number of potential entrants or their degree of competitiveness. In the collusive illustration the counterfactual price is unknown. These elements lead to no inflexion point of $f(p)$ with respect to the equilibrium price p even though it may be concave or convex.

The following relation between p_I and Δ is verified under assumption 1:

$$\frac{\partial p_I}{\partial \Delta} = -\frac{l'_I(\Delta)}{f'(p_I)} < 0$$

Hence, when Δ increases, the equilibrium price after injunctions decreases.

Partial derivatives of p_C w.r.t. the arguments y and Δ are given by:

$$\frac{\partial p_C}{\partial \Delta} = -\frac{l'_{C\Delta}(y,\Delta)}{f'(p_C)} < 0,$$

and:

$$\frac{\partial p_C}{\partial y} = -\frac{l'_{Cy}(y,\Delta)}{f'(p_C)} < 0.$$

This two derivatives give us that the equilibrium price associated to commitments are decreasing with the effort to restore competition and with the firm's gain associated to the practice.

Finally, by definition, p_A is independent of y and Δ.

We solve the game backward, and first present the situation where the CA only uses trial.

3 Benchmark: the Only Trial Subgame

Under the (T) subgame, the firm only chooses between A and NA. Here, the resolution is quiet automatic.

The firm decides not to adopt the practice if and only if $\Pi^t \leqslant \Pi^d = 0$. This may be rewritten:

$$\Delta \leqslant \alpha\beta[l_I(\Delta) + F] \Leftrightarrow \Delta \leqslant \frac{\alpha\beta F}{1-\alpha\beta} \equiv \Delta^0$$

where Δ^0 increases with the elements of the expected fine $\alpha\beta F$. A firm that looses more from competition than from the expected trial sanction cannot be deterred.

In order to analyze a non trivial game, we make the following assumption concerning the deterrent effect of the trial intervention of the CA.[11]

Assumption 2. *We assume that all types of firm would not be deterred even if a preliminary investigation was performed with probability* $\alpha = 1$.

This assumption is equivalent to $\Delta^r \equiv \dfrac{\beta F}{1-\beta} < \overline{\Delta}$ or $F < \dfrac{1-\beta}{\beta}\overline{\Delta}$. Hence, given that $\alpha < 1$, Δ-types in $(\Delta^0, \Delta^r]$ are not deterred but have a negative expected gain in front of the authority: these types *regret* their decision once audited. Assumption 2 implies that all potential Δ-types exist as presented in figure 2.

Figure 2: Types of firm

Let us now present the expected consumers' surplus under this subgame. We denote $S(p)$ the consumers' surplus associated to a price p and defined as follows:

$$S(p) = \int_p^{\infty} D(s)ds$$

where $p \in \{p_A, p_{NA}, p_I\}$. The two latter are assumed to be equal and decreasing with Δ.

For a given Δ, we denote $cs^t(\Delta)$ the expected consumers' surplus:

$$cs^t(\Delta) = \begin{cases} S(p_I) & \text{if } \Delta \leqslant \Delta^0 \\ S(p_A) + \alpha\beta[S(p_I) - S(p_A)] & \text{otherwise} \end{cases}$$

Unless if the firm is deterred, benefits of openness are weighted by the probability of guiltiness in trial $\alpha\beta$. With $S(p_A) < S(p_I)$, the CA would like to set

[11] If Δ is common knowledge, the competition authority can decide an adjusted level of fine. In such a case, $F(\Delta) = \frac{1-\alpha\beta}{\alpha\beta}\Delta$ is sufficient to deter each Δ-type. Under asymmetric information, F cannot be a function of Δ, but the authority can deter all Δ-types if and only if $F \geqslant \frac{1-\alpha\beta}{\alpha\beta}\overline{\Delta}$. We do analyze a game where some types cannot be deterred while using only the trial as a tool of intervention. This is true, for example, if the fine is legally bounded. A more detailed analysis with an increasing fine follows in section 5.

$\Delta^0 = \overline{\Delta}$, which is not feasible under assumption 2 and leaves some place to an interest to introduce the commitments procedure.

When the CA does not observe Δ, the weighted consumers' surplus, denoted CS^t, is given with respect to $\phi(\Delta)$ as follows:

$$CS^t = S(p_A) + \int_0^{\Delta^0} [S(p_I) - S(p_A)]d\Phi(\Delta) + \alpha\beta \int_{\Delta^0}^{\overline{\Delta}} [S(p_I) - S(p_A)]d\Phi(\Delta)$$

Under the first option (T), the weighted consumers' surplus is simply obtained with respect to the firm's decision to adopt or not the practice. In this context, the information on Δ has no particular implication on the consumers' surplus: the CA increases F as much as it can, with $\Delta^0 < \overline{\Delta}$ if complete deterrence is not feasible.

4 Introduction of the Commitments Procedure

Under the $(C+T)$ subgame, where the commitments procedure is an option for the firm prior to formal prosecutions. Here, the firm takes two decisions: A or NA, before a possible audit, and, once audited, to propose commitments or to go to trial.

The trial sanction constitutes the threatening point in negotiations. We will see that assumption 2 gives some potential interest for the CA to announce $y < 1$, whether Δ is commonly known or not.

The information structure on Δ now matters. Indeed, under symmetric information, the CA will be able to announce differentiated efforts according to the value of Δ or even a given subgame for a given Δ-type. When the firm's gain is its private information, the CA can only announce a single effort in the $(C+T)$ subgame or only use trial.

In order to present the firm's decision we denote Π^i_α the expected profits of a firm once it is audited, where $i \in \{d, t, c\}$. They are calculated setting $\alpha = 1$ in Π^i.

We present the equilibrium of the game when Δ is commonly known as a reference frame.

4.1 The Equilibrium under Symmetric Information

As argued before, Δ is a common knowledge if the CA knows the counterfactual situation on the market. In order to justify such a framework, it is possible to consider that the CA is able to determine this element during the investigation. In such a case, the initial announcement of the CA consists in a given y or subgame for a given Δ such as discovered during the audit.

We first suppose that the CA announces the incentive compatible commitments to each Δ-type, and then present the selection of cases that maximizes the consumers' surplus.

4.1.1 Firm's Decisions

An audited firm chooses the commitments procedure rather than trial if and only if $\Pi^t_\alpha \leqslant \Pi^c_\alpha$, which can be rewritten as follows:

$$\Delta - \beta[l_I(\Delta) + F] \leqslant \Delta - l_C(y, \Delta) \Leftrightarrow l_C(y, \Delta) \leqslant \beta[l_I(\Delta) + F],$$

and we denote $\widehat{y}$ the largest effort that make a firm of type Δ indifferent between these two procedures.

However, in order to save the fine, we will find that some Δ-types accept to propose commitments even more constraining than injunctions. Therefore, we need to define $\widehat{y}$ by the following equality:

$$l_C(\widehat{y}, \Delta) = \min\{l_I(\Delta), \beta[l_I(\Delta) + F]\}$$

We first present the main properties of $\widehat{y}$ in the following lemma which is demonstrated in appendix.

Lemma 1. *The incentive compatible effort $\widehat{y}$ is such that:*

1. $\widehat{y} = 1$ *if* $\Delta \leqslant \Delta^r$ *and* $\widehat{y} \in (0, 1)$ *otherwise,*
2. $l_C(\widehat{y}, \Delta) > \beta l_I(\Delta)$*: any type of firm proposes commitments with an higher loss on the market than the one expected with injunctions, and*
3. $l_C(\widehat{y}, \Delta)$ *increases everywhere in* Δ*.*

First, $l_I(\Delta) < \beta[l_I(\Delta) + F]$ if $\Delta \leqslant \Delta^r$: for these types, Δ does not worth the expected trial sanction so that the CA is able to ask for commitments to a complete effort. Otherwise, the firm has a positive expected gain in trial and a partial effort is necessary for commitments to be more attractive than the possible dismissal in trial. This is summarized in figure 3.

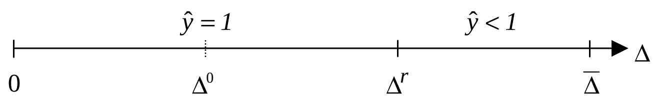

Figure 3: $\widehat{y}$ and thresholds of Δ

The second part of lemma 1 directly comes from the definition of $l_C(\widehat{y}, \Delta)$, where $\beta[l_I(\Delta) + F] > \beta l_I(\Delta)$. Intuitively, the firm avoids the trial fine and accept to propose commitments that implies a larger loss on the market than expected injunctions.

Finally, while it comes straightforward that $l_C(\widehat{y}, \Delta)$ is increasing in Δ if $\widehat{y} = 1$, we find that it is also true when incentive compatibility ask for smaller and smaller efforts (for $\Delta > \Delta^r$): the firm's loss with commitments increases faster than the one with the expected injunctions.

Note that we assume that the indifference of a type leads to a commitments proposal. This assumption eliminates a possible mixed strategy of a type. Under symmetric information, this corresponds to a simplification but does not change results when y is constrained by one. This is discussed with the authority's decision.

We now consider the initial arbitration of the firm concerning the adoption of the practice.

Proposition 1. *When the CA proposes the commitments procedure to any type of firm, the deterrence effect of the trial fine collapses.*

A systematic use of the commitments procedure has for immediate consequence the disappearance of the trial fine for the firm: for any $y \in [0, 1]$, $\Pi^c = \Delta - \alpha l_C(y, \Delta) > \Pi^d = 0$. Any Δ-type adopts a *wait-and-see strategy*, since it might not be audited (with probability $1 - \alpha$) and runs at most the same opportunity cost as under the NA decision.[12]

With this decisions of the Δ-types in mind, we can analyze the decision of the competition authority.

4.1.2 CA's Decisions

We determine in this subsection the cases that should be treated with the commitments procedure in order to maximize the consumers'surplus.

When the CA proposes the largest incentive compatible commitments to any Δ-type, the expected consumers' surplus for a given Δ is written as follows:

$$cs^c(\Delta) = S(p_A) + \alpha[S(\widehat{p}_C) - S(p_A)]$$

where $\widehat{p}_C \equiv p(\widehat{y}, \Delta) \leqslant p_I$, with $p(\widehat{y}, \Delta)$ the competitive price associated to the incentive compatible effort for a type Δ. This surplus consists in the consumers' surplus observed with the practice added to the one in case of an audit where commitments are used.

The optimal selection of cases is defined in the following proposition.

Proposition 2. *Under symmetric information, the competition authority never introduces the commitments procedure if the trial threat is sufficient to obtain the deterrence of the firm ($\Delta \leqslant \Delta^0$). For larger types of firm, the CA never uses trial whenever the firm accept to commit to a complete effort ($\Delta \in (\Delta^0, \Delta^r]$). Finally, depending on the convexity of the* residual profit, *commitments with a too partial effort may push for trial against largest Δ-types ($\Delta > \Delta^*$).*

Figure 4 summarizes these results.[13]

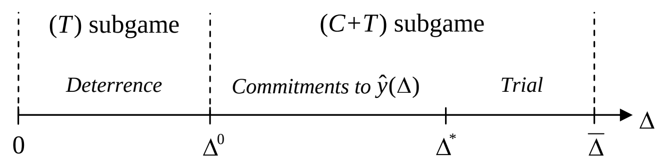

Figure 4: The equilibrium under symmetric information

Intuitively, the CA prefers the trial to the commitments procedure to the trial if the former would induce the deterrence of the practice.[14]

In addition, if a type of firm accepts to propose a complete effort in commitments as opposed to an expected injunction, it clearly represents a efficiency gain to consumers if $\Delta > \Delta^0$.

[12]The term *wait-and-see strategy* refers to Fenn and Veljanovski (1988), where the authority uses either sanction or monitoring.

[13]Note that as opposed to Shavell (1989), there might be some trial in equilibrium.

[14]If such an announcement of the CA is not credible during the audit, it comes immediately that the CA accepts any proposition $\widehat{y} = 1$ of types $\Delta \leqslant \Delta^0$, which brings us back to proposition 1. This will be discussed in the following section.

Finally, a decrease of $\widehat{y}$ on the set of types that do not regret their initial decision does not necessarily imply that trial should be chosen. More precisely, we show in appendix that the authority always negotiate with larger Δ-types next to Δ^r. However, we also find that there may exist a marginal type, denoted Δ^*, above which the negotiation would be harmful to consumers. Such a type exists if the equilibrium residual profit $f(p)$ is not enough concave or convex.[15]

Back to the examples given with the discussion of assumption 1, this may happen with a Cournot competition where there exists a dominant firm. On the opposite, negotiation is only refused for small practices in a Bertrand competition or a collusion case with symmetric firms.

Finally, the assumption of a pure strategy of a commitments proposal by an indifferent firm under the the $(C+T)$ subgame does not affect the equilibrium described in proposition 1. Indeed, for $\Delta < \Delta^r$, $\widehat{y} = 1$, and the firm strictly prefers the commitments procedure. For $\Delta \geqslant \Delta^r$, the CA would have a strict interest to announce $\widehat{y} - \epsilon$ for these types, with $\epsilon > 0$ sufficiently next to zero, in order to obtain their certain proposal.

4.1.3 Commitments with a Fine Increasing in Δ

We now discuss the fact that the trial fine F could depend directly or indirectly on the level of Δ in this symmetric information game. We denote $F(\Delta)$ the fine incurred by a type Δ and assume that $F'(\Delta) > 0$, so that the authority chooses a larger fine for a larger gain of the firm or damage to consumers.[16]

Note that in front of the French Competition Authority, the economic damage is generally introduced in the level of fines. The European Commission has established some guidelines where the determination of the fine is a function of parameters such as the type of practices but the CA the final fine amount may be raised to its "deterring value". However, at the end, the fine increases with Δ as long as it cannot exceed 10 % of the total sales: $F'(\Delta) > 0$ whenever this cap is reached for some Δ-types.

The following proposition presents the consequences of such a practice of the CA.

Proposition 3. *As long as complete deterrence can not be achieved with the trial, the authority introduces the commitments procedure in some or any of the cases where deterrence is not achievable initially.*

Proposition 3 formally establishes the same results as proposition 2. Specifically, we find that the commitments procedure is even more interesting for consumers when $F(\Delta)$ increases less faster than Δ starting from its value F considered in the main model.

Intuitively, types of firm that regret their decision to adopt the practice accept commitments to a complete effort, and higher Δ-types accept to propose commitments where the incentive compatible effort decreases less rapidly than in

[15] More specifically, if $f(p)$ is as much concave as the profit with the practice, $D(p)(p-c)$, for any $p \in [c, p_A]$, then the consumers gain with negotiated commitments increases for any $\Delta > \Delta^0$. Otherwise, there may exist a marginal type $\Delta^* > \Delta^r$ above which trial is chosen by the authority.

[16] Under asymmetric information, F could be for example a function of the mean value of Δ, depending on $\phi(.)$. However, in such a case, there exists by definition a unique amount of fine, equivalent to F.

the previous model. Starting from the previous game, Δ^0, Δ^r and Δ^* increase: commitments are less probable for a given $\overline{\Delta}$, but have more effect on the market.

4.2 The Equilibrium under Asymmetric Information

When the CA does not observe Δ, it cannot associate a given subgame to a given Δ-type. In addition, it can only announce a *unique* effort, denoted $\widetilde{y}$, in the $(C+T)$ subgame or choose the (T) one.

It immediately appears that the set of $\widetilde{y}$ is made of the possible values of $\widehat{y}$: $\widetilde{y}$ is bounded by one and the CA announces at least an effort which let indifferent the type $\overline{\Delta}$, denoted y_{min}, such that all Δ-types accept to commit. More precisely, it will never announce any effort $\widetilde{y} < \widehat{y}(\Delta^*)$ that would incite a type $\Delta > \Delta^*$ to propose commitments while they are harmful to consumers.

For each of these possible values of $\widetilde{y}$ there will be a unique Δ-type indifferent between committing and trial, denoted $\widetilde{\Delta} \in [\Delta^r, \min\{\Delta^*, \overline{\Delta}\}]$ and we assume its commitments proposal.[17]

Finally, note that in this information structure, deterrence is lost when the commitments procedure is introduced, for the reasons presented in Proposition 1, but that this now is a definitive result.

We limit the exposure to the main differences with the prior section and compare the outcome of the $(C+T)$ subgame to the one described in section 3.

4.2.1 Firm's Decisions

For an announced $\widetilde{y}$, the firm's expected gain if it chooses the practice and to propose commitments is $\Pi^c = \Delta - \alpha l_C(\widetilde{y}, \Delta) > 0$.

Once audited, a given Δ-type chooses the commitments procedure rather than trial if and only if:

$$\Pi_\alpha^t \leqslant \Pi_\alpha^c \Leftrightarrow l_C(\widetilde{y}, \Delta) \leqslant \beta[l_I(\Delta) + F]$$

Note that only the marginal type is indifferent between commitments and trial:

$$l_C(\widetilde{y}, \widetilde{\Delta}) = \min\{l_I(\widetilde{\Delta}), \beta[l_I(\Delta) + F]\},$$

which means that $l_C(\widetilde{y}, \widetilde{\Delta})$ has the same properties as $l_C(\widehat{y}, \widetilde{\Delta})$. Hence, $\widetilde{\Delta} = \Delta^r$ if and only if $y = 1$: only deterred firms and regretting ones in the (T) subgame will accept to commit. In addition, with $\widetilde{\Delta} = \overline{\Delta}$ for $y = y_{min}$, the trial probability is null for this level of effort.

Note also that the derivative of $\widetilde{\Delta}$ w.r.t. $\widetilde{y}$ can be shown as negative according to lemma 1. It is given by the following expression:

$$\frac{\partial \widetilde{\Delta}}{\partial \widetilde{y}} = -\frac{l'_{Cy}(\widetilde{y}, \widetilde{\Delta})}{l'_{C\Delta}(\widetilde{y}, \widetilde{\Delta}) - \beta l'_I(\widetilde{\Delta})} < 0$$

Intuitively, as the unique requested effort decreases from one, more Δ-types accept to commit.

[17] Under asymmetric information, this assumption is neutral given that we consider a continuum of types.

Note that the optimal policy under symmetric information is not incentive compatible for a given reduction of the fine (here an immunity). A firm benefits from an informational rent if $\Delta < \widetilde{\Delta}$: it would have no incentive to reveal its real Δ if it could, because it would either be proposed more constraining commitments (for $\Delta \in (\Delta^0, \widetilde{\Delta})$) or would be refused commitments if such an announcement would be credible (for $\Delta \leqslant \Delta^0$). Firms for which $\widetilde{\Delta} < \Delta < \Delta^*$ would have interest to reveal it, because would obtain a certain acceptance of commitments to a lower effort.

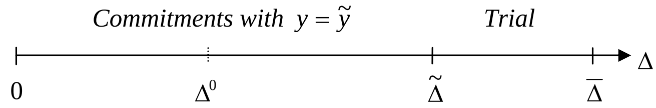

Figure 5: Repartition of types of firm under asymmetric information

Figure 5 summarizes the partial antiselection result that appears: firms with a large Δ are more likely to refuse to commit. However, we will see that the insurance effect associated to Δ-types in $(\Delta^0, \widetilde{\Delta}]$ may overrule the loss of the deterrence of types $\Delta \leqslant \Delta^0$.

4.2.2 CA's Decisions

In this section we provide a sufficient condition under which the introduction of the commitments procedure is beneficial to consumers. In addition, we propose to highlight some elements that increase the trial probability when commitments offer a complete reduction of the fine.

For given Δ and announced $\widetilde{y}$, the expected consumers' surplus is written as follows:

$$cs^c(\widetilde{y}, \Delta) = \begin{cases} S(p_A) + \alpha[S(p_C) - S(p_A)] & \text{if } \Delta \leqslant \widetilde{\Delta} \\ \\ S(p_A) + \alpha\beta[S(p_I) - S(p_A)] & \text{otherwise} \end{cases}$$

where $p_C = p(\widetilde{y}, \Delta)$ is the price associated to a given effort $\widetilde{y}$ and any type Δ. Here, a firm for which $\Delta \leqslant \widetilde{\Delta}$ propose commitments and the complementary set, which is decreasing in $\widetilde{y}$, chooses trial. For these latter, the (T) and $(C+T)$ subgames are equivalent in both the firm and CA points of view.

We denote $CS^c(\widetilde{y})$ the weighted consumers' surplus for a given $\widetilde{y}$:

$$CS^c = S(p_A) + \alpha \int_0^{\widetilde{\Delta}} [S(p_C) - S(p_A)] d\Phi(\Delta) + \alpha\beta \int_{\widetilde{\Delta}}^{\overline{\Delta}} [S(p_I) - S(p_A)] d\Phi(\Delta)$$

The derivative of this function is presented in appendix. First, if the CA announces $\widetilde{y} = 1$ it minimizes the loss associated to the fact that deterrence collapses. In addition, with such an effort it concedes no rent to the Δ-types $(\Delta^0, \Delta^r]$. However, reducing $\widetilde{y}$ induces some proposition by types of firm for which the treatment by trial is a shortfall.[18]

[18] Indeed, for the marginal type $\widetilde{\Delta} \in]\Delta^r, \Delta^*[$, $\widetilde{p}_C = p(\widetilde{y}, \widetilde{\Delta})$ is equal to $\widehat{p}_C = p(\widehat{y}, \widetilde{\Delta})$, so that $S(\widetilde{p}_C) - S(p_A) > \beta[S(\widetilde{p}_I) - S(p_A)]$, where $\widetilde{p}_I$ is the price associated to injunctions.

We provide in the following proposition a simple condition under which the commitments procedure enhances welfare.

Proposition 4. *Under a uniform distribution of Δ-types, the introduction of the commitments procedure is always optimal with a complete effort ($\widetilde{y} = 1$). The form of $l_C(y, \Delta)$ determines whether it is the optimum of the $(C + T)$ subgame.*

Whenever all types of firm are equiprobable, there exists at least one introduction of the commitments procedure that enhances the consumers' surplus. Indeed, we find that the announcement of commitments as constraining as injunctions reduces enough the loss presented above.

In addition, setting $l_C(y, \Delta) = y\Delta$, we find that the optimal requested effort implies that the trial probability is the same as under symmetric information: $\widetilde{\Delta}^* = \min\{\overline{\Delta}, \Delta^*\}$. More generally, we find that this is true whenever a given partial effort has as more effect on the market as Δ increases. Otherwise, the trial probability remains higher than under the symmetric information case: $\widetilde{y}^* \in]\max\{y_{min}, \widehat{y}(\Delta^*)\}, 1]$.

However, for any form of $l_C(y, \Delta)$, commitments are not always optimal when initially deterred types are frequent: the interest of the introduction of the commitments is clearly subjected to the distribution of Δ-types.

Figure 6 provides an illustration of the optimal probability of trial during the audit, when Δ follows a truncated normal distribution with $\mu = 0$, $l_C(y, \Delta) = y\Delta$ and $D(p) = 1 - p$.[19]

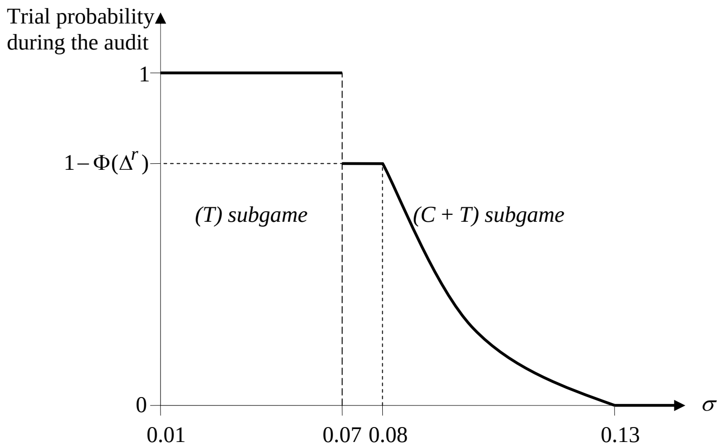

Figure 6: Optimal use of commitments under a normal distribution

Note that an increase of the standard deviation, denoted σ, represents an increase of the frequency of the highest Δ-types. The optimal probability of trial illustrates the decisions of the CA both concerning the subgame and the requested effort.

[19] This example is detailed in the appendix. We assume here the Bertrand competition case with a dominant firm with a null marginal cost and raising its rivals' marginal cost. Hence, $\overline{\Delta} = 0.25$ (monopoly profit), and we assume $\alpha = \beta = 0.5$ with $F = \frac{3}{4}\frac{1-\beta}{\beta}\overline{\Delta}$. With These settings, $\Delta^0 = 0.041$ and $\Delta^r = 0.125$.

The CA never introduces the commitments procedure if the probability of the lowest Δ-types is two important (for $\sigma < 0.07$). Above this value, the CA first announce an effort equivalent to the one associated to injunctions in order to minimize informational rents, so that the trial probability equals $1 - \Phi(\Delta^r)$.

Above $\sigma = 0.08$, the CA reduces the requested effort so that to obtain more frequent proposals: the frequency of intermediary Δ-types becomes sufficient for this marginal loss to be offset by the gain of an additional commitments application. Finally, the trial probability becomes null for $\sigma \geqslant 0.13$.

In this section, we have presented a model in which the competition authority accepts that commitments contains a partial effort of openness and offers an immunity of fine. In symmetric information, we have seen that the firm accept a loss on the market larger than the expected one associated to injunctions. Hence, under symmetric information, commitments enhance the consumers' surplus in most serious cases if consumers' harm and firm's gain are positively correlated. Under asymmetric information, the CA might be better off asking for complete effort in order to compensate the deterrence loss and avoiding too frequent informational rents. However, under a uniform distribution, procedural efficiency dominates such a disadvantage of the negotiation.

What should be reminded here is that the interest of the commitments procedure mainely depends on the distribution of the types of firms. The following extensions allow us to discuss this result.

5 Extensions

We develop in this section two analysis concerning the assumptions on the degree of freedom of the CA and also on the divisible aspect of commitments. In particular, we present the effects of a partial reduction of the fine in the commitments procedure when Δ is a private information. We show that the distribution of types does not influence the interest of the commitments procedure when $l_C(y, \Delta) = y\Delta$: the (T) subgame is strictly dominated.

Finally, relaxing assumption 1, we propose an analysis where commitments are applied on a subset of the market, and characterize the optimal decision of the CA with different correlations between the firm's gain and the consumers' harm.

5.1 Partial Reduction of the Fine

The commitments procedure that we have studied may appear like a constrained tool in the hand of the CA: it offers an immunity of fine when the firm accepts to commit. Intuitively, if the CA has an additional variable, it can at least do as well as with the only effort element.

For simplification purpose, we assume here that $l_C(y, \Delta) = y\Delta$. In addition, we only present formally the game under asymmetric information given that results of proposition 2 remains identical when Δ is common knowledge: the CA offers a complete reduction of the fine with commitments if $\Delta > \Delta^0$, and chooses the only trial subgame otherwise. In this section we show that, coupled with a partial reduction of the fine, it is optimal for the CA to always choose the $(C + T)$ subgame.[20]

[20] With a partial reduction of the fine, the CA is juridically bounded to establish a formal

Here, the CA announces a unique level of effort, denoted $y \in [0,1]$, and the part of the trial fine F that remains in the commitments procedure, denoted $z \in [0,1]$.

We denote Π_z^c the expected gain of a firm that chooses the practice and the commitments procedure once it is audited:

$$\Pi_z^c = \Delta - \alpha(y\Delta + zF)$$

Note that y and z are *substitutable* tools in the sense that, for a given Δ-type, a reduction of z allows for an increase of y without modifying its incentive to commit.

For given y and z, a Δ-type will choose the commitments procedure once audited if and only if:

$$\Pi_{z|\alpha}^c > \Pi_\alpha^t \Leftrightarrow \Delta < \frac{\beta - z}{y - \beta} F \equiv \Delta^c(y,z)$$

where $\Pi_{z|\alpha}^c$ is calculated as Π_z^c setting $\alpha = 1$. $\Delta^c(y,z)$ is the marginal type that is indifferent between commitments and trial for given values of y and z. We will see that in equilibrium, $y \geqslant \beta$, and treat the equality as a special case.

If the set of refusing firms is reduced *a priori*, we also find that a partial reduction of the fine will maintain a certain degree of deterrence for small enough values of Δ. Indeed, some types will prefer not to adopt the practice when the associated gain does not even worth a reduced fine:

$$\Pi_z^c < \Pi^d \Leftrightarrow \Delta < \frac{\alpha z F}{1 - \alpha y} = \Delta^d(y,z)$$

where $\Delta^d(y,z)$ is an increasing function of y and z. Note that this level of deterrence cannot be larger than Δ^0 given that the firm can always choose trial. Indeed, if $\Delta^d(y,z) > \Delta^0$, it is also that $\Delta^c(y,z)$ is smaller than $\Delta^d(y,z)$, which means that no type of firm chooses the commitments procedure.

In figure 7, the hashed zones represents couples that incite some Δ-types to commit, under the constraint that the CA does not accept too poor commitments for a given fine reduction.

Segment $[A]$ is constituted of (y,z) couples for which all types that are not deterred accept to commit. Indeed, for these values, $\Delta^c(y,z) = \overline{\Delta}$ and $\Delta^d(y,z)$ belongs to $[0, \Delta^0]$.[21] (y,z) couples on the left of segment $[A]$ imply for the CA an unnecessary loss of deterrence for a given effort, or a too small effort for a given z.

Segment $[B]$ is constituted of (y,z) couples such that the marginal Δ-type that is indifferent between trial and commitments is also the marginal deterred type.[22] By definition, these couples are such that a slight reduction of y or z insures a positive probability of an application of the commitments procedure

decision that justifies of the conviction of the defendant. However, if the firm accepts to propose commitments, and to pay some part of the trial fine, it should cooperate in the investigation phase. We assume that such a practice is possible for the CA in the commitments procedure, and let to future research the analysis of the no-contest procedure such as used in front of the French competition authority or *plea agreements* in front of the US Department of Justice.

[21] This is achieved setting $z \leqslant \beta - (y - \beta)\frac{\overline{\Delta}}{F}$.

[22] These couples are defined by the equality $\Delta^c(y,z) = \Delta^d(y,z) \Leftrightarrow z = \frac{(1-\alpha y)\beta}{1-\alpha\beta}$. Note that the marginal type of these announcement is always Δ^0. For $z = 0$, any $y \leqslant 1$ implies a total

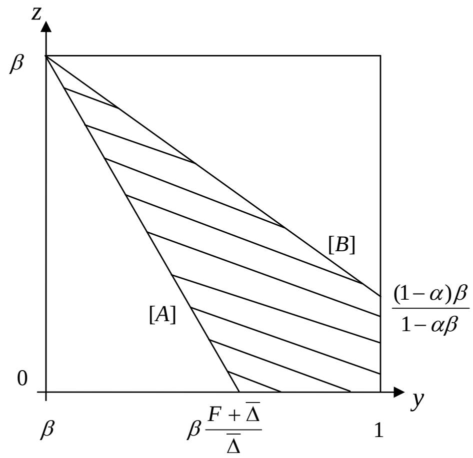

Figure 7: Incentive compatible values of y and z

(because of the continuum of types): segment $[B]$ represents announcements that are formally equivalent to the (T) subgame. Couples on the right of this segment lead to the choice of the trial by all undeterred Δ-types.

To determine the CA's decision, we denote CS_z^c the weighted consumers' surplus associated to the commitments introduction:

$$CS_z^c = S(p_A) + \int_0^{\Delta^d(y,z)} [S(p_I) - S(p_A)]d\Phi(\Delta) + \alpha \int_{\Delta^d(y,z)}^{\Delta^c(y,z)} [S(p_C) - S(p_A)]d\Phi(\Delta)$$
$$+\alpha\beta \int_{\Delta^c(y,z)}^{\overline{\Delta}} [S(p_I) - S(p_A)]d\Phi(\Delta)$$

Here, deterrence is chosen by the smallest Δ-types, while larger ones propose commitments and finally trial. For some (y, z) couples, it now appears that the probability of trial may be null and the loss of deterrence significantly reduced.

We first analyze the particular agreement point $(y, z) = (\beta, \beta)$. This is the one that implies $\Pi_z^c = \Pi^t$ for any Δ (so that $\Delta^c(y, z)$ is undetermined). This indifference means that deterrence is obtained if and only if $\Delta \leqslant \Delta^0$ as in the (T) subgame. Figure 8 represents the Δ-types' decisions for this precise couple.

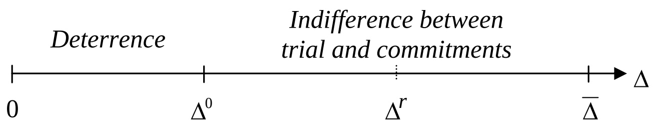

Figure 8: Types' decisions when $y = z = \beta$

The following proposition concerns the interest of an introduction of the commitments procedure that strictly incite some Δ-types to make a proposal.

loss of deterrence as stated in the main model, so that none of these couples is on segment $[B]$.

Proposition 5. *Under any distribution of the types of firms and any form of the residual profits, there exists an introduction of the commitments procedure coupled with a partial reduction of the fine that enhances the consumers' surplus. Moreover, several couples (y, z) insure this result.*

With a sufficiently concave equilibrium residual profit, starting from the couple (β, β) and proposing an additional fine reduction of $\epsilon > 0$, sufficiently next to zero, leads all the types of firm to propose commitments on the set $[\Delta^0, \overline{\Delta}]$ but an insignificant loss of deterrence. The CA remains on segment $[A]$ with a slight increase of y, presented in appendix, given that setting $y = \beta$ would be suboptimal. The demonstration of the concavity of $S(p_C)$ is necessary in such a case.

Even though the couple (β, β) may harm consumers for insufficiently concave or convex residual profit, we find other candidate along the segment $[B]$, implying some trial in expectation.

The following proposition characterizes partially the optimal couples for some particular distributions of the Δ-types.

Proposition 6. *If the weighted gain of the firm, denoted $\Delta\phi(\Delta)$, increases over $[\Delta^d(y, z), \overline{\Delta}]$, the optimal announcement minimizes the trial probability for a given z.*

Here, the optimal couple (y^*, z^*) is on the segment $[A]$ if $f(p)$ is sufficiently concave: there is some deterrence for $z > 0$, so that the insurance against important type-II errors is obtained at a smaller cost.[23] For different form of the residual profit, trial has a positive probability of occurrence.

Hence, by comparison with the introduction of the commitments procedure that offers automatically an immunity of fine, we find that this procedure should always be introduced. The distribution of Δ-types then characterizes its optimal effective frequency of use.

5.2 Commitments on a Subset of the Market

We develop here an analysis where commitments ask for a complete effort of the firm but applied on a subset of the market. The latter is assumed to be divisible and uniformly valued by the firm. We denote $\widetilde{y} \in [0, 1]$ the extent of commitments on the market, and assume that injunctions would concern the whole market. Hence, two prices appear on the market with the commitments procedure whenever $\widetilde{y} < 1$.

We relax here assumption 1 that concerns the link between a marginal increase of the firm's gain with the practice and the consumers' harm. We treat implicitly of potential efficiency gains that could be partially passed on to consumers with the practice. Alternatively, consumers may not only take care of the price but also services that may be developed with the practice. Here, the practice harms consumers but we do not need of a strict correlation. We only analyze the asymmetric information game.[24]

[23]Simulations with a uniform distribution suggest that a complete reduction of fine is often optimal in the case illustrated in appendix.

[24]Under symmetric information, the CA chooses the (T) subgame if and only if $\Delta \leqslant \Delta^0$. Otherwise, commitments are applied to the whole market if $\Delta \in (\Delta^0, \Delta^r]$ and to a part of it for larger Δ-types.

With trial, the expected gain of the firm is unchanged: $\Pi^t = \Delta - \alpha\beta(\Delta + F)$. Under the $(C+T)$ subgame, the expected gain of a firm that adopts the practice and proposes commitments is $\Pi^c = \Delta - \alpha\widetilde{y}\Delta$, where $\widetilde{y} \in [0,1]$ is the part of the market where the firm ends the practice.

While Δ^0 and Δ^r are left unchanged, we now have an explicit definition of the Δ-type that is indifferent between trial and commitments once audited and for a given $\widetilde{y}$:

$$\widetilde{\Delta} = \frac{\beta F}{\widetilde{y} - \beta}$$

Hence, we find that $\widetilde{y} \in [y_{min}, 1]$, where $y_{min} = \beta(\overline{\Delta} + F)/\overline{\Delta} > \beta$, so that $\widetilde{\Delta} \in [\Delta^r, \overline{\Delta}]$.

For a given Δ, commitments now imply an expected consumers' surplus:

$$cs^c(\widetilde{y}, \Delta) = S_A + \alpha\widetilde{y}\,[S_I - S_A]$$

where S_A and S_I are the surplus of consumers with and without the practice. We analyze different link between Δ and S_I.

The weighted consumers' surplus for a given $\widetilde{y}$ can be written as follows:

$$CS^c(\widetilde{y}) = S_A + \alpha \int_0^{\widetilde{\Delta}} h(\widetilde{y}, \Delta) d\Delta + \alpha\beta \int_{\widetilde{\Delta}}^{\overline{\Delta}} h(\Delta) d\Delta$$

where $h(\Delta) = (S_I - S_A)\phi(\Delta)$ is the initial harm to consumers, weighted by the density of Δ, that is "completely repaired" after injunctions. $h(\widetilde{y}, \Delta) = \widetilde{y}h(\Delta)$ is the part of the weighted harm repaired with the commitments procedure. Note that $h(\widetilde{y}, \Delta)$ increases with $\widetilde{y}$.

The following proposition enumerates the optimal announcement of the CA for different shapes of $h(\Delta)$. We restrict the analysis to four "polar" forms of $h(\Delta)$ in order to simplify the exposure.

Proposition 7. *With commitments to a complete effort on a subset $\widetilde{y} \in [0,1]$ of the market, we find that:*

i) the monotonicity of $h(.)$ over $[0, \overline{\Delta}]$ implies that:

1. *the commitments procedure is always introduced and effectively used ($\widetilde{y}^* = y_{min}$) if $h(.)$ is increasing, and that*

2. *the commitments procedure is never introduced if $h(.)$ is decreasing.*

ii) the non-monotonicity of $h(.)$ over $[0, \overline{\Delta}]$ may incite the CA to ask for commitments on the whole the market. More particularly, assuming $h(0) = h(\overline{\Delta})$ and $h'(\Delta^r)$, we find that:

1. *if the highest weighted harm is interior, $\widetilde{y}^*$ might be interior, and the commitments procedure should be effectively introduced if the maximum of $h(.)$ is associated to a sufficiently large value of Δ. Moreover,*

2. *if the smallest weighted harm is interior, the CA chooses $\widetilde{y}^* = 1$ or $\widetilde{y}^* = y_{min}$, but the $(C+T)$ subgame is effectively chosen if the minimum of $h(.)$ is associated to a Δ sufficiently smaller than Δ^r.*

Concerning the part i) of proposition 8, intuitions are given with $\phi(.)$ uniform. If the firm's gain is strictly correlated with the consumer's harm, the deterrence loss of the smallest Δ-types is always dominated by the insurance effects of the commitments procedure in largest cases. Hence, the probability of trial is null.

On the opposite, if the consumers' harm is reduced with an increase of the firm's gain, for example with some efficiency gains that are partially passed on to consumers, the CA never uses the commitments procedure in order to maintain the deterrence of most inefficient practices.[25]

Part ii) 1 of proposition 8 concerns a strictly concave $h(\Delta)$ with a unique interior maximum. As it is presented in figure 9.a) and b), $CS^c(\widetilde{y})$ is also concave in $\widetilde{y}$.

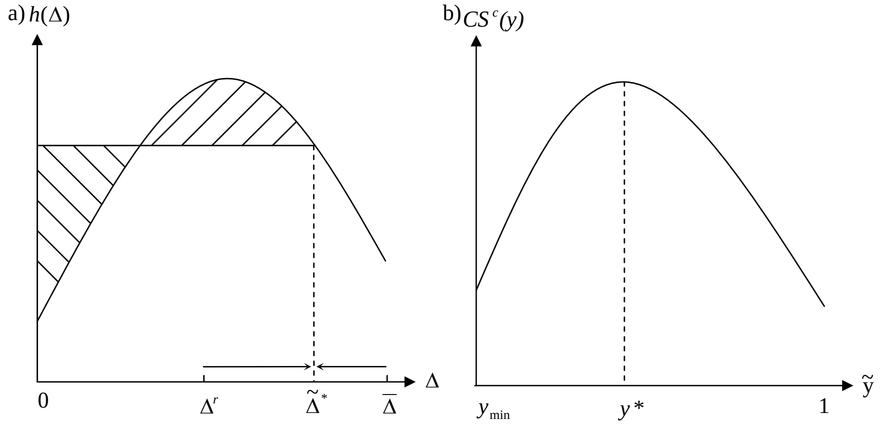

Figure 9: Weighted harm and optimal $\widetilde{\Delta}$ when $h(.)$ is concave

The $(C+T)$ subgame is chosen if the set of $\Delta \in [0, \overline{\Delta}]$ over which $h(\Delta)$ decreases is not too large. Moreover, trial may remain an active procedure, with $\widetilde{y}^* > y_{min}$ when intermediary values of Δ are frequent enough or largest ones not too harmful to consumers.

Formally, the commitments procedure enhances the consumers surplus if there exists a value of $\widetilde{y} \in [y_{min}, 1]$ such that:

$$\alpha \int_0^{\widetilde{\Delta}} \left[h(\Delta) - h(\widetilde{\Delta}) \right] d\Delta = 0$$

Note that an exclusive use of trial might be optimal if this last equation defines a value of $\widetilde{y}$ strictly larger than 1, meaning that $h(\Delta)$ decreases on a too large set of $\Delta \in [0, \overline{\Delta}]$.

Finally, part ii) 2 of proposition 8 concerns a strictly convex $h(\Delta)$ with a unique interior minimum, as shown in figure 10, implying the convexity $CS^c(\widetilde{y})$. The $(C+T)$ subgame is chosen if the set of $\Delta \in [0, \overline{\Delta}]$ over which $h(\Delta)$ increases is sufficiently large.

[25]Note that under assumption 1, with $S(.)$ being a decreasing function of p, the distribution function of Δ may imply these shapes of $h(\Delta)$. The competition authority announces the $(C+T)$ subgame with $\widetilde{y}^* = y_{min}$ or the (T) option.

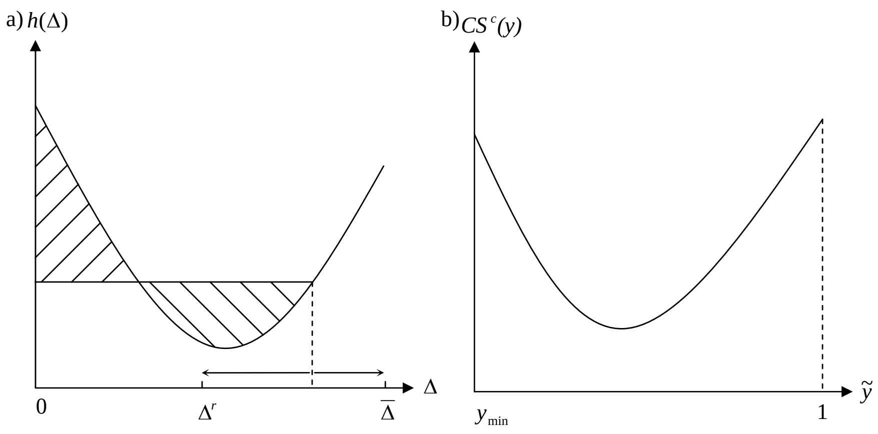

Figure 10: Weighted harm and optimal $\widetilde{\Delta}$ when $h(.)$ is convex

Here, the largest weighted harm are associated to the largest and smallest Δ. Under the $(C+T)$ subgame, the CA decides between $\widetilde{y}^* = 1$ and $\widetilde{y}^* = y_{min}$. However, these two local optima may be dominated by the (T) subgame when the minimum of $h(\Delta)$ is reached for a Δ larger than Δ^r. Otherwise, the sooner this minimum the more likely the $(C+T)$ option is preferred, and $\widetilde{y}^* = y_{min}$ rather than $\widetilde{y}^* = 1$.

Hence, proposing an extension where commitments are used on a subset of the market and with different correlations between the consumers' harm and the firm's gain, we find more variate results concerning the optimal strategy of the CA. Particularly, under asymmetric information and a uniform distribution, the CA may choose not to accept any commitments when efficiency gains are at play, or may optimally accept only very constraining ones.

6 Conclusion

In this paper we analyzed the impact of the introduction of a commitments procedure when the CA does not obtain deterrence from all firms. We have shown that the introduction of the commitments procedure, when used by an authority that have an important number of sectors in its charge, may result in a loss of the deterrent effect of its intervention.

In this sense, and on the contrary to what is generally established in the European practice, the impact of this procedure is negative for firms that causes a small anticompetitive harm— for which the expected trial sanction exceeds the benefits of the practice. Under symmetric information, the commitments procedure, that implies a complete reduction of fine, should only be used when the economic harm to consumer is large enough. However, trial may be chosen by the CA when the incentive compatible negotiation imply too poor commitments.

Under asymmetric information on the degree of competition that would arise without the practice, deterrence is always lost when the CA introduces the commitments procedure. However, this loss can be more than compensated by the insurance effect associated to the commitments procedure. In particular, under a uniform distribution of the possible firm's gain, commitments are always beneficial to consumers.

Note that there are some means to maintain a certain degree of deterrence that can improve the efficiency of that procedure such as a partial reduction of the trial fine. Considering a slightly simplified version of the game, we found that in any case there necessarily exists an introduction of the commitments procedure that enhances the consumers' surplus.

Finally, allowing for diverse correlations between the firm's gain with the practice and the consumers' harm, with some efficiency gains for example, we have characterized the optimal introductions of commitments on a subset of the market.

Concerning research perspectives, note that the analysis in perfect information does not take into account that a firm may be active on different markets, so that it could balance fines' deterrence effect. Future research are needed to determine whether the same commitments policy should be applied in such a setting.

In addition, the CA may take into account that private litigations may be affected by its prosecution decision between trial and commitments. In particular, the latter procedure does not establish the guiltiness of a firm. Assuming that consumers use some Class actions to sue a guilty firm, the CA may reduce its willingness to negotiate with firms. Such a framework requires a particular attention in Europe where the European Commission intends to develop such a private enforcement of the competition law.

7 Appendix

7.1 Discussion of assumption 1

In order to analyze assumption 1, we develop few examples of the function $f(p)$ where it presents no inflexion points. In each case we highlight the source of the possible range of Δ and the intervention tool of the CA concerning y.

• *Bertrand competition and rivals' marginal cost.*

Assume a dominant firm with an essential facility for which access is costly. Assume in addition that there exists a mechanism that establish a competitive price by access unit and no other entry cost. We first consider a practice that excludes any competitor ans that may be prosecuted by the authority.

There exist a range of values of Δ if different access price may result of the competitive mechanism. In addition, we assume that the competition authority may accept commitments with a mechanism less constraining in the firm's point of view and with different effect according to the value of Δ: the authority does not fix the access fee.

Let us consider a null marginal cost for the dominant firm and a marginal cost $c \geqslant 0$ for entrants. With a Bertrand competition, the residuel demand of the dominant firm, denoted $R(p)$, is defined as follows:

$$R(p) = \begin{cases} D(p) & \text{if } p < p_j \\ \frac{D(p)}{n} & \text{if } p = p_j \\ 0 & \text{otherwise} \end{cases}$$

where p is the price set by the dominant firm, p_j the smallest price set by entrants, and n the number of firms that propose this price p_j. Under a Bertrand competition, the dominant firm obtains a null residual demand if an entrants proposes a smaller price.

At the equilibrium, the entrants propose the same price if they have the same marginal cost c, and the dominant firms sets a price $p = c - \epsilon$ to serve the total demand.

Intuitively, in a given case, the smaller the access fee without the practice, denoted $c_I \geqslant 0$, the better the counterfactual situation of consumers. With some commitments implying a less competitive determination of the access fee, we have $c_C \geqslant c_I$. Note that here $l_C(y, \Delta)$ links the situation with the practice and the one with the competitive mechanism, which means that $D(p_A)(p_A - c)$ belongs to $f(p_C)$. However, we will not give an explicit form to $l_C(y, \Delta)$.

Figure 11 presents the residual profit of the dominant firm with some injunctions or commitments: its demand is not affected, so that $f(p_C)$ equals $D(p)(p - c)$ for all $p \in [c, p_A]$.

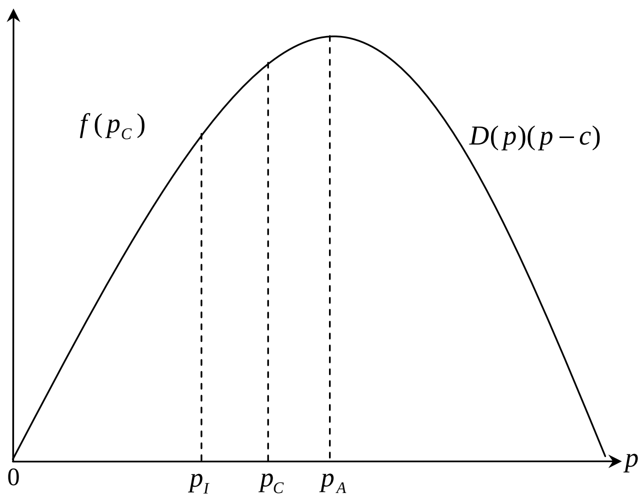

Figure 11: Residual profits and equilibrium price with a Bertrand competition

The main thing to consider in this example is that the firm does not commit in front of the CA to set a given price but on a more or less constraining mechanism that sets the price.

Note that the assumption that $D(p)$ is log-concave will be discussed later but that the residual demand $R(p)$ has the same property by construction. Hence, it would be possible to consider with the practice any price $p_A > c$ smaller than the monopoly price: the dominant firm may initially bear the pressure of potential entrants.

• *Cournot competition and rivals' marginal cost.*

In order to simplify the exposure, we consider here that a single firm may access the market and first that the practice excludes the dominant firm's competitor. The marginal cost of the dominant firm equals zero and the one of the potential entrant, denoted $c \geqslant 0$, may be raised by the incumbent.

With a Cournot competition, there is an effective entry of the competitor if and only if the entrant's marginal cost is not to high: the residual demand is less sensitive to a marginal variation of the quantity than of the price.

We assume here a linear demand function, $D(p) = 1 - p$, and a marginal cost of the entrant $c \in [0, \frac{1}{2}]$. The equilibrium residual profit of the dominant firm can is $(1+c)^2/9$ and the quilibrium price is $(1+c)/3$.

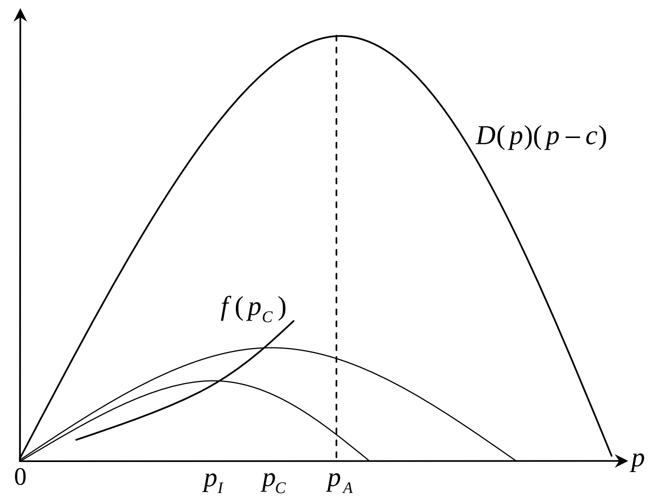

Figure 12: Residual profits and equilibrium price with a Cournot competition

Hence, as it is possible to see in figure 12, $D(p_A)(p_A - c)$ does not belong to $f(p_C)$: a type $\Delta = 0$ does not exist. In addition, the equilibrium residual profit does not tend to zero when c tends to zero: $\overline{\Delta} < D(p_A)(p_A - c)$.

As compared to the previous example, where the dominant firm serves its initial demand for any $c > 0$, openness implies a certain reduction of the residual demand and the price.

It is important to note than we could have a practice that lets the competitor enter the market. The same following analyze applies when the dominant firm does not deter entry with the practice because the equilibrium residual demand remains log-concave. All things else equal, this would simply reduce $\overline{\Delta}$.

• *Cournot competition and number of entrants.*

Here, we consider a dominant firm that prevents the entry of $n \geqslant 0$ firms on the market and that they all are as efficient as the incunbent in terms of marginal cost.

There exist different value of Δ if different number of entrants is possible without the practice. In addition, we assume that the authority may accept commitments to a less contstraining acceptance rule than injunctions in the firms point of view. Finally, we assume that different mechanism has different effect on the loss of the dominant firm according to the value of Δ: the authority does not decide the number of entrants.

We assume that $D(p) = 1 - p$ and denote c the common marginal cost of firms. The equilibrium profit with a Cournot competition is $(1-c)^2/(n+1)^2$ and the price is $(1-c)/(n+1)$.

Assumption 1 is verified because $f(p_C)$ is strictly increasing and convex: when the number of potentially active firms decreases, the increasing rate of the counterfactual profit is positive and larger than the increasing rate of the price. This is illustrated in figure 13, where it can be seen that $f(p_c)$ pass by 0 and $D(p_A)(p_A - c)$.

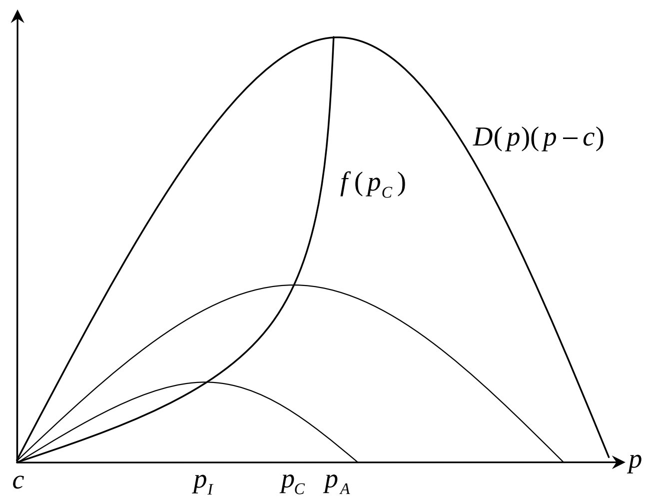

Figure 13: Residual profits and price with Cournot and n entrants

Again, it is possible to start where some firms are active on the market with the practice.

- *Collusion et symmetric firms.*

Assume n symmetric firms with constant marginal cost that compete on the market. Denote $R_{NA}(p)$ their respective demand function without the practice, where p is the equilibrium price on the market. With the practice, let us denote $R_A(p)$ their respective demand function.

The assumption of symmetry implies that in any situation they receive the same individual demand function, in the sense that they share the market in equilibrium:

$$R_{NA}(p) = R_A(p) = \frac{D(p)}{n}$$

For a given value of n, there exist different values of Δ, if different counterfactual price with injunctions are possible. In particular, the equilibrium price without the practice must not always be the marginal cost.

With a log-concave total demand function, it is immediate to see that assumption 1 is verified by the simple fact that $\frac{1}{n} < 1$. Indeed, $f(p)$ is simply:

$$f(p) = R_A(p_A)(p_A - c) - R_{NA}(p_{NA})(p_{NA} - c)$$

where $p_A > p_{NA}$, and as we have seen $R_A(p) = R_{NA}(p)$.

The following inequality then hold for any $n > 1$ and $p \in]p_o, p_A]$:

$$\frac{D(p)}{D(p_o)} > \frac{f'(p)}{f'(p_o)},$$

which means that $f(p)$ is concave as the profit of a firm in monopole and thus has no inflexion point. This is illustrated in figure 14.

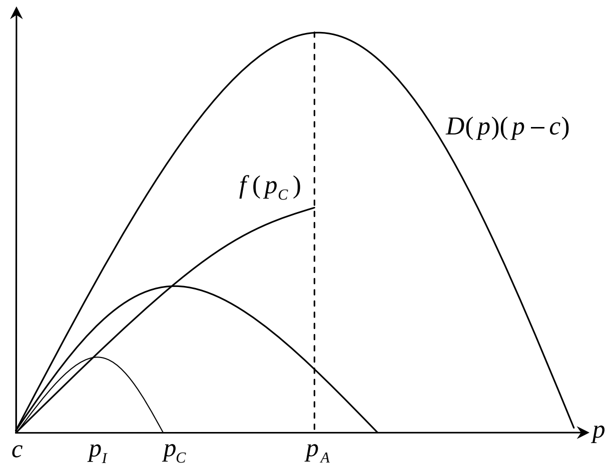

Figure 14: Residual profit of n colluding symmetric firms

With partial commitments, essentially compliance programs in this type of cases concerning no-competition clause or capacity constraints, an higher effort of the firm makes the price tend to the competitive one.

7.2 Proof of lemma 1

A Δ-type is indifferent between commitments and trial if $l_C(\widehat{y}, \Delta) = \beta[l_I(\Delta) + F]$. However, $\widehat{y}$ cannot exceed 1, and $\beta[l_I(\Delta) + F] > l_I(\Delta)$ for $\Delta < \Delta^r$. Hence the CA announces the effort the most important effort incentive compatible for a given Δ-type:

$$l_C(\widehat{y}, \Delta) = \min\{l_I(\Delta), \beta[l_I(\Delta) + F]\}$$

In figure 15, $l_C(\widehat{y}, \Delta)$ is in bold.

For $\Delta \leqslant \Delta^r$, $\widehat{y} = 1$ because $\beta[l_I(\Delta) + F] \geqslant l_I(\Delta)$: $l_C(\widehat{y}, \Delta) = l_I(\Delta)$. For $\Delta > \Delta^r$, a partial effort is necessary to incite the cooperation of the firm. Hence, $\widehat{y} = 1$ if $\Delta \leqslant \Delta^r$ and $\widehat{y} \in]0, 1[$ otherwise.

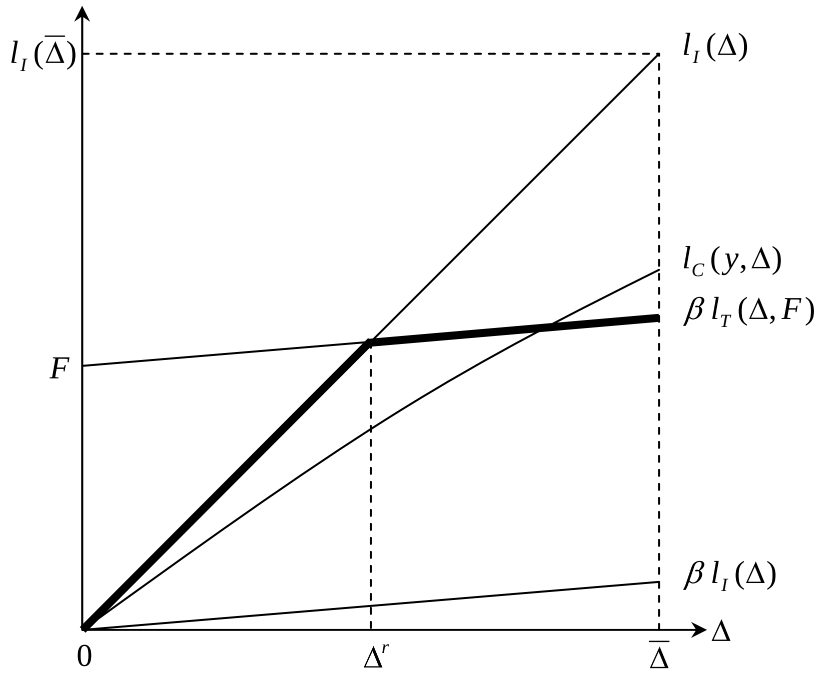

Figure 15: $l_C(\widehat{y}, \Delta)$ when Δ increases

In addition, $l_C(\widehat{y}, \Delta) > \beta l_I(\Delta)$ because $[l_I(\Delta)+F] > l_I(\Delta)$: the firm save the fine and so accept to propose an higher effort on the market than the expected one with injunctions.

However, $l_C(\widehat{y}, \Delta)$ increases for any Δ. This obvious in the last figure. Indeed, $l_C(\widehat{y}, \Delta)$ is first equal to $l_I(\Delta)$ for small values of Δ. For $\Delta > \Delta^r$, $\widehat{y}$ is defined by the intersection points between $l_C(y, \Delta)$ and $\beta[l_I(\Delta) + F]$.

For $\Delta \leqslant \Delta^r$, we have $l'_{C\Delta}(\widehat{y}, \Delta) = l'_I(\Delta)$, because $\widehat{y}$ equals 1 ($\partial\widehat{y}/\partial\Delta = 0$).

For $\Delta > \Delta^r$ the derivative of $l_C(\widehat{y}, \Delta)$ w.r.t. Δ can be written as follows:

$$l'_{C\Delta}(\widehat{y}, \Delta) + l'_{Cy}(\widehat{y}, \Delta)\frac{\partial\widehat{y}}{\partial\Delta} = \beta l'_I(\Delta)$$

With $l'_{C\Delta}(y, \Delta) > 0$, one can see that $l'_{C\Delta}(\widehat{y}, \Delta) > \beta l'_I(\Delta)$ given that $l_C(\widehat{y}, 0) = \beta l_I(0) = 0$ and that $\beta[l_I(\Delta) + F] > \beta l_I(\Delta)$: hence, the derivative of $l_C(y, \Delta)$ w.r.t. Δ, measured at the intersection of $l_C(y, \Delta)$ and $\beta[l_I(\Delta) + F]$, is necessarily larger than the derivative of $\beta l_I(\Delta)$ w.r.t. Δ. Thus, the derivative of $\widehat{y}$ w.r.t. Δ is defined as follows:

$$\frac{\partial\widehat{y}}{\partial\Delta} = -\frac{l'_{C\Delta}(\widehat{y}, \Delta) - \beta l'_I(\Delta)}{l'_{Cy}(\widehat{y}, \Delta)} < 0$$

This establishes that $l_C(\widehat{y}, \Delta)$ increases for any value of Δ, even though $\widehat{y}$ must decrease from unity in negotiations with types $\Delta > \Delta^r$ for incentive compatibility.

7.3 Proof of proposition 2

The difference $sc^c(\Delta) - sc^t(\Delta)$ is written as follows:

$$sc^c(\Delta) - sc^t(\Delta) = \begin{cases} \alpha[S(\widehat{p}_C) - S(p_A)] - S(p_I) + S(p_A) & \text{if } \Delta \leqslant \Delta^0 \\ \\ \alpha\{S(\widehat{p}_C) - S(p_A) - \beta[S(p_I) - S(p_A)]\} & \text{otherwise} \end{cases}$$

Obviously, the commitments procedure means a loss for consumers as compared to trial for $\Delta \leqslant \Delta^0$. Hence, if the CA knows Δ, it announces the (T) subgame for these Δ-types in order to maintain deterrence.

On the complementary set of Δ, the discontinuity of $\widehat{y}$ ask to split the analysis into two parts, where the maximum incentive compatible effort in commitments is bounded to the one in injunctions and when it is not.

If $\widehat{y} = 1$, the difference between $sc^c(\Delta)$ and $sc^t(\Delta)$ is positive:

$$\alpha\{S(\widehat{p}_C) - S(p_A) - \beta[S(p_I) - S(p_A)]\} = \alpha(1-\beta)[S(p_I) - S(p_A)] > 0$$

In addition, given that $S(p_I)$ increases with Δ, so is $sc^c(\Delta) - sc^t(\Delta)$ for Δ-types that accept to propose $\widehat{y} = 1$. Intuitively, the larger is Δ in this set, the more important is the insurance against dismissal in trial.

If $\widehat{y} < 1$, $\widehat{p}_C > p_I$ and we have:

$$\alpha\{S(\widehat{p}_C) - S(p_A) - \beta[S(p_I) - S(p_A)]\} = \alpha \int_{\widehat{p}_C}^{p_A} D(s)ds - \alpha\beta \int_{p_I}^{p_A} D(s)ds$$

The derivative of this expression w.r.t. Δ is $-\alpha D(\widehat{p}_C)\frac{\partial \widehat{p}_C}{\partial \Delta} + \alpha\beta D(p_I)\frac{\partial p_I}{\partial \Delta}$, where:

$$\frac{\partial p_I}{\partial \Delta} = -\frac{l_I'(\Delta)}{f'(p_I)} < 0,$$

and:

$$\frac{\partial \widehat{p}_C}{\partial \Delta} = p_y'(\widehat{y}, \Delta)\frac{\partial \widehat{y}}{\partial \Delta} + p_\Delta'(\widehat{y}, \Delta) = -\frac{\beta l_I'(\Delta)}{f'(\widehat{p}_C)} < 0.$$

Hence, on the set of Δ in $]\Delta^r, \overline{\Delta}]$:

$$\frac{\partial[sc^c(\Delta) - sc^t(\Delta)]}{\partial \Delta} = \alpha\beta l_I'(\Delta)\left(\frac{D(\widehat{p}_C)}{f'(\widehat{p}_C)} - \frac{D(p_I)}{f'(p_I)}\right)$$

The variation of the consumers' gain for $\Delta > \Delta^r$ depends of $D(.)$ and $f'(.)$ measured at p_I and $\widehat{p}_C$. Intuitively, this comes from the fact that the firm takes a decision in function of its residual profit, while the consumers' gain is measured in function of the variation of the price for the total demand. We will see that the sign of the element into brackets depends on the variation of the initial demand with the equilibrium price and the sign of the second derivative of $f(.)$.

Let us denote Δ^* a Δ-type such that $S(p(\widehat{y}, \Delta^*)) - S(p_A) - \beta[S(p(1, \Delta^*)) - S(p_A)] = 0$, so that the incentive compatible effort for this type implies no additional expected consumers' surplus as compared to trial. Assuming a type $\Delta = 0$ exists, we then have $p_I = p_A$. However, the question is to determine whether there exist other types implying the former equality. In addition, for $\Delta \in]\Delta^0, \Delta^r]$, we have seen that $\widehat{y} = 1$: if a $\Delta^* > 0$ exists, it must be in the set $]\Delta^r, \overline{\Delta}]$ where a partial effort is proposed. We denote $\widehat{y}(\Delta^*)$ the effort associated to this type under symmetric information.

The remaining of this proof consists in giving explicit conditions under which we can determine the sign of the derivative of $sc^c(\Delta) - sc^t(\Delta)$ for $\Delta \in]\Delta^r, \overline{\Delta}]$.

If the equilibrium residual profit $f(p)$ is at least as concave as $D(p)(p-c)$ for any $p \in [c, p_A]$, $\Delta^* \to \infty$.

Indeed, under this condition, we can show that the gain of consumers increases with Δ. With, in case of practice, a log-concave demand function served by the dominant firm, the following equality is verified for any $p \in]p_o, p_A]$:

$$\frac{D(p)}{D(p_o)} > \frac{D'(p)(p-c) + D(p)}{D'(p_o)(p_o - c) + D(p_o)}.$$

This means that the initial demand of the dominant firm is less sensitive to a variation of the price than its marginal profit. Note that the exact condition on $D(.)$ is less restrictive than a log-concavity: in the case where the firm adopts the practice, the elasticity-adjusted Lerner index must decrease with the price. This corresponds to the fact that the derivative w.r.t. p of the function $D'(p)(p-c)/D(p)$ is negative.

A "sufficient" concavity of $f(.)$ with $p \in [c, p_A]$ is establishes when we have the following large inequality:

$$\frac{D'(p)(p-c) + D(p)}{D'(p_o)(p_o - c) + D(p_o)} \geqslant \frac{f'(p)}{f'(p_o)}$$

Intuitively, given that the residual profit cannot be larger than the initial profit for a given price, this means that any increase of the price increases more the profit with the practice than without the practice.

For $f'(p) > 0$, the last inequality is strict if:

$$\frac{\partial}{\partial p}\left[\frac{D'(p)(p-c) + D(p)}{f'(p)}\right] > 0 \Leftrightarrow \frac{D''(p)(p-c) + 2D'(p)}{D'(p)(p-c) + D(p)} > \frac{f''(p)}{f'(p)}$$

where the two elements of the second inequality are negative. This is true if $f(p)$ is more concave than $D(p)(p-c)$. Assumption 1 rejects any point where the sign of $f''(p)$ is reversed, but not that $f'(p)$ might tends to zero for a price $p > c$. In such a case, the large inequality holds given that for this same point we have $D'(p)(p-c) + D(p) \geqslant 0$.

If the equilibrium residual profit $f(p)$ is less concave than $D(p)(p-c)$ for $p \in [c, p_A]$, it might be that $D(.)$ diminishes less faster than $f'(.)$. However, if $f(p)$ is convex, it is obvious that the derivative of $sc^c(\Delta) - sc^t(\Delta)$ is negative for $\Delta \in]\Delta^r, \overline{\Delta}]$: $D(p) < D(p_o)$ while $f'(p) > f'(p_o)$.

Still, if $\Delta^* < \overline{\Delta}$, note that $\Delta^* > \Delta^r$ because there exists necessarily a type $\Delta = \Delta^r + \epsilon$, where ϵ is positive and next to zero, such that $S(\widehat{p}_C) \simeq S(p_I) > \beta S(p_I)$. Hence, the value of $\overline{\Delta}$ and the decreasing rate of $sc^c(\Delta) - sc^t(\Delta)$ w.r.t. Δ determines if Δ^* is larger or smaller than $\overline{\Delta}$.

With the previous illustrations of assumption 1 in mind, we are able to obtain the following results of this paper without specification of $\Delta^* > \Delta^r$.

7.4 Proof of proposition 3

As compared to the initial game, the expected fine in trial increases with Δ, so that the expected loss in trial is $l_T(\Delta) = l_I(\Delta) + F(\Delta)$, which increases faster in Δ than the initial sanction $l_I(\Delta) + F$. Moreover, assuming $F(0) = F$, Δ^r is larger. Figure 16 illustrates these elements.

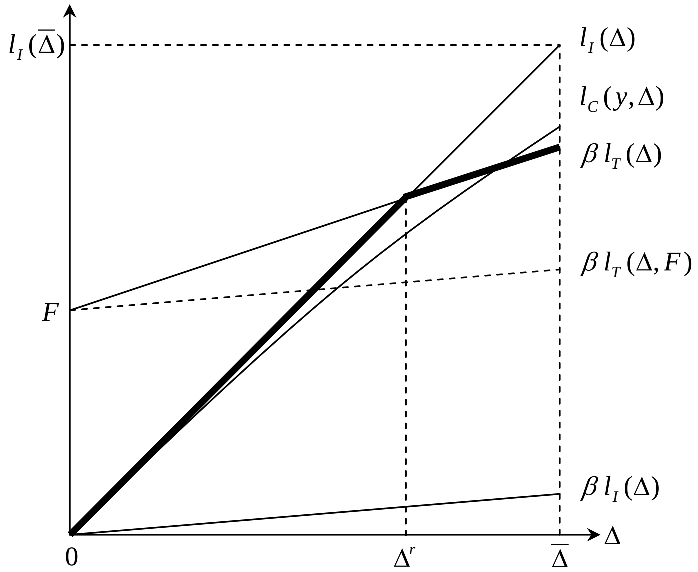

Figure 16: $l_C(\widehat{y}, \Delta)$ when F increases with Δ

It follows that $\widehat{y}$ decreases less faster for types $\Delta > \Delta^r$, and that the derivative of $\widehat{p}_C$ w.r.t. Δ in this set may be written as follows:

$$\frac{\partial \widehat{p}_C}{\partial \Delta} = -\frac{\beta l_T'(\Delta)}{f'(\widehat{p}_C)} < 0$$

Smaller Δ-types accept $\widehat{y} = 1$, so that we find that:

$$\frac{\partial S(\widehat{p}_C)}{\partial \Delta} - \frac{\partial S(p_I)}{\partial \Delta} = \beta l_I'(\Delta) \left(\frac{D(\widehat{p}_C) \frac{l_T'(\Delta)}{l_I'(\Delta)}}{f'(\widehat{p}_C)} - \frac{D(p_I)}{f'(p_I)} \right)$$

where by definition $l_T'(\Delta)/l_I'(\Delta) > 1$. This implies that, as compared to the initial game, consumers benefit more easily of the negotiation with high Δ-types.

Note that assuming a maximal amount of $F(\Delta)$ brings ourselves back to the initial game and would not modify this result.

7.5 Proof of proposition 4

In the $(C + T)$ subgame, we first look for tha value of $\widetilde{y}$ that maximizes SC^c. The derivative of $SC^c(\widetilde{y})$ w.r.t. $\widetilde{y}$ is written as follows:

$$\alpha\left\{\int_0^{\widetilde{\Delta}}\frac{\partial[S(p_C)-S(p_A)]}{\partial\widetilde{y}}\phi(\Delta)d\Delta+\frac{\partial\widetilde{\Delta}}{\partial\widetilde{y}}\left[S(\widetilde{p}_C)-S(p_A)-\beta[S(\widetilde{p}_I)-S(p_A)]\right]\phi(\Delta^c(y))\right\}$$

where $\widetilde{p}_C$ and $\widetilde{p}_I$ are the prices associated to the marginal Δ-type respectively with commitments and injunctions. The integrate represent the interest to increase the requested effort in commitments. As long as $\partial\widetilde{\Delta}/\partial\widetilde{y}$ is negative, $S(\widetilde{p}_C)-S(p_A)$ represents the associated loss due to the reduction of Δ-types that proposes commitments (exclusively in the set that cannot be deterred), diminished by the interest to treat these ones with trial implying a consumers' gain $\beta[S(\widetilde{p}_I)-S(p_A)]$. Note that the treatment of these Δ-types in trial represents an opportunity cost given that for the marginal type $\widetilde{y}=\widehat{y}$, which insures the symmetric game result for this unique Δ-type: $S(\widetilde{p}_C)-S(p_A)>\beta[S(\widetilde{p}_I)-S(p_A)]$. Hence, the sign of this derivative is not undetermined *a priori*.

Under asymmetric information, $\widetilde{y}$ is the unique variable of the authority, and the two following definitions are usefull in demonstrations:

$$\frac{\partial\widetilde{p}_C}{\partial\Delta}=p'_\Delta(\widetilde{y},\widetilde{\Delta})$$

and:

$$\frac{\partial\widetilde{p}_C}{\partial\widetilde{y}}=p'_y(\widetilde{y},\widetilde{\Delta})+p'_\Delta(\widetilde{y},\widetilde{\Delta})\frac{\partial\widetilde{\Delta}}{\partial\widetilde{y}}$$

Using integration by parts, the derivative of $SC^c(\widetilde{y})$ is written as follows:

$$\alpha\left\{-\int_0^{\widetilde{\Delta}}[S(p_C)-S(p_A)]\frac{\partial}{\partial\Delta}\left[\frac{p'_y(\widetilde{y},\Delta)}{p'_\Delta(\widetilde{y},\Delta)}\phi(\Delta)\right]d\Delta\right.$$

$$\left.+\frac{\partial\widetilde{\Delta}}{\partial\widetilde{y}}\left[\left(1+\beta\frac{l'_I(\widetilde{\Delta})}{l'_{C\Delta}(\widetilde{y},\widetilde{\Delta})}\right)[S(\widetilde{p}_C)-S(p_A)]-\beta[S(\widetilde{p}_I)-S(p_A)]\right]\phi(\widetilde{\Delta})\right\}$$

The sign of the first element depends of the derivative of $\frac{p'_y(\widetilde{y},\Delta)}{p'_\Delta(\widetilde{y},\Delta)}\phi(\Delta)$ w.r.t. Δ that measures the weighted relative effect of a slight increase of Δ on the equilibrium price, and equals $\frac{l'_{Cy}(\widetilde{y},\Delta)}{l'_{C\Delta}(\widetilde{y},\Delta)}\phi(\Delta)$. As long as $\frac{l'_I(\widetilde{\Delta},F)}{l'_{C\Delta}(\widetilde{y},\widetilde{\Delta})}>0$, the second element is negative.

It then appears that this derivative is negative if $\frac{\partial}{\partial\Delta}\left[\frac{p'_y(\widetilde{y},\Delta)}{p'_\Delta(\widetilde{y},\Delta)}\phi(\Delta)\right]\geqslant 0$ over $[0,\widetilde{\Delta}]$. Whenever this condition is verified over $[0,\overline{\Delta}]$, the optimal announcement is $\widetilde{y}^*=y_{min}$, that corresponds to the highest effort for which all Δ-types accept to commit.

Note that this is true for example for $l_C(y,\Delta)=y\Delta$ and a weighted value of Δ, denoted $\Delta\phi(\Delta)$, increasing over $[0,\overline{\Delta}]$. This is the case under an increasing or uniform distribution or a truncated normal distribution with $\frac{\mu+\sqrt{\mu^2+\sigma^2}}{2}>\overline{\Delta}$, where μ is the mean and σ the standard deviation.

After this analysis of the derivative of $SC^c(\widetilde{y})$, we can determine the sign of $SC^c(\widetilde{y})-SC^t$ under a uniform distribution. We have:

$$SC^c(\widetilde{y}) - SC^t = \alpha \left[\int_0^{\widetilde{\Delta}} [S(p_C) - S(p_A)]\phi(\Delta)d\Delta + \beta \int_{\widetilde{\Delta}}^{\overline{\Delta}} [S(p_I) - S(p_A)]\phi(\Delta)d\Delta \right]$$

$$- \int_0^{\Delta^0} [S(p_I) - S(p_A)]\phi(\Delta)d\Delta - \alpha\beta \int_{\Delta^0}^{\overline{\Delta}} [S(p_I) - S(p_A)]\phi(\Delta)d\Delta$$

where the element into brackets increases with $\widetilde{y}$ if $p'_y(\widetilde{y}, \Delta)\phi(\Delta)/p'_\Delta(\widetilde{y}, \Delta) > 0$ for any Δ, and the other elements are independent of $\widetilde{y}$.

Let us denote $G(\Delta)$ a primitive of $S(p_I) - S(p_A)$ and $G_y(\Delta)$ a primitive of $S(p_C) - S(p_A)$, so that under a uniform distribution of Δ:

$$SC^c(\widetilde{y}) - SC^t = \frac{1}{\overline{\Delta}}[\alpha G_y(\widetilde{\Delta}) - \alpha\beta G(\widetilde{\Delta}) - (1 - \alpha\beta)G(\Delta^0) + (1 - \alpha)G(0)]$$

which is non negative if and only if $G_y(\widetilde{\Delta}) - \beta G(\widetilde{\Delta}) \geqslant \frac{1-\alpha\beta}{\alpha} G(\Delta^0) - \frac{1-\alpha}{\alpha} G(0)$.

We then find that $\widetilde{y} = 1$ enhances the consumers' surplus assuming $G(0) \geqslant 0$. Indeed, as long as $G(\Delta)$ is increasing and convex with Δ, $G(t) - G(t_0) \geqslant (t - t_0)G'(t_0)$. Setting $t = \Delta^r$ and $t_0 = \Delta^0$ we find that:

$$G(\Delta^r) - G(\Delta^0) \geqslant \frac{(1 - \alpha)\beta F}{(1 - \beta)(1 - \alpha\beta)} g(\Delta^0)$$

This last inequality is more constraining than $SC^c(1) - SC^t \geqslant 0$ if and only if:

$$\frac{(1 - \alpha)\beta F}{(1 - \beta)(1 - \alpha\beta)} \geqslant \left(\frac{1 - \alpha\beta}{\alpha} - 1 \right) G(\Delta^0) \Leftrightarrow \Delta^0 g(\Delta^0) \geqslant G(\Delta^0)$$

Given that this is a large inequality, the first part of proposition 4 is demonstrated.

For $l_C(y, \Delta) = y\Delta$, we have seen that $\widetilde{y} = 1$ is the worst decision of the CA under a uniform distribution. Hence, in such a case, we have $SC^c(y_{min}) - SC^t > 0$. A necessary condition for the optimal effort $\widetilde{y}^*$ to be interior is that the ratio $\frac{p'_y(\widetilde{y},\Delta)}{p'_\Delta(\widetilde{y},\Delta)}$ is not monotonic with Δ.

7.6 A simple example with a normal distribution of Δ

Assume a Bertrand competition and a dominant firm raising its rivals' marginal cost. We propose here a numerical example of equilibria under a Normal distribution truncated over $[0, \overline{\Delta}]$ with $l_C(y, \Delta) = y\Delta$, $D(p) = 1 - p$ and $\mu = 0$. We set a null dominant firm's marginal cost assuming an initial monopoly so that $\overline{\Delta} = 0.25$, $\alpha = \beta = 0.5$ and $F = \frac{3}{4}\frac{1-\beta}{\beta}\overline{\Delta}$.

With these settings, we find that $\Delta^0 = 0.041$, $\Delta^r = 0.125$, $S(p_I) - S(p_A) = \frac{\Delta + \sqrt{\Delta}}{2}$, and that $\frac{p'_y(\widetilde{y},\Delta)}{p'_\Delta(\widetilde{y},\Delta)}\phi(\Delta)$ is increasing over $[0, \overline{\Delta}]$ if and only if $\sigma > 0.5$. Finally, all types accept to commit if $\widetilde{y} \leqslant 0.75$.

Figure 17 represents the value of $CS^c(\widetilde{y}) - CS^t$ for $\widetilde{y} \in [0.75, 1]$ and $\sigma \in [0.01, 0.25]$, where CS^t does not depend on $\widetilde{y}$. The height gives us the highest $CS^c(\widetilde{y}) - CS^t$ for a given σ, and whether or not the commitments procedure

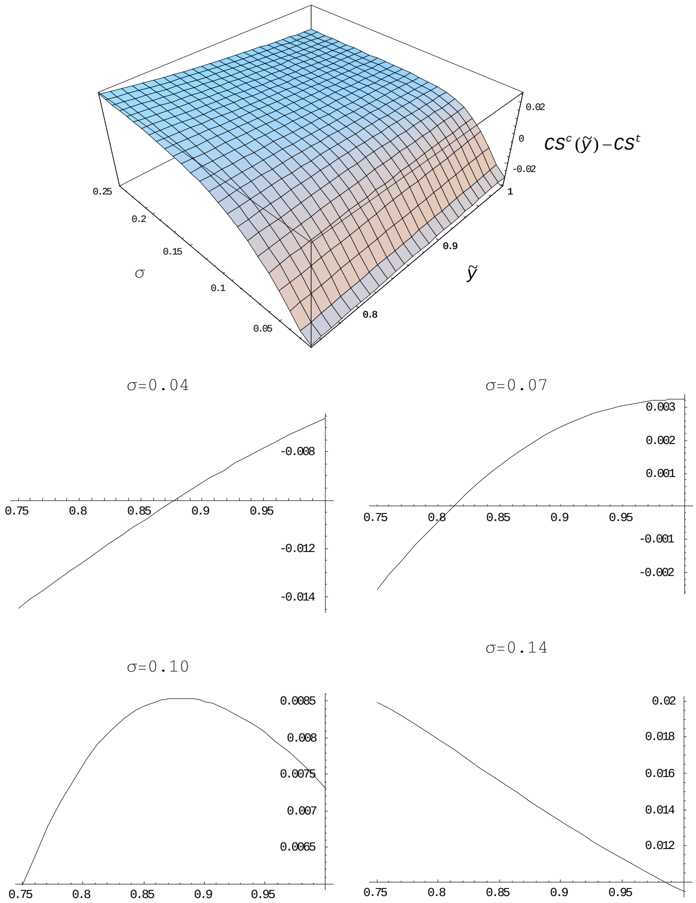

Figure 17: $CS^c(\widetilde{y}) - CS^t$ under a Normal distribution with $\mu = 0$

should be introduced by the CA (whether $CS^c(\widetilde{y}^*) - CS^t$ is positive or negative). The three dimensional plot is followed by details of selected values of σ.

Figure 18 summarizes equilibria. A complete effort should be requested only for low values of σ (< 0.08), but this may result in a reduction of the consumers' surplus when initially deterred firms are very probable (< 0.07, represented with dashes). For largest values, the optimal $\widetilde{y}$ decreases until all types should be handled with the commitments procedure.

7.7 Proof of proposition 5

To establish proposition 5 we distinguish the cases where the equilibrium residual profit is sufficiently concave and where it is not.

As shown in figure 8, Δ-types in $[\Delta^0, \overline{\Delta}]$ are indifferent between trial and commitments if $y = z = \beta$. Offering a slight additional reduction of the fine ϵ implies that $\Delta^c(y, z)$ is higher than $\overline{\Delta}$. The CA has an interest to increase y in order to remain on segment [a]: it announces $y = \beta + \epsilon \frac{F}{\overline{\Delta}}$.

For these couples, we have by definition $\Delta^c(y, z) = \overline{\Delta}$, but also $\Delta^d(y, z) =$

y \ σ	< 0.07	$[0.07, 0.08]$	$]0.08, 0.12[$	≥ 0.12
0.75	T	T	$C+T$	$C+T$
⋮	T	$C+T$	$C+T$	$C+T$
1	T	$C+T$	$C+T$	$C+T$

increasing probability of commitments ↑

increasing frequency of high Δ-types →

Figure 18: The optimal strategy of the CA

$\Delta^0 + o(\epsilon)$, where $o(.)$ is negative and of first order w.r.t. ϵ.

Indeed:

$$\Delta^d(\beta + \epsilon\frac{F}{\overline{\Delta}}, \beta - \epsilon) = \frac{\alpha\beta F - \alpha\epsilon F}{1 - \alpha\beta - \alpha\epsilon\frac{F}{\overline{\Delta}}},$$

which is written using limited development:

$$\Delta^d(\beta + \epsilon\frac{F}{\overline{\Delta}}, \beta - \epsilon) = \Delta^0 - \frac{\epsilon\alpha F}{(1-\alpha\beta)^2}\left[1 - \alpha\beta\left(1 + \frac{F}{\overline{\Delta}}\right)\right]$$

where the element into brackets is positive if and only if $\Delta^0 < \overline{\Delta}$.

With an effort $y \simeq \beta$ requested for all Δ-types, a sufficient concavity of $f(p_C)$ implies an increase of the consumers' surplus with commitments: $S(p_c)$ is concave w.r.t. y and Δ when $l_C(y, \Delta) = y\Delta$. The concavity of $S(p_I)$ follows and is not demonstrated.

We must show that the second derivatives of $S(p_C)$ are negative and that the hessien matrice is negative semi-definite.

We have the following expressions:

$$\frac{\partial^2 S(p_C)}{\partial \Delta^2} = -D'(p_C)\left(\frac{\partial p_C}{\partial \Delta}\right)^2 - D(p_C)\frac{\partial^2 p_C}{\partial \Delta^2}$$

and:

$$\frac{\partial^2 S(p_C)}{\partial y^2} = -D'(p_C)\left(\frac{\partial p_C}{\partial y}\right)^2 - D(p_C)\frac{\partial^2 p_C}{\partial y^2}$$

where, with the function $\Psi(p_C, y, \Delta) = f(p_C) + l_C(y, \Delta) - D(p_A)(p_A - c) = 0$, we obtain the following equality:

$$\frac{\partial^2 p_C}{\partial \Delta^2} = -\frac{\left[\Psi''_{\Delta^2} + \Psi''_{\Delta p}\frac{\partial p_C}{\partial \Delta}\right]\Psi'_p - \Psi'_\Delta\left[\Psi''_{p^2}\frac{\partial p_C}{\partial \Delta} + \Psi''_{p\Delta}\right]}{\left[\Psi'_p\right]^2}$$

$$= -\frac{l''_{C\Delta^2}(y,\Delta)}{f'(p_C)} - \left(\frac{\partial p_C}{\partial \Delta}\right)^2 \frac{f''(p_C)}{f'(p_C)}$$

where by assumption $l''_{C\Delta^2}(y,\Delta) = 0$.

In addition:

$$\frac{\partial^2 p_C}{\partial y^2} = -\frac{l''_{Cy^2}(y,\Delta)}{f'(p_C)} - \left(\frac{\partial p_C}{\partial y}\right)^2 \frac{f''(p_C)}{f'(p_C)}$$

The two second derivatives of $S(p_C)$ are negative given that with $f(p_C)$ sufficiently concave the following inequality holds:

$$\frac{D'(p_C)}{D(p_C)} < -\frac{f''(p_C)}{f'(p_C)}$$

We then define the crossed derivatives of $S(p_C)$. For $l''_{Cy\Delta}(y,\Delta) = l''_{C\Delta y}(y,\Delta) = 0$, it appears that:

$$\frac{\partial^2 S(p_C)}{\partial y \partial \Delta} = \frac{\partial^2 S(p_C)}{\partial \Delta \partial y} = \frac{\partial p_C}{\partial y}\frac{\partial p_C}{\partial \Delta}\left[-D'(p_C) + D(p_C)\frac{f''(p_C)}{f'(p_C)}\right]$$

The hessien of $S(p_C)$ is null and the two second derivatives are negative so that we establish the concavity of $S(p_C)$.

Hence, for any Δ-type, with $l_C(y,\Delta) = y\Delta$, the authority prefers commitments to an effort β rather than expected injunctions:

$$S(p(\beta,\Delta)) > \beta S(p(1,\Delta)) = \beta S(p_I)$$

The following of the demonstration of proposition 5 is obtained with the analysis of the derivative of SC^c_z w.r.t. z and y.

The derivative of SC^c_z w.r.t. z is written:

$$\frac{\partial \Delta^d(y,z)}{\partial z}\left\{[S(p^d_I) - S(p_A)] - \alpha[S(p^d_C) - S(p_A)]\right\}\phi(\Delta^d(y,z))$$
$$+\alpha\frac{\partial \Delta^c(y,z)}{\partial z}\left\{[S(p^c_C) - S(p_A)] - \beta[S(p^c_I) - S(p_A)]\right\}\phi(\Delta^c(y,z))$$

where $p^i_I = p(1,\Delta^i(y,z))$ and $p^i_C = p(y,\Delta^i(y,z))$, for $i = d,c$. In addition, $\frac{\partial \Delta^d(y,z)}{\partial z} = \frac{\alpha F}{1-\alpha y}$ and $\frac{\partial \Delta^c(y,z)}{\partial z} = -\frac{F}{y-\beta}$.

We then analyze its sign along segment [b], where $\Delta^d(y,z) = \Delta^c(y,z) = \Delta^0$:

$$\frac{\partial SC^c_z}{\partial z} = -\frac{\alpha F(1-\alpha\beta)}{(1-\alpha y)(y-\beta)}\left\{[S(p^0_C) - S(p_A)] - y[S(p^0_I) - S(p_A)]\right\}\phi(\Delta^0)$$

If $S(p_C)$ is concave, the element into brackets is positive. Hence, the derivative of SC^c_z w.r.t. z is negative for the concerned couples, $(y, \frac{(1-\alpha y)\beta}{1-\alpha\beta})$ where $y \in [\beta, 1]$. $y = z = \beta$ is not the only mean to enhance the consumers' surplus: this result is obtain with any slight reduction of z from segment [b].

If $S(p_C)$ is not concave, we first have to analyze the effect of a given effort y applied to all types $\Delta \in [0, \Delta^c(y,z)]$. The form of $l_C(y,\Delta)$ is not crucial in this part of the demonstration.

The derivative of $S(p_C) - S(p_A) - \beta[S(p_I) - S(p_A)]$ w.r.t. Δ is equal to:

$$\frac{\partial[S(p(y,\Delta)) - \beta S(p(1,\Delta))]}{\partial\Delta} = S'(p(y,\Delta))p'_\Delta(y,\Delta) - S'(p(1,\Delta))p'_\Delta(1,\Delta)$$

Given that in $\Delta = 0$, $p(y,0) = p(1,0) = p_A$, it appears that $S(p_C) - S(p_A) - \beta[S(p_I) - S(p_A)] = 0$. In addition, at this point $\partial S(p(y,\Delta)) - \beta S(p(1,\Delta))/\partial\Delta > 0$. Recall also that in Δ^*, $S(p_C) - S(p_A) = \beta[S(p_I) - S(p_A)]$, so that the maximal incentive compatible effort of type Δ^* is just high enough to compensate the uncertainty assotiated to injunctions. Finally, note that $S(p(y,\Delta)) - \beta S(p(\Delta))$ has no inflexion point whenever the equilibrium residual profit has none.

Hence, it is possible to say that the consumers' gain associated to a given effort in commitments as compared to injunctions expected in trial is positive for any $\Delta \leqslant \Delta^*$. Figure 19 presents this result for a concave or convex residual profit, and $\Delta^* \leqslant \overline{\Delta}$. It presents the curves of $S(p(y,\Delta))$ and $\beta S(p(1,\Delta))$ associated to the effort $\widehat{y}(\Delta^*)$.

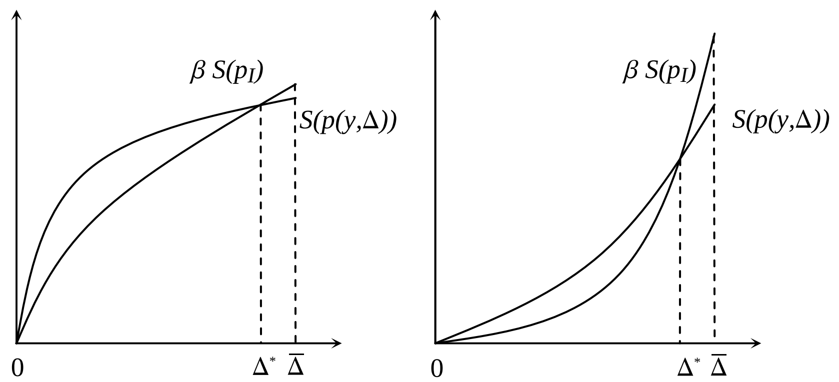

Figure 19: Partial effort and form of the equilibrium residual profit

We now are able to analyze the sign of the derivative of SC^c_z w.r.t. y. Integrating by parts, it is written as follows:[26]

$$\begin{aligned}\frac{\partial SC^c_z}{\partial y} = \frac{\partial\Delta^d(y,z)}{\partial y}&\left[S(p^d_I) - S(p_A) - \left(1 + \frac{1}{\alpha y}\right)[S(p^d_C) - S(p_A)]\right]\phi(\Delta^d(y,z)) \\ +\alpha\frac{\partial\Delta^c(y,z)}{\partial y}&\left[\left(1 + \frac{\beta}{y}\right)[S(p^c_C) - S(p_A)] - \beta[S(p^c_I) - S(p_A)]\right]\phi(\Delta^c(y,z)) \\ &-\frac{\alpha}{y}\int_{\Delta^d(y,z)}^{\Delta^c(y,z)}[S(p_C) - S(p_A)]\frac{\partial}{\partial\Delta}[\Delta\phi(\Delta)]d\Delta\end{aligned}$$

where $\frac{\partial\Delta^d(y,z)}{\partial y} = \frac{\alpha^2 zF}{(1-\alpha y)^2} > 0$ and $\frac{\partial\Delta^c(y,z)}{\partial y} = -\frac{(\beta-z)F}{(y-\beta)^2} < 0$.

[26] The following defintions are needed: $\frac{\partial p^c_C}{\partial y} / \frac{\partial p^c_C}{\partial\Delta} = \frac{l'_{Cy}(y,\Delta^c) + l'_{C\Delta}(y,\Delta^c)\frac{\partial\Delta^c(y,z)}{\partial y}}{l'_{C\Delta}(y,\Delta^c)}$, and $\frac{\partial p^d_C}{\partial y} / \frac{\partial p^d_C}{\partial\Delta} = \frac{l'_{Cy}(y,\Delta^d) + l'_{C\Delta}(y,\Delta^d)\frac{\partial\Delta^d(y,z)}{\partial y}}{l'_{C\Delta}(y,\Delta^d)}$.

Knowing that $\Delta^* > \Delta^r$, we only analyze its sign for couples $(y, \frac{(1-\alpha y)\beta}{1-\alpha\beta})$, where $y \in [\widehat{y}(\Delta^*), 1]$, and such that $\Delta^c(y,z) = \Delta^d(y,z) = \Delta^0$:

$$\frac{\partial SC_z^c}{\partial y} = \left\{ [S(p_I^0) - S(p_A)] \left(\frac{\partial \Delta^d(y,z)}{\partial y} - \alpha\beta \frac{\partial \Delta^c(y,z)}{\partial y} \right) \right.$$
$$\left. + [S(p_C^0) - S(p_A)] \left(-\left(1 + \frac{1}{\alpha y}\right) \frac{\partial \Delta^d(y,z)}{\partial y} + \alpha \frac{\partial \Delta^c(y,z)}{\partial y} \left(1 + \frac{\beta}{y}\right) \right) \right\} \phi(\Delta^0)$$

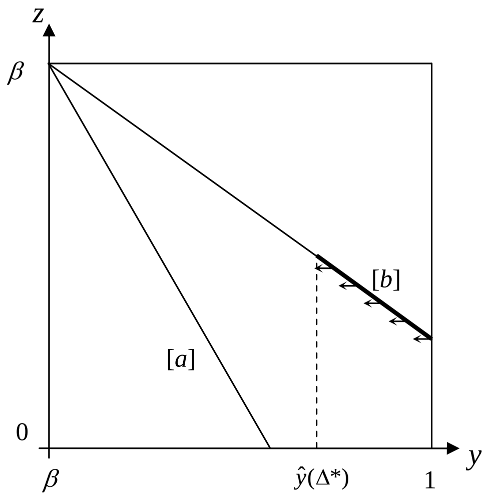

Figure 20: Fine and effort with $\Delta^* < \overline{\Delta}$

In $y = 1$ and $z = \frac{(1-\alpha)\beta}{1-\alpha\beta}$, this derivative is negative, and a more precise analysis shows that some other couples increase the consumers' surplus[27]. Indeed, on the right side of the point on segment $[b]$ where $y = \widehat{y}(\Delta^*)$, we know that $S(p_C^0) - S(p_A) > \beta[S(p_I^0) - S(p_A)]$, which implies that the derivative of SC_z^c w.r.t. y is negative if and only if:

$$\frac{\frac{\partial \Delta^d(y,z)}{\partial y} - \alpha\beta \frac{\partial \Delta^c(y,z)}{\partial y}}{\left(1 + \frac{1}{\alpha y}\right) \frac{\partial \Delta^d(y,z)}{\partial y} - \alpha \frac{\partial \Delta^c(y,z)}{\partial y} \left(1 + \frac{\beta}{y}\right)} < \beta \Leftrightarrow \frac{\alpha\beta^2(1-\alpha y)}{[\alpha y(1-\beta) - \beta](y-\beta)} > 1$$

where the values of $\partial\Delta^d(y,z)/\partial y$ and $\partial\Delta^c(y,z)/\partial y$ for the considered value of z allow the second inequality which is a function of y, α and β. If the denominator in this inequality is negative or null, the first is always verified. Otherwise, it is as easily verified as y is small: for any $y \in [\widehat{y}(\Delta^*), 1[$, the fact that $S(p_C^0) - S(p_A) > \beta[S(p_I^0) - S(p_A)]$ implies that the derivative of SC_z^c w.r.t. y is negative on segment $[b]$ as it is the case in $y = 1$.

[27] In $y = 1$, $p_C^0 = p_I^0$, and $\frac{\partial SC_z^c}{\partial y} = [S(p_I^0) - S(p_A)] \left[-\frac{\partial \Delta^d(y,z)}{\partial y} \frac{1}{\alpha} + \alpha \frac{\partial \Delta^c(y,z)}{\partial y} \right] \phi(\Delta^0) < 0$.

Hence, there always are different (y, z) couples such that the consumers' surplus is enhanced with commitments. These points are next to the bold segment presented in figure 20.

The distribution of Δ-types will allow for other points increasing the consumers' surplus. Here, it is sufficient to see that some couples necessarily exist with $l_C(y, \Delta) = y\Delta$.

7.8 Proof of proposition 6

Figure 21 presents the potential optimal couples when $f(p_C)$ is sufficiently concave or not. The presented frontier in the latter case is not necessarily linear but does not joined segment $[a]$.

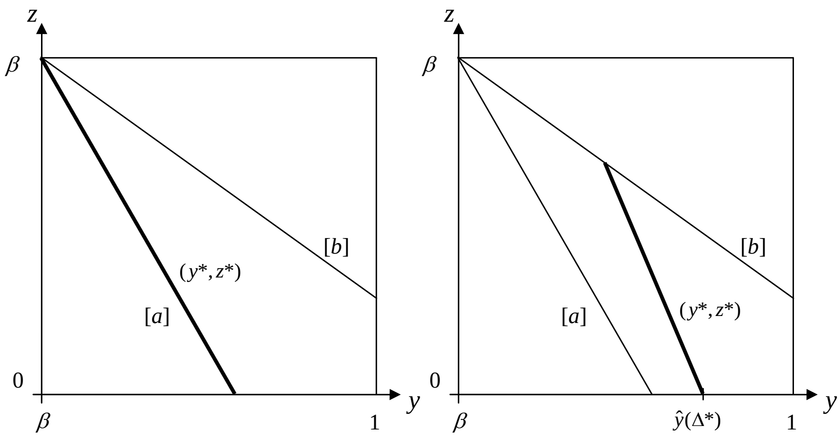

Figure 21: Potential optima with a partial reduction of the fine and an increasing weighted gain of the firm

The analysis of the derivative of SC_z^c w.r.t. y shows that an increase of $\Delta\phi(\Delta)$ over $[\Delta^d(y, z), \Delta^c(y, z)]$ insures that it is negative. If $f(p_C)$ is sufficiently concave, the optimal couple is then on segment $[a]$.

Otherwise, for $z = 0$, $\Delta^c(y, z) = \Delta^*$. For $z > 0$, the optimal couple implies smaller effort in commitments and a trial probability larger than in symmetric information. This probability increases with the degree of the maintained fine in negotiations.

7.9 Proof of proposition 7

$h(\Delta)$ is the weighted harm to consumers with a practice that implies a gain Δ to the firm: $h(\Delta) = [S_I - S_A]\phi(\Delta) > 0$, where S_I is larger than S_A. In this extension, we have $h(y, \Delta) = yh(\Delta)$.

We have seen that for a given $\widetilde{y}$, the weighted consumers' surplus is given by the following expression:

$$SC^c(\widetilde{y}) = S_A + \alpha \int_0^{\widetilde{\Delta}} h(\widetilde{y}, \Delta)d\Delta + \alpha\beta \int_{\widetilde{\Delta}}^{\overline{\Delta}} h(\Delta)d\Delta$$

where $\widetilde{\Delta} = \frac{\beta F}{\widetilde{y}-\beta}$.

The derivative of $SC^c(\widetilde{y})$ w.r.t. $\widetilde{y}$ is written as follows:

$$\frac{\partial SC^c(\widetilde{y})}{\partial \widetilde{y}} = \alpha \int_0^{\widetilde{\Delta}} \left[h(\Delta) - h(\widetilde{\Delta}) \right] d\Delta$$

In addition, the second derivative of $SC^c(\widetilde{\Delta})$ w.r.t. $\widetilde{y}$ has the same sign as $h'(\widetilde{\Delta})$:

$$\frac{\partial^2 SC^c(\widetilde{y})}{\partial \widetilde{y}^2} = -\alpha h'(\widetilde{\Delta})\widetilde{\Delta}\frac{\partial \widetilde{\Delta}}{\partial \widetilde{y}}$$

where $\partial\widetilde{\Delta}/\partial\widetilde{y}$ is negative.

The difference $SC^c(\widetilde{y}) - SC^t$ is given by:

$$SC^c(\widetilde{y}) - SC^t = -(1-\alpha\widetilde{y}) \int_0^{\Delta^0} h(\Delta)d\Delta + \alpha(\widetilde{y}-\beta)\int_{\Delta^0}^{\widetilde{\Delta}} h(\Delta)d\Delta$$

where the first element represents the loss associated to the no deterrence of the smallest Δ-types. The second element measures the gain associated to the insurance effect of the commitments procedure, knowing that $\widetilde{y} > \beta$ given that the firm saves the fine.

We analyze different form of $h(\Delta)$.

• If $h(\Delta)$ is strictly increasing, the primitives of $h(\Delta)$, denoted $H(\Delta)$, are increasing and convex.

Given this form of $h(.)$, the optimal solution is y_{min}. In particular, this offer dominates $\widetilde{y} = 1$. For the latter value, we find that:

$$SC^c(1) - SC^t > 0 \Leftrightarrow \frac{\alpha(1-\beta)}{1-\alpha}[H(\Delta^r) - H(\Delta^0)] > H(\Delta^0) - H(0),$$

where we know that $\Delta^0 h(0) < H(\Delta^0) - H(0) < \Delta^0 h(\Delta^0)$ and that $(\Delta^r - \Delta^0)h(\Delta^0) < H(\Delta^r) - H(\Delta^0) < (\Delta^r - \Delta^0)h(\Delta^r)$.

As long as $\Delta^r - \Delta^0 = \frac{\beta F(1-\alpha)}{(1-\beta)(1-\alpha\beta)}$, we find that $SC^c(1) - SC^t$ is positive if:

$$\Delta^0 h(\Delta^0) > H(\Delta^0) - H(0)$$

which is always true and insures that $SC^c(y_{min}) - SC^t > 0$.

• If $h(\Delta)$ is strictly decreasing, $H(\Delta)$ is increasing and concave.

Here $\widetilde{y}^* = 1$, and by definition of $SC^c(1) - SC^t$, we find that this difference cannot be positive if:

$$\Delta^0 h(\Delta^0) < H(\Delta^0) - H(0)$$

which is always true and insures that the $(C+T)$ subgame is strictly dominated by (T) for any $\widetilde{y} \leqslant 1$.

• If $h'(\Delta) > 0$ for small values of Δ and then $h'(\Delta) < 0$, $H(\Delta)$ is increasing and convex then concave.

With $h'(\Delta^r) \geqslant 0$ and $h(0) = h(\overline{\Delta})$, we find that $\widetilde{y}^*$ is unique and defined by the following equality:

$$\int_0^{\widetilde{\Delta}} [h(\Delta) - h(\widetilde{\Delta})]d\Delta = 0 \Leftrightarrow H(\widetilde{\Delta}^*) - H(0) = \widetilde{\Delta}^* h(\widetilde{\Delta}^*)$$

With this form of $SC(\widetilde{y})$, $h'(.) < 0$ at $\widetilde{\Delta}^*$, and we must distinguish the cases where $\widetilde{y}^*$ is bounded by y_{min} or unity.

First assume that the previous equality defines a $\widetilde{y}^* \in [y_{min}, 1]$, for which $\widetilde{\Delta}^* \in [0, \overline{\Delta}]$, so that $SC^c(\widetilde{y}^*) - SC^t$ is positive if and only if:

$$SC^c(\widetilde{y}^*) - SC^t > 0 \Leftrightarrow \frac{1-\alpha\beta}{1-\alpha\widetilde{y}^*}[H(\widetilde{\Delta}^*) - H(\Delta^0)] > \widetilde{\Delta}^* h(\widetilde{\Delta}^*)$$

which is less certain as Δ^0 is large. It is sufficient to show that Δ^0 is enough large for that $h'(\Delta^0)$ tends to zero, for example with $\alpha = 1$ and $h'(\Delta^r) = 0$: For such a value, $H(.)$ is concave over $[\Delta^0, \widetilde{\Delta}^*]$.

Defining $\widetilde{\Delta} - \Delta^0$, we have $SC^c(\widetilde{y}^*) - SC^t > 0$, knowing that $(\widetilde{\Delta}^* - \Delta^0)h(\widetilde{\Delta}^*) < H(\widetilde{\Delta}^*) - H(\Delta^0) < (\widetilde{\Delta}^* - \Delta^0)h(\Delta^0)$.

Second, assume that $\widetilde{y}^*$ is a corner solution in $\widetilde{y}^* = y_{min}$, with $\widetilde{\Delta}^* = \overline{\Delta}$. By definition of $\widetilde{y}^*$, the following inequality holds:

$$H(0) > H(\overline{\Delta}) - \overline{\Delta} h(\overline{\Delta})$$

So that $SC^c(y_{min}) - SC^t$ is positive:

$$SC^c(y_{min}) - SC^t > 0 \Leftrightarrow \frac{\alpha(y_{min} - \beta)}{1 - \alpha y_{min}}[H(\overline{\Delta}) - H(\Delta^0)] > H(\Delta^0) - H(0),$$

which is true for $h'(\Delta^0)$ next to zero and therefore for any value of Δ^0.

Finally, assume that the value of $\widetilde{y}^*$ is bounded to unity, with $\widetilde{\Delta}^* = \Delta^r$. In Δ^r, $H(0) < H(\Delta^r) - \Delta^r h(\Delta^r)$. Here, nothing insures that $SC^c(1) - SC^t$ is positive. Intuitively, we are close to the case where $h(.)$ strictly decreases with Δ.

The third part of proposition 7 is established given that $\widetilde{\Delta}^*$ defines an upper bound of the maximum of $h(\Delta)$.

• If $h'(\Delta) < 0$ for small values of Δ and then $h'(\Delta) > 0$, $H(\Delta)$ is increasing and concave then convex.

With $h'(\Delta^r) \geqslant 0$, $SC^c(\widetilde{y})$ is strictly convex for the considered form of $h(.)$. We find that $\widetilde{y}^* = y_{min}$ or 1. It then is possible to determine according to the value of Δ that minimizes $h(.)$ if one of these solutions imply a positive $SC^c(\widetilde{y}) - SC^t$. More precisely, we can define an upper bound of this Δ above which commitments enhance the consumers' surplus.

First note that a necessary condition nécessaire for $SC^c(1) - SC^t$ to be positive is that $h'(\Delta^r) > 0$. Otherwise, the concavity of $H(.)$ on the set $[0, \Delta^r]$ insures the opposite, even though it is a local optima of $SC^c(\widetilde{y})$.

Denote m the value of $\widetilde{y}$ such that $\int_0^{\widetilde{\Delta}(m)}[h(\Delta) - h(\widetilde{\Delta}(m))]d\Delta = 0$, where $\widetilde{\Delta}(m)$ is the marginal type indifferent between commitments and trial when $\widetilde{y} = m$. Given that $h'(0) < 0$, $\widetilde{\Delta}(m)$ is an upper boundof the value of Δ for which $h(.)$ is the smallest. Hence, $\widetilde{\Delta}(m) > 0$, meaning that $m > \beta$. However, given that $h'(\overline{\Delta}) > 0$, $\widetilde{\Delta}(m)$ may be larger than $\overline{\Delta}$. Therefore, $m \in]\beta, +\infty[$.

By definition of m, we have $H(0) = H(\widetilde{\Delta}(m)) - \widetilde{\Delta}(m)h(\widetilde{\Delta}(m))$. It then is necessary to determine the effect of an increase of m on $SC^c(\widetilde{y}) - SC^t$, when $\widetilde{y} = \{y_{min}, 1\}$:

$$\frac{\partial[SC^c(\widetilde{y}) - SC^t]}{\partial m} = -(1 - \alpha\widetilde{y})\widetilde{\Delta}(m)h'(\widetilde{\Delta}(m))\frac{\partial\widetilde{\Delta}(m)}{\partial m} > 0$$

Intuitively, this expression indicates that these two consumers' surplus increase with m, which means whenever the minimum of $h(\Delta)$ is reached for a small value of Δ. Note also that the second derivative of $SC^c(\widetilde{y}) - SC^t$ w.r.t. m is negative.

It is then sufficient to determine $SC^c(1)$, $SC^c(y_{min})$ and SC^t for the different possible ranges of m. We show hereafter that there exists a value of m, denoted $m_t \in]y_{min}, 1+\beta\frac{1-\alpha}{\alpha}[$, under which the (T) subgame is prefered and above which $\max\{SC^c(1), SC^c(y_{min})\} > SC^t$. We also find that there exists a value of m, denoted $m_1 \in]y_{min}, 1[$, under which $SC^c(1) > SC^c(y_{min})$ and above which $\widetilde{y}^* = y_{min}$. Figure 22 illustrates these results.

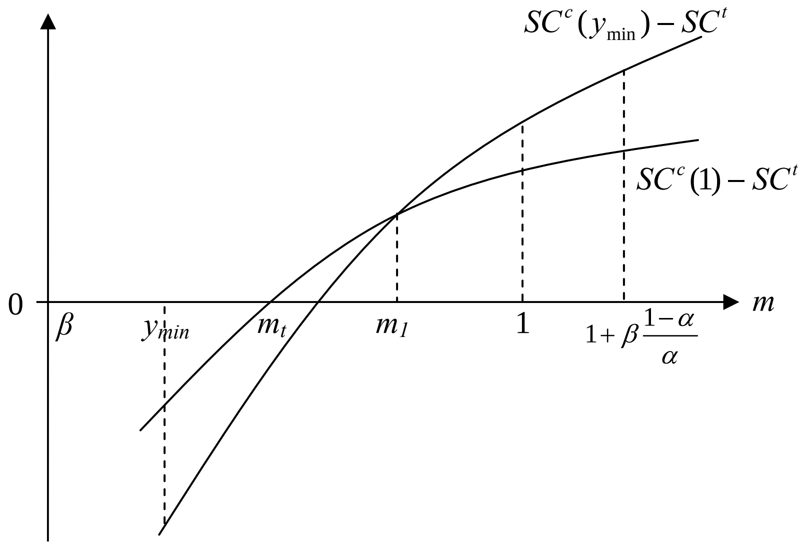

Figure 22: Optimal effort and subgame according to m

Hence, for a value of $\widetilde{\Delta}(m)$ large enough (above $\widetilde{\Delta}(m_t)$), the commitments procedure is never introduced, while it is certain if $\widetilde{\Delta}(m)$ is sufficiently small (if m is defined larger m_1).

Indeed, if $m > 1+\beta\frac{1-\alpha}{\alpha}$, we have $\widetilde{\Delta}(m) < \Delta^0$, and by definition $h'(\Delta^0) > 0$. This also implies that $h'(\Delta^r) > 0$ and therefore $H(\Delta^0) - H(0) - \Delta^0 h(\Delta^0) < 0$. Hence $SC^c(1) > SC^t$. In addition, with $m \geqslant 1$ the derivative of $SC^c(\widetilde{y})$ w.r.t. $\widetilde{y}$ is negative, so that $\widetilde{y}^* = y_{min}$. Therefore, $SC^c(y_{min}) > SC^c(1) > SC^t$.

If $m \leqslant y_{min}$ we have $\widetilde{\Delta}(m) \geqslant \overline{\Delta}$, which implies $h(0) > h(\overline{\Delta})$. Even assuming that $h'(\Delta^0) < 0$, implying that $H(\Delta^0) - H(0) > \Delta^0 h(\Delta^0)$, we find that $SC^c(1) < SC^t$, while $\widetilde{y}^* = 1$. Therefore, $SC^c(y_{min}) < SC^c(1) < SC^t$.

The continuity of $SC^c(\widetilde{y}) - SC^t$ w.r.t. m, for $\widetilde{y} = \{y_{min}, 1\}$, implies that there exists a m_t above which one of these two introduction enhances the consumers' surplus. Note that the order of m_t and m_1 may be reversed as compared to figure 22.

8 References

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