Legal Uncertainty – an Effective Deterrent in Competition Law?*

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Abstract

This article considers legal uncertainty in competition law. Contrary to perceived wisdom, I show that the uncertainty itself might have positive welfare effects, if it is sufficiently small. Legal uncertainty acts as a screening device, if the threshold of legality is uncertain. Then, near the threshold, firms decide contingent on their type whether to pursue controversial business practices. This allows mitigating the policy restrictions, as the competition authority cannot perfectly observe the types of the firms. Such an effect might influence the trade-off between per-se rules and rules of reason in competition law. In an extension, I discuss the effects of introducing ambiguity about the fine and prove that this mitigates enforcement problems, if auditing costs are sufficiently high.

JEL classifications: D8, K2, K4, L5

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1 Introduction

Given the complexity of legal procedures in competition law legal uncertainty is a major issue.[1] Previous literature has shown that legal uncertainty yields disproportionate deterrence – over-deterring socially beneficial actions, while under-deterring socially detrimental ones.[2] This article sets up a formal model to study whether the legal uncertainty inherent in a legal rule can advance the objectives of the policymaker. Our analysis shows that the legal uncertainty itself might have positive effects, if it is not too large. The reason is

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[1] With legal uncertainty, I refer here to circumstances where it is not clear whether a certain business practice is legal. This is similar to the notion of D'Amato (1983).

[2] See for instance Calfee and Craswell (1984), Craswell and Calfee (1986), Schinkel and Tuinstra (2006) or Katsoulacos and Ulph (2010).

that it allows to mitigate the policy restrictions of the competition authority and to get closer to the optimal deterrence level that is contingent on aspects unobservable by the competition authority. Thus, the competition authority may use the legal uncertainty as a screening device. A certain business practice can be pro- or anticompetitive depending on the circumstances and the competition authority cannot perfectly distinguish between them. In this case, uncertainty about the threshold of legality deters firms with few gains from the business practices that are close to the threshold. Yet, firms with large gains still pursue the new business practices, even if they are near the legal threshold. This allows to screen firms according to some unobservable characteristics and increases social welfare, as the inherent legal uncertainty makes the rules more selective. If the welfare weight on firms's profits is low or zero, the result still holds, as the legal uncertainty increases the probability of a conviction and thus allows the competition authority to impose fines.

There are different reasons for this kind of legal uncertainty. According to Calfee and Craswell (1984, p. 968) 'it is difficult to predict ... how an antitrust court will distinguish between 'predatory' and 'competitive' price cuts.' Alternative reasons are the existence of different procedures, measurement errors by the competition authority, different assessments of efficiency defenses or uncertainty about what kind of evidence will be allowed. Consider two examples. In the European Union vertical restraints, like resale price maintenance or exclusive dealings, are prohibited under Article 101 (TFEU), formerly Article 81 (EC).[3] However there is a Block Exemption, so that this rule does not apply, if the market shares of the involved parties are below 30%. Although the European Commission gives guidelines how the relevant market shares are to be determined, it is extremely difficult to predict correctly the market share determined by the competition authorities, in particular, if these are close to 30%. The reasons are discrepancies in the definition of the relevant market, information asymmetries or imprecision in the measurement of sales and other factors. This creates the kind of uncertainty analyzed in the model.

The second example is the case of Microsoft tying its operating system with additional software, in particular, a web browser and a media player.[4] In both instances the European Commission found an abuse of a dominant market position under Article 102 (TFEU), formerly Article 82 (EC). Think of a scale beginning with products where the bundling with the operating system is socially beneficial, as the integration allows for new features or higher speeds and independent competing products are non-existent. On the other end of the scale are products where the bundling yields few or no efficiency gains, but competition is harmed considerably. While there is legal certainty on both ends of the scale, in the middle there is a region where it is very difficult to exclude legal uncertainty completely and according to the model in this paper this might actually be socially beneficial.[5]

[3] See European Commission (2010a) for details.

[4] These are the cases COMP/39.530 and T-201/04 Microsoft vs. Commission. The commission summarizes its findings in the former case in European Commission (2010b).

[5] Although both examples are from Europe, the results of this paper are also valid for U.S. competition law. Yet in the United States courts have frequently interpreted legal uncertainty in favor of the investigated

In an extension of the model I analyze uncertainty about the size of the fine that is imposed on firms in case of a conviction. After a conviction by the competition authority, firms might turn to the courts that can change the fine. If the competition authority is more concerned about enforcement costs than its income from fines, this additional uncertainty is beneficial, as long as there is no change in the expected value of the fines. The reason is that the legal uncertainty about the fine makes enforcement easier for the competition authority.

A caveat applies here. Although this model points out positive effects of legal uncertainty, too much legal uncertainty actually decreases social welfare. Furthermore, there may be negative effects of legal uncertainty that are not captured in our analysis.[6] Consequently, legal uncertainty is no panacea. However, the policymakers might positively influence the effects of legal uncertainty and the direction of the deterrence effects towards anticompetitive behavior by complementing the rule of reason with per-se exceptions, like safe harbors, or detailed information with respect to some aspects of the procedure.[7]

The effects of legal uncertainty discussed in this paper might also influence the trade-off between per-se rules and rules of reason. With per-se rules, some clearly specified practices, like, e. g., certain rebates or resale price maintenance, are prohibited. A rule of reason, on the other hand, judges anticompetitive behavior as illegal, where the test of legality is whether competition was promoted or hindered.[8] Therefore a business practice may be legal in some cases, but not in others, depending on its consequences. Recently, there has been a major shift away from per-se rules — exemplified by the case Leegin vs. PSKS, as it allowed resale price maintenance, if it does not impede competition.[9] Also the competition authorities in the European Union aim to pursue a more economic approach. This approach focuses more on the market effects of the business practice under consideration. An example is the discussion of the European Commission about the enforcement of Article 102 (TFEU), formerly Article 82 (EC).[10] Previous literature has argued that rules of reason allow to differentiate the competition law in a more selective way at the price of some inherent legal uncertainty, because firms sometimes do not know whether their conduct is legal. Katsoulacos and Ulph (2010, p. 3) summarize this issue as follows: the 'legal uncertainty induced by effects-based procedures [i. e., rules of reason] is harmful and should lead [the competition] authority to favor per-se procedures.' This paper shows that the conclusion depends very much on the amount of legal uncertainty. If it is sufficiently small, it could even improve the balance in favor of rules of reason.

The rest of the article is organized as follows. Section 2 discusses the relevant lit-

party, thereby reducing overall deterrence.

[6] For instance, the legal uncertainty may reduce the possibility to control the enforcement agency, here the competition authority, as it becomes more difficult to detect incompetent or corrupt behavior.

[7] Ahlborn et al. (2004) and Christiansen and Kerber (2006) propose such modified or structured rules of reason.

[8] Kaplow and Shapiro (2007, p. 54ff) provide a good discussion of rules of reason in antitrust.

[9] Supreme Court of U.S. 'Leegin Creative Leather Products, Inc. vs. PSKS, Inc.' Decision No. 06-480, June 28th, 2007.

[10] Cf. European Commission (2009), Kroes (2006), European Commission (2005) or Gual et al. (2005).

erature. Section 3 sets up the model with the competition authority choosing a policy that determines which actions will be punished if the competition authority detects a firm implementing them. The competition authority can observe only one of two dimensions of the action, and thus cannot distinguish pro- and anticompetitive actions perfectly. Section 4 provides the analysis why legal uncertainty may be beneficial. The reason is a screening effect, as the legal uncertainty allows to influence firms' behavior depending on unobservable aspects of their type. Section 5 discusses an extension of the model, allowing firms to turn to the courts to repeal the decision of the competition authority. Finally, Section 6 contains the concluding remarks.

2 The Literature

This paper focuses on the legal uncertainty inherent in legal rules. This is closely related to the trade-off between rules of reason and per-se rules that has been studied before. Ehrlich and Posner (1974) discuss the advantages and disadvantages of having per-se rules replaced by rules of reason, as these could better distinguish beneficial from harmful actions, but provide less guidance for the concerned parties. However they do not analyze the overall effects on welfare. Kaplow (1995) assumes mutual ignorance about the nature of the considered action, because firms do not know the exact rules and the competition authority does not know the specific circumstances of the firm. Therefore both parties have to invest, if they want to get the information. Thus, he models the trade-off between compliance costs and selectivity of rules. He shows that compliance costs are often low, even for quite complex rules. However, there is no legal uncertainty if a firm decides to invest in order to learn the rules.

Katsoulacos and Ulph (2009) model the trade-off between different procedures in competition law. They characterize the conditions, when the distinction of pro- and anticompetitive procedures by rules of reason is welfare-enhancing compared to per-se rules. In many circumstances rules of reason are welfare superior. The extension of Katsoulacos and Ulph (2010) scrutinizes, in particular, the arising legal uncertainty by introducing a second dimension of uncertainty about the nature of the considered business practice similarly to my model. They find that the selectivity of a rule of reason often outweighs the losses due to the arising legal uncertainty. I concentrate on the legal uncertainty and do not consider the comparison between per-se rules and rules of reason. My model allows to vary continuously the legal uncertainty inherent in legal rules and shows that the uncertainty itself might increase social welfare, if the amount of uncertainty is sufficiently small.

Calfee and Craswell (1984) discuss the kind of legal uncertainty I consider here and Craswell and Calfee (1986) formalize it. However, in their model, there is no information asymmetry about the nature of the considered action. Therefore, the legal uncertainty only hinders the implementation of the optimal legal threshold and either causes too much

or too little deterrence. I show that the legal uncertainty is beneficial and has positive effects on welfare.

The beneficial effects of legal uncertainty have appeared in different contexts. Strausz (2011) points out that regulatory risk might be advantageous and studies the necessary market structures. Lang and Wambach (2010) show for insurance fraud that uncertainty about enforcement might have a beneficial deterrence effect. Furthermore, the deterrence effect of uncertainty is already used in tax enforcement. According to Reinganum and Wilde (1988, p. 794), the Internal Revenue Service (IRS) in the U.S. confirms that 'one of the tools in the arsenal of the IRS which promotes voluntary compliance is the uncertainty in the minds of the taxpayers.' However, there are additional aspects to consider, as also individuals and not only firms are affected.[11]

Finally, there is a literature on costly state verification. Besanko and Spulber (1989) use such a model to analyze optimal enforcement of antitrust laws, but do not touch on the issue of legal uncertainty. The problem of costly state verification (Townsend, 1979) considered here is not limited to competition law, but also appears in different settings, like regulation (Baron and Besanko, 1984), financing (Gale and Hellwig, 1985), or accounting (Border and Sobel, 1987).

3 The Model

In the model there is a competition authority facing a continuum of firms with mass one.[12] The competition authority sets competition policy by choosing two enforcement parameters $\hat{x}$ and p in order to maximize total welfare, weighting the firm's profits by α with $0 \leq \alpha < 1$. $\hat{x} \in [0,1]$ captures the threshold of legality. In the first example of vertical restraints this is a specific market structure, e.g. a market share of 30%. p is the fraction of firms audited. Every firm has the binary choice whether to implement a specific, new business practice, like, e. g., bundling, retail price maintenance or rebates, or to abstain from it. Depending on its choice, I refer to a firm as active or deterred.

The nature of the available action depends on the firm's type (x, b) that is two-dimensional as in Katsoulacos and Ulph (2010). The firm knows its type (x, b) in both dimensions. The first dimension x captures the aspects that the competition authority can observe with its auditing technology at a cost c per audit. Coming back to my examples from the introduction, this refers to the market structure, for example, market shares in the case of vertical restraints. In the case of Microsoft x denotes the kind of software added to the operation system and whether the integration is socially beneficial or harmful. The value x of the firm's type is drawn from a uniform distribution G on the interval $[0,1]$.[13]

[11] For a discussion, see Cronshaw and Alm (1995), Snow and Warren (2005), Osofsky (2011), and Gergen (2011). In a different realm Teitelbaum (2007) discusses the effects of ambiguity on individuals in tort law.

[12] This is equivalent to a single firm. Both interpretations are valid.

[13] This assumption is only a small restriction, as I could redefine the units of x to match any distribution that admits a density and has connected support.

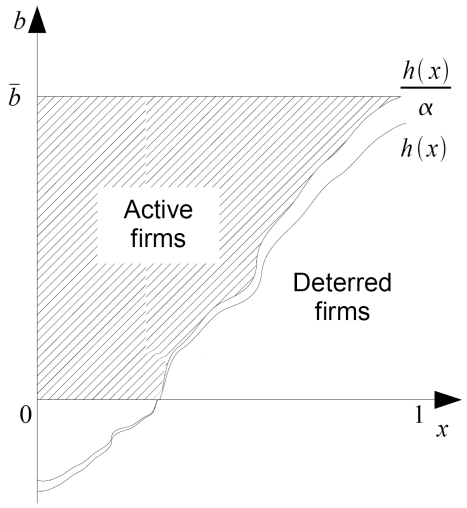

Figure 1: Active firms in the first-best policy

The value b is unobservable by the competition authority. As b influences the decision of the firm, I will call b the private benefits of the company, which are independently and uniformly distributed on $[0, \bar{b}]$ with $\bar{b} > 0$.[14]

Social welfare remains unchanged, if the firm continues its previous market behavior. If a firm of type (x, b) adopts the new business practice, it generates negative externalities of $-h(x)$ given by the function $h(\cdot)$ and private benefits of b.[15] Thus, weighted welfare changes by $\alpha b - h(x)$. The first dimension of the firm's type x is ordered in such a way that a higher x signifies higher social harm, i. e., $h'(x) > 0$. However, for some types implementing the business practice is socially beneficial and for some it is socially harmful, such that $h(0) < 0$ and $h(1) > 0$ to make the problem interesting. Thus, the practice can be pro- or anticompetitive depending on the firm's type. There are many examples for business practices that can have pro- as well as anticompetitive effects. Price reductions, e. g., might reflect lower costs or an attempt at predatory pricing. The same holds for bidding patterns in procurement contests or standardization efforts, which might have beneficial effects or be part of some collusive agreement in order to harm other market participants. For the case of vertical restraints a simplification might be just to consider the market shares. If these are very low, the restraints do not harm other market participants, $h(0) < 0$. However, they could be socially very harmful, $h(1) > 0$, if the firms involved dominate their respective markets. In the case of Microsoft, $h(1) > 0$ corresponds to implementing a web browser in order to acquire a dominant position in the browser market by abusing its dominance in the market for operating systems. $h(0) < 0$, on the other hand, corresponds to integrating new and socially beneficial features, like a basic firewall, touchscreen support or improved USB drivers.

As a benchmark consider the first-best policy, where the firm's type is observable and verifiable. In this case a firm of type (x, b) should be active, whenever $\alpha b - h(x) \geq 0$. Then

[14]The model is robust to the introduction of some correlation between b and x. As long as the correlation with x is not too perfect, the mechanism in this model works. With perfect correlation the competition authority could infer the value of b from the x value and therefore does not need the legal uncertainty as a screening device.

[15]The externality function $h(x)$ should be differentiable on the whole domain, i.e., $h'(x) < \infty$.

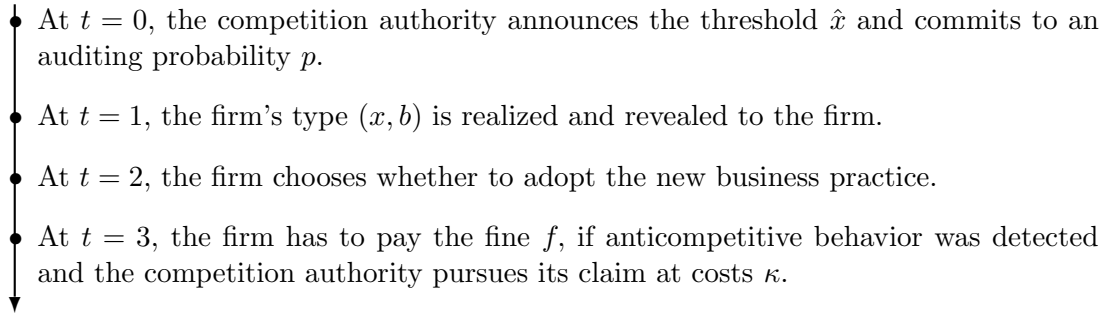

Figure 2: The timing of the model

only the firms depicted in Figure 1 are active and implement the new business practice. In the model the competition authority cannot perfectly observe and verify the firms' type. In particular, the competition authority might allow using the business practice for some firms depending on the observable aspects, i.e., the first dimension, of their type. Due to the monotonicity of $h(x)$, it is optimal for the competition authority to forbid all actions above a threshold, i. e. $x \geq \hat{x}$. Therefore restricting competition policy to setting a threshold of legality $\hat{x}$ is without loss of generality. If the competition authority finds the firm to have x below $\hat{x}$, its actions may well be socially efficient and therefore the competition authority allows the firm to continue. However, above $\hat{x}$, actions are judged as anticompetitive and are prohibited. If the competition authority detects a firm violating this policy, it can make the firm pay a fine $f > \bar{b}$. Yet for this purpose it has to make the observed x verifiable at costs κ with $\kappa < (1 - \alpha)f$.[16] The costs κ capture experts' testimonies, reports and other expenses to prove the competition authority's case. Figure 2 summarizes the timing of the model.

The legal uncertainty makes the auditing technology of the competition authority imperfect. By an audit, the competition authority does not learn the first dimension x of the type exactly, but receives only a noisy signal $x^M = x + \delta$ with δ uniformly distributed on $[-\Delta, \Delta]$ with a small Δ. In the case of vertical restraints this captures the difficulty in determining, whether the market share is 29.8% or 30.1%. With Microsoft the uncertainty might arise for products, like anti-virus software, where tying might offer great benefits, but also has the potential to harm other market participants considerably. This uncertainty about the legal threshold or this measurement error is implied by the structure of the legal rules and is exogenous to the competition authority.[17] Therefore the case without legal uncertainty, i.e., $\Delta = 0$, serves only as a benchmark. Proposition 1 will

[16]The fine f is exogenous. However, making the fine endogenous does not change the model. The results in this paper just require a jump in the fine at $\hat{x}$ which is optimal, even if the fine is completely endogenous. Section A.1 in the Appendix discusses this case. For a discussion of the setting of fines in Europe see European Commission (2006) and Wils (2007).

[17]Even in very formalistic approaches such uncertainty can arise. An example is the case of Michelin II, where the European Commission fined Michelin for using pure quantity rebates, although most scholars at that time believed these to be legal. The details can be found in European Commission 'Manufacture Française des pneumatiques Michelin v. Commission' Decision 2002/405, June 20th, 2001. See Motta (2006) for a discussion.

show that the legal uncertainty increases welfare compared to a rule of reason without legal uncertainty.

To sum up, the expected change in weighted welfare is

$$w(x,b) = \begin{cases} -cp & \text{if the firm } (x,b) \text{ is deterred} \\ \alpha b - h(x) - cp & \text{if the firm } (x,b) \text{ is active and not fined} \\ \alpha b - h(x) + (1-\alpha)pf - (c+\kappa)p & \text{if the firm } (x,b) \text{ is active and fined.} \end{cases}$$

On the other hand, the expected profits of the firm are

$$\pi(x,b) = \begin{cases} 0 & \text{if the firm } (x,b) \text{ is deterred} \\ b & \text{if the firm } (x,b) \text{ is active and not fined} \\ b - pf & \text{if the firm } (x,b) \text{ is active and fined.} \end{cases}$$

Thus, the firm's type is two-dimensional. Yet the competition authority can only observe a costly and imprecise signal about one dimension of the firms' types. Given its policy constraints, it cannot enforce the first-best policy. To make the problem interesting, I assume that on average the new behavior is harmful

$$\alpha \frac{\bar{b}}{2} < \int_0^1 h(x) \mathrm{d}G(x). \tag{1}$$

Therefore a social planner restricted to a binary rule, i.e., $\hat{x} = 0$ or 1, would prohibit all the business practices in the class considered here. This assumption is not crucial for the results, but allows to avoid a corner solution with the competition authority approving all actions. Furthermore, I assume that the costs of auditing are not excessively high, so that auditing is worthwhile. That is

$$c < \frac{f}{\bar{b}} \max_{\hat{x}} \int_0^{\hat{x}} \alpha \frac{\bar{b}}{2} - h(x) \mathrm{d}G(x). \tag{2}$$

To guarantee an interior solution of the competition authority's optimization, another, more technical assumption is required

$$\int_{x'-\Delta}^{x'+\Delta} 2f^2(1 - x' - \frac{1}{3}\Delta)h'(x) - fh(x) - \bar{b}((1-\alpha)f - \kappa)\mathrm{d}x > 0, \tag{3}$$

for all $x' \in [\Delta, 1-\Delta]$, which is equivalent to the slope of the externality function being sufficiently high, such that the policy of the competition authority matters. The next section shows that the legal uncertainty allows to mitigate the limitations of policy enforcement.

4 Analysis

First, we consider the decision process of the firm. The firm faces the fine f, if it is caught by the competition authority implementing the controversial business practice and $x^M \geq \hat{x}$. Therefore the firm will only take action, if its private benefits b are high enough. Thus, there is a cut-off level $\hat{b}(x)$, such that only firms above $\hat{b}(x)$ will become active. The cut-off for the private benefit $\hat{b}(x)$ will vary with x. If the firm's type is low, $x < \hat{x} - \Delta$, the firm will always implement the new business practice, as long as the private benefits are positive. Beginning at $x = \hat{x} - \Delta$, $\hat{b}(x)$ will be increasing in x. Consequently, the cut-off level will be

$$\hat{b}(x) = \begin{cases} pf & \text{if } x > \hat{x} + \Delta \\ \frac{x - \hat{x} + \Delta}{2\Delta} pf & \text{if } \hat{x} - \Delta \leq x \leq \hat{x} + \Delta \\ 0 & \text{if } x < \hat{x} - \Delta. \end{cases} \tag{4}$$

Therefore the legal uncertainty created by the imprecise measurement allows some screening of firms. If they are close to the policy threshold $\hat{x}$, firms with low private benefits will already abstain from taking action for lower values of x than firms with a high value of private benefits. If the rule of reason would provide complete legal certainty, the measure of the competition authority would be perfect, i. e., $\Delta = 0$, and the policy threshold would be sharp. This means that below $\hat{x}$, all firms will take action, and above, only those with a private value above the cut-off $\hat{b}(x)$. Figure 3 depicts this situation. It is easy to recognize that the right-hand side is closer to the optimal schedule of Figure 1 than the left-hand side, as there is no discontinuous jump at the threshold $\hat{x}$. Consequently, a rule of reason with some inherent legal uncertainty about the policy threshold is beneficial, as this makes the behavior of the firm more gradual around the threshold. Before the next proposition formalizes this argument, we turn to the competition authority. The competition authority chooses the threshold $\hat{x}$ and the auditing probability p to maximize total

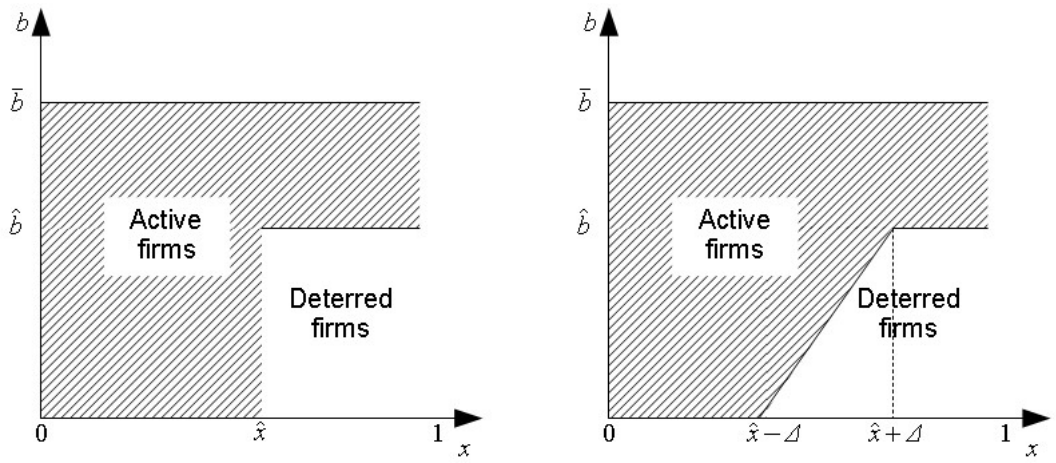

Figure 3: Changes in active firms due to legal uncertainty

welfare $W(\hat{x}, p)$ which is given by

$$\int_0^{\hat{x}-\Delta} \alpha\frac{\bar{b}}{2} - h(x)\mathrm{d}G(x) +$$
$$+ \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} \frac{\bar{b}-\hat{b}(x)}{\bar{b}} \left(\alpha\frac{\bar{b}+\hat{b}(x)}{2} - h(x) + \frac{x-\hat{x}+\Delta}{2\Delta}(f(1-\alpha)-\kappa)p\right)\mathrm{d}G(x) +$$
$$+ \int_{\hat{x}+\Delta}^{1} \frac{\bar{b}-pf}{\bar{b}} \left(\alpha\frac{\bar{b}+pf}{2} - h(x) + (f(1-\alpha)-\kappa)p\right)\mathrm{d}G(x) - cp. \tag{5}$$

The first term in equation (5) captures the region where the competition authority judges all business practices as procompetitive. In the intermediate region, there is legal uncertainty which decreases the probability of a firm implementing the new business practices while increasing its expected benefits. The second integral represents this behavior. Finally, in the anticompetitive region, activity is limited to the firms with the highest benefits. The intuition is that the active firms under the policy implemented by the competition authority match closer the active firms in the first-best policy, because firms' behavior allows some inference about the second dimension. Payments of the fine enter the considerations of the competition authority, if the weight of firms' profit α in the objective function of the competition authority is smaller than one. As legal uncertainty increases the probability of a conviction, this effect can make legal uncertainty attractive, if the weight α is very low. Proposition 1 summarizes these arguments.

Proposition 1. *Legal uncertainty with $\Delta > 0$, i. e., uncertainty about the legal threshold à la Craswell and Calfee (1986), increases social welfare compared to a regime with $\Delta = 0$ and legal certainty, if the uncertainty is not too large.*

Proof: Taking the derivative with respect to Δ results by the envelop theorem in

$$\frac{\partial W(\hat{x},p)}{\partial\Delta} = -\alpha\frac{\bar{b}}{2} + h(\hat{x}-\Delta) + \alpha\bar{b} - \alpha\frac{(pf)^2}{3\bar{b}} - h(\hat{x}+\Delta)\frac{\bar{b}-pf}{\bar{b}} + \frac{\bar{b}-pf}{\bar{b}}h(\hat{x}+\Delta) -$$
$$- h(\hat{x}-\Delta) - \int_{-\Delta}^{\Delta} h(\hat{x}+x)\frac{x}{2\Delta^2}\frac{pf}{\bar{b}}\mathrm{d}x - \alpha\frac{\bar{b}^2-(pf)^2}{2\bar{b}} + (f(1-\alpha)-\kappa)\frac{p^2 f}{3\bar{b}}.$$

Rearranging gives

$$\alpha\frac{\bar{b}}{2} + (f(1-\alpha)-\kappa)p\frac{pf}{3\bar{b}} - \alpha\frac{(pf)^2}{3\bar{b}} - \int_{-\Delta}^{\Delta} h(\hat{x}+x)\frac{x}{2\Delta^2}\frac{pf}{\bar{b}}\mathrm{d}x - \alpha\frac{\bar{b}^2-(pf)^2}{2\bar{b}} =$$
$$= \alpha\frac{(pf)^2}{6\bar{b}} + ((1-\alpha)f-\kappa)\frac{fp^2}{3\bar{b}} - \frac{pf}{2\Delta^2\bar{b}}\int_{-\Delta}^{\Delta} h(\hat{x}+x)x\mathrm{d}x.$$

In order to approximate the integral, a second-order Taylor extension around $x = 0$ is

used for $h(\hat{x}+x)x$. Accordingly, the term $h(\hat{x}+x)x$ equals

$$h(\hat{x}+x)x = h(\hat{x})0 + x(h'(\hat{x})0 + h(\hat{x})) + x^2 h'(\hat{x}+\epsilon) = xh(\hat{x}) + x^2 h'(\hat{x}+\epsilon) \tag{6}$$

with $\epsilon \in (0,x)$. We define $h'_{\max} = \max_{\epsilon\in[-\Delta,\Delta]} h'(\hat{x}+\epsilon) > 0$ to derive an upper bound in the next step. Consequently, the integral is bounded by

$$\int_{-\Delta}^{\Delta} h(\hat{x}+x)x\mathrm{d}x \leq \int_{-\Delta}^{\Delta} xh(\hat{x}) + x^2 h'_{\max}\mathrm{d}x = \frac{2}{3}\Delta^3 h'_{\max}. \tag{7}$$

Therefore we get a lower bound for the derivative.

$$\frac{\partial W(\hat{x},p)}{\partial \Delta} \geq \frac{pf}{3\bar{b}}\left(p((1-\frac{\alpha}{2})f-\kappa) - \Delta h'_{\max}\right) \tag{8}$$

Under a per-se rule or with a perfect auditing technology, there would be no measurement error and $\Delta = 0$. Yet having some legal uncertainty and some imprecision in the measurement increases social welfare as the derivative is positive for small Δ.

$$\Delta < \frac{(2-\alpha)f - 2\kappa}{2h'_{\max}}p \Rightarrow \frac{\partial W(\hat{x},p)}{\partial \Delta} > 0. \tag{9}$$

Notice that the assumption $\kappa < (1-\alpha)f$ ensures that $(2-\alpha)f - 2\kappa$ is positive and the condition is feasible.[18] This concludes the proof and shows how legal uncertainty might increase social welfare. □

The next step considers the effects of the legal uncertainty on the policy threshold $\hat{x}$. The legal uncertainty Δ about the threshold of legality inherent in the legal rules allows to have a lower policy threshold without hurting too many procompetitive business practices, if the externality function is convex.

Proposition 2. *If the externality function, $h(x)$, is convex, legal uncertainty decreases the policy threshold $\hat{x}$. With a concave externality function legal uncertainty raises the policy threshold.*

Proof: For social welfare stated in (5), the first derivative with respect to the policy threshold $\hat{x}$ equals

$$\frac{\partial W(\hat{x},p)}{\partial \hat{x}} = \alpha\frac{\bar{b}}{2} - h(\hat{x}-\Delta) - h(\hat{x}+\Delta)\frac{\bar{b}-pf}{\bar{b}} + h(\hat{x}-\Delta) - \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} h(x)\frac{pf}{2\Delta\bar{b}}\mathrm{d}x -$$

[18] To make a statement about welfare, if the uncertainty increases, consider the second derivative

$$\frac{\partial^2 W(\hat{x},p)}{\partial \Delta^2} = \frac{pf}{\Delta^3\bar{b}}\int_{-\Delta}^{\Delta} h(\hat{x}+x)x\mathrm{d}x - \frac{pf}{2\Delta\bar{b}}(h(\hat{x}+\Delta)-h(\hat{x}-\Delta)) \leq \frac{pf}{\Delta\bar{b}}\int_{-\Delta}^{\Delta}\frac{1}{3}h'_{\max} - \frac{1}{2}h'(\hat{x}+x)\mathrm{d}x \tag{10}$$

It is negative, if, e.g., $h(x)$ is linear. In this case the benefits of legal uncertainty decrease in its size.

$$- \alpha \frac{\bar{b}^2 - (pf)^2}{2\bar{b}} - ((1-\alpha)f - \kappa)p\frac{\bar{b} - pf}{\bar{b}} + \frac{\bar{b} - pf}{\bar{b}} h(\hat{x} + \Delta). \tag{11}$$

Rearranging the terms delivers

$$\frac{\partial W(\hat{x}, p)}{\partial \hat{x}} = \alpha \frac{(pf)^2}{2\bar{b}} - p((1-\alpha)f - \kappa)\frac{\bar{b} - pf}{\bar{b}} - \frac{pf}{2\Delta\bar{b}} \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} h(x)\mathrm{d}x. \tag{12}$$

Then the first-order condition corresponds to

$$\alpha pf\Delta - 2\frac{(1-\alpha)f - \kappa}{f}\Delta(\bar{b} - pf) = \int_{\hat{x}-\Delta}^{\hat{x}+\Delta} h(x)\mathrm{d}x, \tag{13}$$

if an interior solution exists.[19] As, the externality function h is increasing, there is a unique solution to (13). Due to assumption (1) and the increasing externality function, equation (13) shows that there is consequently never a corner solution at the right end and $\hat{x} < 1 - \Delta$. However there might be a corner solution at the left end with $\hat{x} = 0$, if the business practice under consideration is very harmful. In this case the competition authority uses a per-se prohibition. Using Taylor's theorem there is a function $\epsilon \colon [-\Delta, \Delta] \mapsto (\hat{x} - \Delta, \hat{x} + \Delta)$, such that the integral equals

$$\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} h(x)\mathrm{d}x = \int_{-\Delta}^{\Delta} h(\hat{x}) + h'(\hat{x})x + \frac{1}{2}h''(\epsilon(x))x^2\mathrm{d}x = 2\Delta h(\hat{x}) + \frac{1}{2}\int_{-\Delta}^{\Delta} h''(\epsilon(x))x^2\mathrm{d}x.$$

The sign of the last term corresponds to the sign of the second derivative of the externality function. Consequently, the curvature of the externality function determines the effect of legal uncertainty and for the policy threshold $\hat{x}$ it holds that

$$h(\hat{x}) = \frac{1}{2}\alpha pf - \frac{(1-\alpha)f - \kappa}{f}(\bar{b} - pf) - \frac{1}{4\Delta}\int_{-\Delta}^{\Delta} h''(\epsilon(x))x^2\mathrm{d}x. \tag{14}$$

On the other hand, the condition for the policy threshold is

$$h(\hat{x}) = \frac{1}{2}\alpha pf - \frac{(1-\alpha)f - \kappa}{f}(\bar{b} - pf), \tag{15}$$

if there is no legal uncertainty, $\Delta = 0$, and the measurement is sharp. The same policy threshold results if the externality function h is linear. Making the externality function concave increases the policy threshold $\hat{x}$ above (15). On the contrary, a convex externality function results in a lower threshold. □

Depending on the curvature of the externality function, the policy threshold $\hat{x}$ is changed by the legal uncertainty of the legal rule. If the externality function is con-

[19] The Appendix A.4 shows that the conditions of a maximum are satisfied.

vex, there is a low threshold of legality prohibiting more actions than in the case with an exact measurement and legal certainty about the policy threshold. Thus, the competition authority adapts its policy to the uncertainty. Finally, I scrutinize the optimal auditing probability and show that there will always be some auditing. In contrast to the common costly state verification models, even deterministic auditing is optimal, if auditing costs are small. Lemma 1 in the Appendix A.3 determines the optimal auditing level for interior solutions.

Proposition 3. *Auditing occurs with positive probability. Moreover, if the auditing costs are sufficiently low, $c < \frac{1}{3}\Delta((1-5\alpha)f - \kappa)$, complete auditing is optimal.*

Proof: To show that it is never optimal to have no auditing, I compare social welfare with complete deterrence to welfare without auditing. Without any auditing, social welfare is independent of the policy threshold $\hat{x}$

$$\alpha\frac{\bar{b}}{2} + \int_0^1 h(x)\mathrm{d}x < 0, \tag{16}$$

which is negative by assumption (1). With complete deterrence, $p \geq \frac{\bar{b}}{f}$, the policy threshold $\hat{x}$ matters and social welfare $W(\hat{x}, p)$ equals

$$\int_0^{\hat{x}-\Delta} \frac{\alpha\bar{b}}{2} - h(x)\mathrm{d}x + \Delta\frac{2\alpha\bar{b}^2}{3pf} - \int_{-\Delta}^{\Delta(2\frac{\bar{b}}{pf}-1)} \frac{\Delta - x}{2\Delta}h(\hat{x}+x)\mathrm{d}x + ((1-\alpha)f - \kappa)\frac{\Delta p}{3} - cp.$$

By assumption (2) the last expression is positive for the optimal $\hat{x}$ and $\Delta = 0$. Therefore social welfare is lower without auditing, $W(\hat{x}, 0) < W(\hat{x}, \frac{\bar{b}}{f})|_{\Delta=0}$. Increasing Δ raises social welfare by Proposition 1. Consequently, having a small $\Delta > 0$ increases the objective function of the competition authority further. Therefore social welfare is higher with auditing, $W(\hat{x}, 0) < W(\hat{x}, \frac{\bar{b}}{f})$, and it is never optimal to abstain from auditing, $p = 0$.

On the other hand, it is sometimes optimal to implement complete auditing, $p = 1$. Increasing the value of the externality function in the interval $[1-\Delta, 1]$ increases the optimal p according to Lemma 1. Therefore using complete deterrence, $p \geq \frac{\bar{b}}{f}$, is optimal, if some actions with positive mass are very harmful. This does not imply complete auditing. Yet if, in addition, the auditing costs are small, $c < \frac{1}{3}\Delta((1-5\alpha)f - \kappa)$, the competition authority prefers to audit every firm.[20] The reason is that complete deterrence just holds for firms in the illegal region, $x > \hat{x} + \Delta$, in contrast to firms close to the threshold of legality, $\hat{x} - \Delta \leq x \leq \hat{x} + \Delta(2\frac{\bar{b}}{pf} - 1)$, that are still active, although they might face a fine. However, this extreme case is impossible, if the legal uncertainty is sufficiently small. In that case decreasing p from $p = 1$ slightly allows the competition authority to save on auditing costs, while at the same time the deterrence effect does not change for

[20] This is derived in Lemma 2 in the Appendix A.3.

$x \notin (\hat{x} - \Delta, \hat{x} + \Delta)$, because $f > \bar{b}$. □

This completes the analysis of the model.

5 Extensions

So far I abstracted from the interaction between the judicial system and the competition authority. In the following I want to model it explicitly. Thus, a convicted firm might turn to the courts. If the courts decide in favor of the firm and repeal the assessment of the competition authority, the fine is reassessed. Therefore there is an additional stage, $t = 5$, at which the courts can overturn the decision of the competition authority and change the fine f to f_L or f_H with $f_L < f \leq f_H$. It seems plausible that there are no objective probabilities available of the courts setting the fine because of few precedents and inconsistent decisions. This creates ambiguity, as it is unclear how severely the courts assess the infringement of the firm.[21] The firm exhibits ambiguity aversion à la Gilboa and Schmeidler (1989). Therefore it has a set of priors Π and maximizes its expected utility under the worst possible distribution. The preferences of the firm are represented by

$$\min_{\pi \in \Pi} \int u(y) \mathrm{d}\pi(y). \tag{17}$$

Due to the risk neutrality of the firm, the von-Neumann Morgenstern utility function u is linear. I assume that the subjective probability of a high fine f_H can take one of the following values $\{q_1, \ldots, q_n\}$. An ambiguity-neutral agent is not concerned by this ambiguity and behaves as if she anticipates the high fine with a subjective probability q_N, such that

$$\min_{1 \leq i \leq n} q_i < q_N < \max_{1 \leq i \leq n} q_i.$$

I write $\mathrm{E}(f) = q_N f_H + (1 - q_N) f_L$ for the fine expected by an ambiguity-neutral agent. This completes the model set-up and allows to turn to the analysis. Ambiguity-averse firms will turn to the courts whenever $\max_i (q_i f_H + (1 - q_i) f_L) = f^A < f$. Assume this is the case. Then the cut-off level for firms to be active is

$$\hat{b}(x) = \begin{cases} \hat{p} f^A & \text{if } x > \hat{x} + \Delta \\ \frac{x - \hat{x} + \Delta}{2\Delta} p f^A & \text{if } \hat{x} - \Delta \leq x \leq \hat{x} + \Delta \\ 0 & \text{if } x < \hat{x} - \Delta. \end{cases} \tag{18}$$

The competition authority takes this modified cut-off level into account when setting its policy. The next proposition shows that the additional uncertainty surrounding the judicial decision has positive welfare effects, because the ambiguity about the fine makes the firm implement less anticompetitive business practices than in the absence of ambiguity.

[21] The notion of ambiguity is introduced in the Appendix A.2 which also discusses the assumption of ambiguity-averse firms.

Proposition 4. *Having ambiguous procedures increases social welfare compared to a regime without ambiguity, if auditing costs are higher than the expected income due to fines collected by the competition authority:*

$$c > ((1-\alpha)E(f) - \kappa)(\frac{1}{3}pf^A\Delta + (1-\hat{x})(\bar{b} - pf^A))/\bar{b}.$$

Proof: Obviously the competition authority can implement any policy $\hat{x}, p$ with a strictly lower probability of auditing than in the absence of ambiguity. As it is easier to enforce compliance, the competition authority can save on auditing costs. The reason is that, at a given probability of auditing, an ambiguity-averse firm will have a higher cut-off value $\hat{b}(x)$ than an ambiguity-neutral firm in the region with intervention of the competition authority $x > \hat{x} - \Delta$. This is caused by overvaluing the fine. Notice that the ambiguity aversion does not change the beliefs, but the firms only act, as if the probability for a high fine f_H were higher. Yet at the same time the fine income of the competition authority decreases. This trade-off is captured in the following derivative with respect to the ambiguity tolerance, here the ratio of the fine expected by an ambiguity-neutral agent and an ambiguity-averse one, which equals

$$\frac{\partial W(\hat{x},p)}{\partial \mathrm{E}(f)/f^A} = \frac{1}{\mathrm{E}(f)}\left(\frac{(1-\alpha)\mathrm{E}(f)-\kappa}{\bar{b}}\left(\frac{\Delta}{3}pf^A + (1-\hat{x})(\bar{b}-pf^A)\right)pf^A - cpf^A\right) \quad (19)$$

by the envelop theorem.[22] This derivative is negative, if the change in income of fines imposed by the competition authority is smaller than the marginal auditing costs

$$\frac{(1-\alpha)\mathrm{E}(f)-\kappa}{\bar{b}}\left(\frac{\Delta}{3}pf^A + (1-\hat{x})(\bar{b}-pf^A)\right) < c. \qquad \square$$

To understand this result, notice that the ambiguity has no negative effects on firms' behavior, because the competition authority can always balance the ambiguity by reducing the auditing probability and the firms can avoid the ambiguity.[23] Moreover the ambiguity only affects firms violating the rules of competition law. If the firm is in the procompetitive region, $x \leq \hat{x} - \Delta$, it does not care about the ambiguity in the fine. If it is in the prohibited region, it has always the possibility to abstain from the controversial business practice and avoid the ambiguity.

6 Conclusion

This article sets up a costly state verification model to scrutinize the effects of legal uncertainty in competition law. It is commonly assumed that a rule of reason approach is more selective than a per-se rule and is hence welfare-enhancing. Yet the inherent legal

[22] Formally, I substitute the choice variable p by $p' = pf^A$, then take the derivative and resubstitute again. The reason is that the competition authority takes the ambiguity into account when setting p.

[23] The first part of the argument requires homogeneous attitudes towards ambiguity of the firms.

uncertainty of the rule of reason might be a drawback. This paper points out why legal uncertainty itself might further raise social welfare.

The uncertainty about the threshold of legality might be due to imprecision in the measurement of the competition authority, missing precedents or unclear rules. The model considered here shows that this uncertainty may be welfare-enhancing as the deterrence becomes more selective without having more selective rules. Firms self-select according to the nature of the business practice under consideration. This is beneficial, as firms have better knowledge and information to assess the nature of the business practice than the competition authority.

Yet there are limitations to the benefits of legal uncertainty. If there is too much legal uncertainty, this will decrease social welfare, because it will deter procompetitive business practices and encourage anticompetitive ones. In a dynamic framework additional effects may appear. Legal uncertainty might give firms incentives to experiment and therefore implement more anticompetitive business practices than under legal certainty. On the other hand, the costs of such behavior, e. g., possible fines, are incurred by a single firm, while the benefits spill over to all market participants, as they learn the court decision and the legal uncertainty is reduced. Therefore competition might decrease the experimentation.

A Appendix

A.1 Endogenous fines

Assume the fine f is endogenous and a function of the observed dimension of the type x. Then the competition policy consists of the auditing probability and the fine, as the threshold of legality is implicitly defined by the schedule of the fine. To guarantee existence of a solution, there has to be an upper bound $\bar{f}$ for the fines. Otherwise fines would be raised to infinity to save on auditing costs. For a discussion of this effect, see, e. g., Kaplow and Shavell (1994, p. 586) and Polinsky and Shavell (1979). The upper bound corresponds to wealth constraints of the firm or legal considerations. If $\bar{f}$ is sufficiently high, pointwise optimization yields an optimal fine of

$$f(x) = \begin{cases} 0 & \text{for } x < \hat{x} \\ \frac{\kappa}{(1-\alpha)} & \text{for } x \in [\hat{x}, x_1] \\ \frac{(1-\alpha)\bar{b}+h(x)+p\kappa}{(2-\alpha)p} & \text{for } x \in (x_1, x_2) \\ \bar{f} & \text{for } x \geq x_2 \end{cases}$$

with $\hat{x} \leq x_1 \leq x_2 < 1$.[24] x_1 is determined by $h(x_1) = \frac{p\kappa}{(1-\alpha)} - (1-\alpha)\bar{b}$ and x_2 by $h(x_2) = (2-\alpha)p\bar{f} - (1-\alpha)\bar{b} - p\kappa$. Thus, for socially beneficial types, i.e., low x, the

[24] This result holds for the case of a rule of reason without legal uncertainty.

competition authority tolerates active firms without setting fines. As the threshold of legality $\hat{x}$ is passed, a fine of $\kappa/(1-\alpha)$ is imposed, because lower fines do not justify spending the costs κ to make x verifiable. Then there is a region $x \in (x_1, x_2)$ where f is strictly increasing, until it reaches $\bar{f}$ at x_2. Finally, the fine equals the upper bound, $f(x) = \bar{f}$ for all $x \geq x_2$.

A.2 Ambiguity and why it matters

The ambiguity in the extensions captures the vagueness of procedures in legal rules. In general, ambiguity denotes uncertainty about probabilities resulting from missing relevant information. Thus, ambiguity aversion describes a preference for lotteries where the firm has more confidence in its probability assessment. A classical example is due to Ellsberg (1961) and considers two urns. In both urns there are red and black balls. Yet the distribution of the balls is only known in the first urn. The ratio of red to black balls in the second urn is unknown. Individuals can place bets on either a red ball or a black ball drawn from one of the urns. Most subjects prefer to take the bet on balls drawn from the first urn, the familiar one, no matter what color was specified. Thus, under ambiguity, the exact probabilities are unknown. Missing precedents might imply this lack of reliable probabilities for a conviction. Savage (1954) and Schmeidler (1989) have developed two axiomatized approaches to this problem. The Subjective Expected Utility of Savage requires agents to be ambiguity-neutral. The representation of Schmeidler (1989) allows agents to have preferences with respect to ambiguity.[25]

In order to guarantee that ambiguity matters in competition law firms have to be ambiguity-averse. Yet in contrast to risk aversion, ambiguity-averse firms seem more plausible. First, there are empirical observations, like Kunreuther et al. (1995) or Cabantous (2007), that even professionals in firms behave in an ambiguity-averse way. Ambiguity makes insurers usually more restrictive, i. e., they request higher premiums and reject to offer an insurance policy in more cases than in the absence of ambiguity. Second, even from a theoretical point of view firms may be ambiguity-averse. Marinacci (1999) shows that ambiguity changes the law of large numbers and the ambiguity does not vanish when different ambiguous random variables are combined. Therefore even perfect diversification does not protect an investor from ambiguity in the underlying assets. Third, joint decision-making on boards or on committees does not mitigate the effects of ambiguity aversion. On the contrary, Keller et al. (2007) show that collaboration even amplifies ambiguity aversion and joint decision making exhibits a higher degree of ambiguity aversion than the average member. Consequently, the assumption of ambiguity-averse firms seems plausible.[26]

[25] Following Schmeidler (1989) different representations with ambiguity preferences have been proposed by, e. g. Gilboa and Schmeidler (1989) or Klibanoff et al. (2005).

[26] The assumption has been made previously, e. g., in Mukerji (1998) for incomplete contracting or Tor and Rinner (2010) for retail price maintenance.

A.3 Additional proofs

Lemma 1 determines the optimal auditing level for interior solutions.

Lemma 1. *The optimal auditing probability is determined by* (21).

Proof: For social welfare stated in (5) the first derivative with respect to the auditing probability p equals

$$\frac{\partial W(\hat{x},p)}{\partial p} = -\alpha\Delta\frac{2pf^2}{3\bar{b}} + f\int_{-\Delta}^{\Delta}\frac{x+\Delta}{2\Delta\bar{b}}h(\hat{x}+x)\mathrm{d}x - \alpha(1-\hat{x}-\Delta)\frac{pf^2}{\bar{b}} + \frac{f}{\bar{b}}\int_{\hat{x}+\Delta}^{1}h(x)\mathrm{d}x + \\ + ((1-\alpha)f-\kappa)(\frac{2pf}{3\bar{b}}\Delta + (1-\hat{x})\frac{\bar{b}-2pf}{\bar{b}}) - c. \tag{20}$$

Consequently, the first-order condition for an interior solution equals

$$p = \frac{((1-\alpha)f-\kappa)(1-\hat{x})\bar{b} + f\int_{-\Delta}^{\Delta}\frac{x+\Delta}{2\Delta}h(\hat{x}+x)\mathrm{d}x + f\int_{\hat{x}+\Delta}^{1}h(x)\mathrm{d}x - c\bar{b}}{f((2-\alpha)f-2\kappa)(1-\hat{x}-\frac{1}{3}\Delta)}. \tag{21}$$

By Proposition 3 the optimal p is always positive. Therefore only the corner solution $p=1$ exists for values of equation (21) bigger than 1. □

The next lemma shows the optimality of complete auditing for small costs.

Lemma 2. *If the costs of auditing are sufficiently small, $c < \frac{1}{3}\Delta((1-5\alpha)f-\kappa)$, the competition authority implements complete auditing, $p=1$.*

Proof: Proposition 3 shows that with complete deterrence, $p \geq \frac{\bar{b}}{f}$, social welfare equals $W(\hat{x},p) =$

$$\int_{0}^{\hat{x}-\Delta}\frac{\alpha\bar{b}}{2} - h(x)\mathrm{d}x + \Delta\frac{2\alpha\bar{b}^2}{3pf} - \int_{-\Delta}^{\Delta(2\frac{\bar{b}}{pf}-1)}\frac{\Delta-x}{2\Delta}h(\hat{x}+x)\mathrm{d}x + ((1-\alpha)f-\kappa)\frac{\Delta p}{3} - cp.$$

The derivative of the social welfare above with respect to p is

$$-\Delta\frac{4\alpha\bar{b}^2}{3p^2f} + (1-\frac{\bar{b}}{pf})h(\hat{x}+\Delta(2\frac{\bar{b}}{pf}-1))4\Delta\frac{\bar{b}}{p^2f} + ((1-\alpha)f-\kappa)\frac{\Delta}{3} - c \tag{22}$$

Rearranging yields

$$\frac{1}{3}\Delta\left((1-\alpha)f-\kappa-\frac{4\alpha\bar{b}^2}{p^2f}\right) + (1-\frac{\bar{b}}{pf})h(\hat{x}+\Delta(2\frac{\bar{b}}{pf}-1))4\Delta\frac{\bar{b}}{p^2f} - c \tag{23}$$

For $p \geq \frac{\bar{b}}{f}$ it holds $\frac{4\alpha\bar{b}^2}{p^2f} \leq 4\alpha f$. Therefore the first term is bigger than $\frac{1}{3}\Delta((1-5\alpha)f-\kappa)$, which is positive for small welfare weights α of firms' profits. Moreover it is possible to neglect the second term in the derivative, as it is positive. This shows that (23) is positive, if $c < \frac{1}{3}\Delta((1-5\alpha)f-\kappa)$. □

A.4 Second-order conditions

Given the two-dimensional optimization, there are three conditions in order to assure that the paper characterizes the optimum. First, I compute the second derivatives. From (12) it follows

$$\frac{\partial^2 W(\hat{x},p)}{\partial \hat{x}^2} = -\frac{pf}{2\Delta\bar{b}}(h(\hat{x}+\Delta)-h(\hat{x}-\Delta)) < 0, \tag{24}$$

and differentiating (20) with respect to p yields

$$\frac{\partial^2 W(\hat{x},p)}{\partial p^2} = -\frac{1}{\bar{b}}(1-\hat{x}-\frac{1}{3}\Delta)f((2-\alpha)f-2\kappa) < 0 \tag{25}$$

for small Δ.

Finally, the following steps prove that the discriminant is positive in the relevant range. Differentiating (12) with respect to p results in the cross-derivative

$$\begin{aligned}\frac{\partial^2 W(\hat{x},p)}{\partial \hat{x}\partial p} &= \alpha\frac{pf^2}{\bar{b}} - ((1-\alpha)f-\kappa)\frac{\bar{b}-2pf}{\bar{b}} - \frac{f}{2\Delta\bar{b}}\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} h(x)\mathrm{d}x = \\ &= \frac{pf^2}{2\bar{b}}\alpha + \frac{pf}{\bar{b}}((1-\alpha)f-\kappa) = \frac{pf}{2\bar{b}}((2-\alpha)f-2\kappa). \end{aligned}\tag{26}$$

The second equality here follows from the first-order condition (13). The determinant of the Hessian equals

$$\begin{aligned}&\frac{\partial^2 W(\hat{x},\hat{b})}{\partial \hat{x}^2}\frac{\partial^2 W(\hat{x},\hat{b})}{\partial \hat{b}^2} - \left(\frac{\partial^2 W(\hat{x},\hat{b})}{\partial \hat{b}\partial \hat{x}}\right)^2 = \\ &= \frac{pf}{2\Delta\bar{b}^2}(h(\hat{x}+\Delta)-h(\hat{x}-\Delta))(1-\hat{x}-\frac{1}{3}\Delta)f((2-\alpha)f-2\kappa) - \left(\frac{pf}{2\bar{b}}((2-\alpha)f-2\kappa)\right)^2 = \\ &= \frac{pf^2}{2\bar{b}^2}((2-\alpha)f-2\kappa)\left(\frac{1}{\Delta}(h(\hat{x}+\Delta)-h(\hat{x}-\Delta))(1-\hat{x}-\frac{1}{3}\Delta) - \frac{p}{2}((2-\alpha)f-2\kappa)\right) = \\ &= \frac{p}{4\Delta\bar{b}^2}((2-\alpha)f-2\kappa)\int_{\hat{x}-\Delta}^{\hat{x}+\Delta} 2f^2(1-\hat{x}-\frac{1}{3}\Delta)h'(x) - fh(x) - \bar{b}((1-\alpha)f-\kappa)\mathrm{d}x.\end{aligned}$$

For the simplification in the third line again the first-order condition (13) is used. The assumption $\kappa < (1-\alpha)f$ ensures that $(2-\alpha)f-2\kappa$ is positive. Finally, the determinant is positive, if the slope of the externality function h is sufficiently high, as this lowers $\hat{x}$ and increases h'. By assumption (3) this is satisfied. The assumption holds for example, if the harm function $h(\cdot)$ is linear and the slope is sufficiently high. The reason is that $\hat{x}$ is decreasing in the slope, while $h'(\cdot)$ is increasing. Therefore $(h(\hat{x}+\Delta)-h(\hat{x}-\Delta))(1-\hat{x}-\frac{1}{3}\Delta)$ in the second line increases, while $\frac{p}{2}((2-\alpha)f-2\kappa)$ is bounded from above by $\frac{1}{2}((2-\alpha)f-2\kappa)$.

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