Menu pricing with reference-dependent preferences

Kris De Jaegher and Najmeh Rezaei-Khavas
Utrecht University School of Economics

Preliminary version, February 2012

Abstract
This paper offers a model of menu pricing by a monopolist who does not observe the consumers' valuations, when consumers have reference-dependent preferences. Assuming that the monopolist can make consumers expect to buy the desired variety of the good, and that these expectations determine the consumers' reference points, we obtain two main results. *First*, with reference-dependent preferences, menu pricing is *possible* even if the single-crossing property is violated (high-valuation consumers do not have a larger marginal utility of quality than low-valuation consumers). *Second*, that when consumers facing consumers with reference-dependent preferences, menu pricing may become *more desirable* to the monopolist compared to the screening of high-valuation consumers.

Keywords: price discrimination, menu pricing, reference-dependent preferences.
JEL codes: D82, D42, D03.

1. Introduction

Consider an airline which faces poor and rich passengers, but is not able to tell them apart. The airline attempts to design a business class ticket and an economy class ticket, where the latter is cheaper but has lower quality, in such a way that rich passengers self-select the business class ticket and poor passengers self-select the economy class ticket. Is it easier or more difficult for the airline to do this when passengers have reference-dependent preferences, and are subject to loss aversion (Köszegi and Rabin, 2006)? The following intuition suggests that reference-dependence preferences make self-selection easier. Let it be the case that rich consumers expect to buy business class tickets, and poor consumers expect to buy economy class tickets, and let this also determine their respective reference points. When considering to buy the economy class ticket instead of the business class ticket, the loss averse rich consumer focuses more on the loss in quality than on the gain of paying a lower price, and is thereby less inclined to switch to economy class. In the same way, when considering to buy the business class ticket instead of the economy class ticket, the loss averse poor consumer focuses more on the loss caused by paying a higher price than on the gain in quality, and will be less inclined to upgrade to business class.

We construct a simple version of Mussa and Rosen's (1978) model of monopolistic menu pricing, with the added feature that consumers have reference-dependent preferences. We follows Köszegi and Rabin's (2006) model of reference-dependent preferences, where consumers total utility consists of a standard, intrinsic utility part, and of a gain-loss part, where losses are subject to loss aversion. Gains and losses are with respect to the reference point determined by the consumers' expectations. It is assumed that consumers are always in a *personal equilibrium*, where they expectations are fulfilled. In Köszegi and Rabin's model, there may be multiple personal equilibria, where for the same price and quality it may both be a personal equilibrium that the consumer buys the good because she expects to buy it, and that she does not buy it because she does not expect to buy it. If there are multiple personal equilibria, we assume that the monopolist can ensure that the consumer forms expectations such that she expects to buy a particular variety of the good. This may take place if the monopolist is able to influence the consumers' expectations, and thereby their reference points.

Our model shows two ways in which the fact that consumers have reference-dependent preferences may facilitate menu pricing compared to the case where they have standard preferences. *First*, with reference-dependent preferences, menu pricing becomes possible even if the consumers' intrinsic utility does not satisfy the single-crossing property, where this property requires that richer consumers do not only have a higher absolute willingness-to-pay for quality, but also have a higher marginal utility of quality. With reference-dependent preferences, sorting of consumers arises naturally once rich consumers expect to buy high quality at a high price, and poor consumers expect to buy low quality at a low price. Rich consumers who consider buying low quality consider this as a loss, and for this reason attach a higher overall marginal utility (including the gain-loss part of their utility) to quality. Poor consumers who consider buying high quality focus on the loss of having to pay a higher price, and therefore attach a lower overall marginal utility to quality.

Second, it is possible that when facing consumers *without* reference-dependent preferences, the monopolist prefers offering a single high price and thus prefers screening the high-valuation consumers rather than menu pricing, whereas when facing consumers *with* reference-dependent preferences, the monopolist prefers menu pricing. In general, the self selection necessary for menu pricing to work succeeds only if the monopolist offers high quality at a discount. The monopolist may shy away from menu pricing if the discount needed to make menu pricing work is too large. However, a rich consumer with reference-dependent preferences is less inclined to choose the low-quality variant of the monopolist's product, given her aversion to quality loss. For this reason, the monopolist does not need to offer such a high discount, and is more likely to prefer menu pricing.

Several recent research relates to this paper. In general, the paper is part of a strand of research that investigates firm behavior when firms face naïve consumers (see Armstrong and Huck (2010)). Within this literature, our paper is part of a growing body of research that integrates consumers with prospect-theory preferences or reference-dependent preferences into standard economic models. The difference between this literature and our paper is that we find an argument for price variation with reference-dependent preferences, whereas the literature on the contrary shows how reference-dependence can lead to price stickiness or to less price variation (see e.g. Heidhues and Köszegi (2005, 2008)). The paper closest to ours is Hahn et al. (2010). These authors also investigate the menu pricing model with reference-dependent consumers. As essential difference with our paper is that Hahn et al. assume that consumers form their expectations, and therefore also form their reference point, before they find out their own valuation of the good. Intuitively, price is then sticky around this average expectation, so that it becomes more likely that the monopolist does not prefer menu pricing, but instead offers a single price. Our model instead is based on the assumption that rich and poor consumers form different expectations, leading us to a result that is diametrically opposed to the one of Hahn et al.

In Section 2, we introduce our model of menu pricing where consumers have reference-dependent preferences. Section 3 treats the case of symmetric information between the monopolist and consumers as a benchmark, followed by the case of asymmetric information in Section 4. We end with a discussion in Section 5.

2. The model

At stage 1, Nature decides on the type of the consumer. The consumer is of type θ_1 with probability $(1-\lambda)$, and of type θ_2 with probability λ. The consumer finds out her type, but the monopolist does not. At stage 2, the monopolist offers the consumer the possibility a menu of price-quality combinations, which allow the consumer to buy a single unit of quality *s* at price *p*. As there are only two types, the monopolist sets at most two price-quality combinations. These are denoted as (s_1, p_1) and (s_2, p_2), where $s_2 \geq s_1$. The price-quality combinations may either be identical or different. When they are different, the monopolist will design them such that consumer θ_1 chooses (s_1, p_1), and consumer θ_2 chooses (s_2, p_2). As the consumer is better off the higher the quality she gets (see below), it only makes sense for the monopolist to offer two price-quality combinations if $p_2 \geq p_1$, as the consumer otherwise opts for one of the price-quality combinations, independent of her type.

At stage 3, the consumer either chooses one price-quality combination from the menu, or does not buy the good at all. At stage 4, if the consumer chose one of the price-quality combinations from the menu, the monopolist provides the good with quality and price as agreed on, and the players obtain their payoffs. The payoff of the monopolist equals $p - C(s)$, where $C(s)$ is the monopolist's cost function of quality, with $C'(s) > 0$ (including $C'(0) > 0$) and $C''(s) > 0$. The consumer's utility contains an intrinsic-utility part, and a gain-loss part, where her total utility is additively separable in her intrinsic utility and her gain-loss utility. The intrinsic utility of any choice that the consumer makes equals $U(\theta, s) - p$, where $U(\theta, s)$ is the consumer's utility of obtaining quality *s* when she is of type θ. We assume throughout that $U(\theta, 0) = 0$, $U_s(\theta, s) > 0$, $U_{ss}(\theta, s) < 0$ and $U_s(\theta, 0) = \infty$. $U(\theta, 0) = 0$ means that for a consumer who does not pick any of the price-quality combinations, her intrinsic utility is zero. Further, we assume throughout that

(A1) For any *s*, $U(\theta_2, s) > U(\theta_1, s)$ (i.e. $U_\theta(\theta, s) > 0$),

meaning that type θ_2 receives a higher utility from quality than type θ_1. This gives the monopolist a motive for attempting to price discriminate. Finally, we formulate the standard single-crossing property, where we immediately note that we will not systematically assume this property:

(A2) For any $s_2 > s_1$, $U(\theta_2, s_j) - U(\theta_2, s_k) > U(\theta_1, s_j) - U(\theta_1, s_k)$ (i.e. $U_{s\theta}(\theta, s) > 0$)

The consumer's gain-loss utility is determined by the reference price p_r and the reference quality s_r that she forms. Following Köszegi and Rabin (2006), we assume that the consumer's reference price and quality are each time determined by the price-quality combination that she expects to choose. When the consumer expects to buy at price p_r but instead buys it at a lower price *p*, she experiences a gain $\beta_p(p_r - p)$, which is added to her total utility, where $\beta_p \geq 0$ is a parameter reflecting how much she cares about price gains/losses. When the price at which she buys is higher than p_r, she experiences a loss $\beta_p \delta(p - p_r)$, which is subtracted from her total utility. The parameter δ, with $\delta \geq 0$ reflects the consumer's degree of loss aversion. In the same way, when the consumer expects to buy a quality of s_r but instead buys higher quality s, she experiences a gain $\beta_s(s-s_r)$ which is added to her intrinsic utility. If instead she buys a lower quality she experiences a loss $\beta_s\delta(s_r-s)$, which is subtracted from her intrinsic utility. Each time $\beta_q \geq 0$ is a parameter reflecting how much she cares about quality gains/losses. Thus, for instance a consumer of type θ_2 who expects to choose high quality s_2 and pay high price p_2 for it, but instead ends up with lower quality s_1 and lower price p_1, gets overall utility $U(\theta_2,s_1)-p_1-\beta_s\delta(s_2-s_1)+\beta_p\delta(p_2-p_1)$. Note that for $\beta_s=\beta_p=0$, we have the standard model of monopolistic menu pricing.

Following Köszegi and Rabin (2006), we adopt a rational expectations approach, where the consumer's expectations about what she will choose are also fulfilled. The resulting choice of the consumer is then referred to as a *personal equilibrium*. This raises the possibility of multiple personal equilibria. For one and the same price-quality combination offered by the monopolist, there may be both a personal equilibrium where the consumer prefers to buy the good because she expects to buy it, and a personal equilibrium where the consumer prefers not to buy the good because she prefers not to buy it. For two price-quality combinations offered by the monopolist, it is possible that there both is a personal equilibrium where the consumer always chooses one price-quality combination, and a personal equilibrium where each type of consumer chooses a different price-quality combination. In this case, Köszegi and Rabin assume that the consumer can ex ante compare the different personal equilibria, and picks out the personal equilibrium best to her, then referred to as the *preferred personal equilibrium*. Unless indicated otherwise, we here assume that the monopolist has the power to influence the consumer's expectations, and thereby her reference point, to its advantage.

3. Symmetric information

As a benchmark, we first look at the case where not only the consumer, but also the monopolist finds out the consumer's type. We then have a simple model of third-degree price discrimination, where the monopolist can offer a different price-quality combination to each type, with as an additional feature that the consumer has reference-dependent preferences. The monopolist maximizes its expected profit with respect to each consumer's type participation constraint, where the assumption is made that the monopolist can make sure that the consumer expects to buy the good.

$$\underset{p_1,p_2,s_1,s_2}{Max}\ (1-\lambda)[p_1-C(s_1)]+\lambda[p_2-C(s_2)] \quad (1)$$

such that

$$U(\theta_1,s_1)-p_1 \geq -\beta_s\delta U(\theta_1,s_1)+\beta_p p_1 \quad (2)$$

$$U(\theta_2,s_2)-p_2 \geq -\beta_s\delta U(\theta_2,s_2)+\beta_p p_2 \quad (3)$$

The left-hand side of the participation constraint takes a standard form: as pointed out above, if the consumer chooses the equilibrium price-quality combination, she achieves her reference price and quality, and does not experience gains or losses. If she chooses not to buy the good, given our assumption $U(\theta,0)=0$, the consumer obtains intrinsic utility zero. However, she also experiences a loss of not getting the quality she expected, and a gain of not having to pay a price.

As shown in Proposition 1, reference-dependent preferences, as long as there is loss aversion and as long as the consumer does not overweigh gain-losses in prices too much compared to gain-losses in quality, results in higher quality and a corresponding higher price in both price-quality combinations. Intuitively, a consumer who has as a reference point that she buys the good, and who is loss averse, when considering not to buy the good focuses more on the loss of not receiving the good than on the gain of not having to pay any price, whereby the consumer's willingness to pay increases. Further, whatever the outcome, reference-dependent preferences do not cause a distortion of the quality offered to one type of consumer versus the other type of consumer. The higher loss aversion, the more weight the consume puts on the quality gain-loss part, and the less weight she puts on the price gain-loss part, the higher quality in both price-quality combinations. Denote $s_i^{FB,R}$ and $p_i^{FB,R}$ ($s_i^{FB,NR}$ and $p_i^{FB,NR}$) as respectively the optimal quality and price offered to the consumer of type *i* when there is symmetric information (FB = first best), and when the consumer has reference-dependent preferences (no reference-dependent preferences).

Proposition 1. In the model with symmetric information:

(i) It is always better to offer two price-quality combinations rather than one.

(ii) $\beta_s\delta >=< \beta_p$ if and only if $s_i^{FB,R} >=< s_i^{FB,NR}$, $p_i^{FB,R} >=< p_i^{FB,NR}$.

(iii) Both with and without reference-dependent preferences, it is the case that $\frac{C'(s_1^{FB})}{C'(s_2^{FB})} = \frac{U_s(\theta_1, s_1^{FB})}{U_s(\theta_2, s_2^{FB})}$.

(iv) Further, $\partial s_i^{FB,R} / \partial\delta > 0$ and $\partial p_i^{FB,R} / \partial\delta > 0$, $\partial s_i^{FB,R} / \partial\beta_s > 0$ and $\partial p_i^{FB,R} / \partial\beta_s > 0$, $\partial s_i^{FB,R} / \partial\beta_p < 0$ and $\partial p_i^{FB,R} / \partial\beta_p < 0$.

Proof:

Note first that the price-quality combination offered to one type of consumer does not have any consequences for the price-quality combination offered to the other type of consumer. Letting $(\gamma_1, \gamma_2) \geq 0$ be the multipliers on constraints (i), (ii), respectively, the Kuhn-Tucker conditions for this problem can be written

$$(1-\lambda) - \gamma_1(1+\beta_p) = 0 \tag{4}$$

$$\lambda - \gamma_2(1+\beta_p) = 0 \tag{5}$$

$$-(1-\lambda)C'(s_1) + \gamma_1(1+\delta\beta_s)U_s(\theta_1, s_1) \leq 0 \tag{6}$$

$$-\lambda C'(s_2) + \gamma_2(1+\delta\beta_s)U_s(\theta_2, s_2) \leq 0 \tag{7}$$

along with the complementary slackness conditions for constraints (2) and (3).

Solving (4) and (5) we get $\gamma_1 = \frac{1-\lambda}{1+\beta_p}$ and $\gamma_2 = \frac{\lambda}{1+\beta_p}$, which are both are strictly positive so that the participation constraints (2) and (3) are binding. Substituting these values of γ_1, γ_2 into conditions (6) and (7) we obtain

$$C'(s_1^{FB,R}) = \frac{1+\beta_s\delta}{1+\beta_p} U_s(\theta_1, s_1^{FB,R}) \tag{8}$$

$$C'(s_2^{FB.R}) = \frac{1+\beta_s\delta}{1+\beta_p} U_s(\theta_2, s_2^{FB,R}) \tag{9}$$

Dividing the left-hand side of (8) by the left-hand side of (9), and the right-hand side of (8) by the right-hand side of (9), we obtain that:

$$\frac{C'(s_1^{FB,R})}{C'(s_2^{FB,R})} = \frac{U_s(\theta_1, s_1^{FB,R})}{U_s(\theta_2, s_2^{FB,R})} \tag{10}$$

This same condition is valid whether or not the consumer has reference-dependent preferences, showing that the presence of reference-dependent preferences does not distort one quality level with respect to the other. Conditions (8) and (9) characterize the optimal values $s_1^{FB,R}$ and $s_2^{FB,R}$, respectively. It is clear now that for $\beta_s\delta >=< \beta_p$, $s_i^{FB,R} >=< s_i^{FB,NR}$ for $i = 1,2$. Also, quality increases in δ and β_s, but decreases in β_p.

The optimal values for $p_1^{FB,R}$ and $p_2^{FB,R}$ are then determined from constraints (2) and (3), which we have seen hold with equality at the optimal solution:

$$p_1 = \frac{1+\beta_s\delta}{1+\beta_p} U(\theta_1, s_1^{FB,R}) \tag{11}$$

$$p_2 = \frac{1+\beta_s\delta}{1+\beta_p} U(\theta_2, s_2^{FB,R}) \tag{12}$$

It follows that, if $\beta_s\delta \geq \beta_p$, as s_1 and s_2 increase compared to the case without reference-dependent preferences, p_1 and p_2 also increase. If $\beta_s\delta < \beta_p$, as s_1 and s_2 decrease compared to the case without reference-dependent preferences, p_1 and p_2 also decrease.
QED

The assumption that the monopolist can always ensure that the consumer expects to buy the good, and therefore accordingly forms her reference point, is not a trivial one. A personal equilibrium where the consumer does not buy because she expects not to buy exists if:

$$0 \geq \beta_s U(\theta_1, s_1) - \beta_p \delta p_1 \tag{13}$$

$$0 \geq \beta_s U(\theta_2, s_2) - \beta_p \delta p_2 \tag{14}$$

In this case, the equilibrium obtained in Proposition 1 is a preferred personal equilibrium if additionally:

$$U(\theta_1, s_1) - p_1 \geq 0 \tag{15}$$

$$U(\theta_2, s_2) - p_2 \geq 0 \tag{16}$$

Given (11) and (12), constraints (15) and (16) are only slack if $\beta_s\delta \leq \beta_p$. Thus, only if the consumer cares disproportionally about price is the personal equilibrium preferred by the monopolist also the consumer's personal preferred personal equilibrium. This makes sense, as for $\beta_s\delta < \beta_p$, the consumer with reference-dependent preferences is better off with than without these preferences. If on the contrary it is the case that $\beta_s\delta > \beta_p$, and the monopolist cannot influence expectations, the true constraints that the monopolist faces are (15) and (16), the equilibrium is indistinguishable from the one without reference-dependent preferences, and both consumer and monopolist are exactly equally well off with and without reference-dependent preferences.

3. Asymmetric information

We now look at the case of asymmetric information. We first look at the optimal prices and qualities in three cases, namely where the monopolist offers a different quality level to both consumers (Proposition 2), where it offers the same low-quality level to both consumers (Proposition 3), and where it only offers high quality, which is not bought by the low-valuation consumers (Proposition 4). The next step is then to check which of these outcomes is best to the monopolist (Proposition 5).

We first look at the case where the firm sets *two* price-quality combinations, and adapts prices and qualities such that each consumer type self-selects the right combination. The maximization problem is now the same as in (1) to (3), but we now additionally have two incentive compatibility constraints:

$$U(\theta_1,s_1)-p_1 \geq U(\theta_1,s_2)-p_2+\beta_s\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right]-\beta_p\delta(p_2-p_1) \quad (17)$$

$$U(\theta_2,s_2)-p_2 \geq U(\theta_2,s_1)-p_1-\beta_s\delta\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]+\beta_p(p_2-p_1) \quad (18)$$

Denote $s_i^{SB,R}$ and $p_i^{SB,R}$ ($s_i^{SB,NR}$ and $p_i^{SB,NR}$) for $i=1,2$ the optimal price-quality combinations under asymmetric information (SB = second-best) with reference-dependent preferences (without reference-dependent preferences). A first key result of Proposition 1 is that with reference-dependent preferences, the single-crossing property is more easily satisfied, and applies even if the high- and low-valuation consumers have the same marginal utility of quality. From this perspective, it is more likely that menu pricing is possible if consumers have reference-dependent preferences. A second key result is that the effect of reference-dependent preferences on the price paid the high valuation consumers is ambiguous. Just as in the case without reference-dependent preferences, self-selection is achieved by offering the high-valuation consumer a discount, where the low-valuation consumer is offered lower quality than in the first best in order to limit the size of this discount. As shown in Proposition 2, the effect of reference-dependent preferences on this discount is ambiguous. Intuitively, the high-valuation consumer who considers buying the low-quality low-price combination, when she is loss averse focuses more on the loss in quality that she will suffer from this than on the gain because of the reduction in price. For this reason, she is less inclined to choose the low-quality low-price combination, and self selection may take place for a lower discount on the high-quality high-price combination.

Proposition 2. Consider the case of asymmetric information. For any $s_2 > s_1$, let $\frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right] > \frac{1+\beta_s}{1+\beta_p\delta}\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right]$. Then the monopolist can make the consumer self-select contracts (s_1,p_1) and (s_2,p_2). We first compare prices and qualities in case of reference-dependent preferences with and without asymmetric information, and we then compare prices and qualities in case of asymmetric information with and without reference-dependent preferences.

(i) For $U_{s\theta} > 0$, $s_1^{SB,R} < s_1^{FB,R}$ and $p_1^{SB,R} < p_1^{FB,R}$, for $U_{s\theta} = 0$, $s_1^{SB,R} = s_1^{FB,R}$ and $s_1^{SB,R} = s_1^{FB,R}$. Further, $s_2^{SB,R} = s_2^{FB,R}$, and $p_2^{SB,R} < p_2^{FB,R}$.

(ii) $\beta_s\delta >=< \beta_p$ iff $s_i^{SB,R} >=< s_i^{SB,NR}$ for $i = 1,2$, and iff $p_1^{SB,R} >=< p_1^{SB,NR}$. The relation between $p_2^{SB,R}$ and $p_2^{SB,NR}$ is ambiguous.

Proof:

Step 1. We first show that participation constraint (2) is slack. Using (18), and (A1), it follows that

$$U(\theta_2,s_2)-p_2 \geq U(\theta_1,s_1)-p_1-\beta_s\delta\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]+\beta_p(p_2-p_1) \tag{19}$$

Given participation constraint (2), given that (18) is valid, it is then certainly true that

$$U(\theta_2,s_2)-p_2 \geq -\beta_q\delta U(\theta_1,s_1)+\beta_p p_1-\beta_s\delta\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]+\beta_p(p_2-p_1)$$

$$\Leftrightarrow$$

$$U(\theta_2,s_2)-p_2 \geq -\beta_s\delta U(\theta_2,s_2)+\beta_p p_2+\beta_q\delta\left[U(\theta_2,s_1)-U(\theta_1,s_1)\right] \tag{20}$$

Given that by (A1), $\left[U(\theta_2,s_1)-U(\theta_1,s_1)\right] > 0$, it follows that (3) is slack. It follows that we only need to consider constraints (2), (17) and (18). Letting $(\gamma_1,\phi_1,\phi_2) \geq 0$ be the multipliers on constraints (2), (17) and (18), respectively, the Kuhn-Tucker conditions for this problem can be written

$$(1-\lambda)-\gamma_1(1+\beta_p)-\phi_1(1+\beta_p\delta)+\phi_2(1+\beta_p)=0 \tag{21}$$

$$\lambda+\phi_1(1+\beta_p\delta)-\phi_2(1+\beta_p)=0 \tag{22}$$

$$-(1-\lambda)C'(s_1)+\gamma_1(1+\beta_s\delta)U_s(\theta_1,s_1)+(1+\beta_s)\phi_1 U_s(\theta_1,s_1)-\phi_2(1+\beta_s\delta)U_s(\theta_2,s_1) \leq 0 \tag{23}$$

$$-\lambda C'(s_2)-\phi_1(1+\beta_s)U_s(\theta_1,s_2)+\phi_2(1+\beta_s\delta)U_s(\theta_2,s_2) \leq 0 \tag{24}$$

along with the complementary slackness conditions for constraints (2), (17) and (18).

Step 2. By condition (22), if $\phi_2 = 0$, then $\phi_1 < 0$, which is not possible. Thus, $\phi_2 > 0$ and constraint (18) is binding.

Step 3.

Constraints (17) and (18) can be rewritten as:

$$p_2-p_1 \geq \frac{1+\beta_s}{1+\beta_p\delta}\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right] \tag{25}$$

$$\frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right] \geq p_2-p_1 \tag{26}$$

If $\frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]\leq\frac{1+\beta_s}{1+\beta_p\delta}\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right]$, then the monopolist is not able to achieve self-selection, and the two proposed price-quality combinations are necessarily identical. If $\frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]>\frac{1+\beta_s}{1+\beta_p\delta}\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right]$, it follows that either (17) or (18) is slack. Since in Step 2 we showed that (18) is binding, it follows that (17) is slack, so that $\phi_1=0$. Note that $\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]>\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right]$ is not a necessary condition for $\phi_1=0$. If $\left[U(\theta_2,s_2)-U(\theta_2,s_1)\right]\leq\left[U(\theta_1,s_2)-U(\theta_1,s_1)\right]$, the monopolist can still achieve self-selection if the degree of loss aversion is sufficiently large $(\delta>>1)$.

Step 4. By adding conditions (21) and (22), we obtain that $\gamma_1=1/(1+\beta_p)$. It follows that constraint (2) is binding.

Step 5.

Given that $\phi_1=0$, $\gamma_1=1/(1+\beta_p)$, it follows from (21) that $\phi_2=\lambda/(1+\beta_p)$. Plugging these values into (23) and (24), and evaluating at respectively $s_1=0$ and $s_2=0$, we obtain

$$-(1-\lambda)\underbrace{C'(0)}_{>0}+\frac{1+\beta_s\delta}{1+\beta_p}\underbrace{U_s(\theta_1,0)}_{=+\infty}-\lambda\frac{1+\beta_s\delta}{1+\beta_p}\underbrace{U_s(\theta_2,0)}_{=+\infty} \tag{27}$$

$$-\lambda\underbrace{C'(0)}_{>0}+\lambda\frac{1+\beta_s\delta}{1+\beta_p}\underbrace{U_s(\theta_2,0)}_{=+\infty} \tag{28}$$

It follows that both constraints are positive for zero levels of quality, so that neither of the qualities is optimally put at zero.

Step 6. Given that the optimal qualities are non-zero, for the qualities under asymmetric information and reference-dependent preferences, we have:

$$-(1-\lambda)C'(s_1^{SB,R})+\frac{1+\beta_s\delta}{1+\beta_p}U_s(\theta_1,s_1^{SB,R})-\lambda\frac{1+\beta_s\delta}{1+\beta_p}U_s(\theta_2,s_1^{SB,R})=0$$

$\Leftrightarrow$

$$C'(s_1^{SB,R})=\frac{1+\beta_s\delta}{1+\beta_p}U_s(\theta_1,s_1^{SB,R})-\frac{\lambda}{1-\lambda}\frac{1+\beta_s\delta}{1+\beta_p}\left[U_s(\theta_2,s_1^{SB,R})-U_s(\theta_1,s_1^{SB,R})\right]=0 \tag{29}$$

$$-\lambda C'(s_2^{SB,R})+\lambda\frac{1+\beta_s\delta}{1+\beta_p}U_s(\theta_2,s_2^{SB,R})=0$$

$\Leftrightarrow$

$$C'(s_2^{SB,R})=\frac{1+\beta_s\delta}{1+\beta_p}U_s(\theta_2,s_2^{SB,R}) \tag{30}$$

Given (29), it is clear that $s_1^{SB,R}<s_1^{FB,R}$ iff $\left[U_s(\theta_2,s_1^{SB,R})-U_s(\theta_1,s_1^{SB,R})\right]>0$ (i.e., the standard single-crossing property (A2)), $s_1^{SB,R}=s_1^{FB,R}$ iff $\left[U_s(\theta_2,s_1^{SB,R})-U_s(\theta_1,s_1^{SB,R})\right]=0$, and $s_1^{SB,R}>s_1^{FB,R}$ iff $\left[U_s(\theta_2,s_1^{SB,R})-U_s(\theta_1,s_1^{SB,R})\right]>0$. Further, as (30) is identical to (9), it follows that $s_2^{SB,R}=s_2^{FB,R}$. Comparing asymmetric information with and without reference-dependent preferences, it is further clear that for $\beta_s\delta >=< \beta_p$, $s_i^{SB,R} >=< s_i^{SB,NR}$ for $i = 1,2$.

We now look at the optimal prices. Given that (2) and (18) are binding, we have that:

$$p_1 = \frac{1+\beta_s\delta}{1+\beta_p} U(\theta_1, s_1) \tag{31}$$

$$p_2 = p_1 + \frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2, s_2) - U(\theta_2, s_1)\right]$$

$\Leftrightarrow$

$$p_2 = \frac{1+\beta_s\delta}{1+\beta_p} U(\theta_2, s_2) - \frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2, s_1) - U(\theta_1, s_1)\right] \tag{32}$$

From (31), given that $\beta_s\delta >=< \beta_p$ iff $s_1^{SB,R} >=< s_1^{SB,NR}$, it follows that $\beta_s\delta >=< \beta_p$ iff $p_1^{SB,R} >=< p_1^{SB,NR}$.

The effect of reference-dependent preferences on p_2 in the case of asymmetric information depends on several factors. Consider the case $\beta_s\delta > \beta_p$, $U_{s\theta} > 0$ (see assumption (A2)). Then as we have seen, $s_1^{SB,R} > s_1^{SB,NR}$. Given that $U_{s\theta} > 0$, this makes $\left[U(\theta_2, s_1) - U(\theta_1, s_1)\right]$ increase. At the same time, as $s_2^{SB,R} > s_2^{SB,NR}$, $U(\theta_2, s_2)$ increases. Finally, as $\frac{1+\beta_s\delta}{1+\beta_p} > 1$, ceteris paribus this increases p_2. It follows that the effect of reference-dependent preferences on p_2 is ambiguous. Consider the case $\beta_s\delta > \beta_p$, $U_{s\theta} \le 0$. Then p_2 unambiguously increases. Consider the case $\beta_s\delta < \beta_p$, $U_{s\theta} \le 0$. Then p_2 unambiguously decreases.
QED

We next consider the monopolist's optimal price and quality when offering the same low-quality to both types of consumers. It is easy to see that the monopolist then sets the same price and quality as offered to the low-valuation consumer in the first best.

Proposition 3. Consider the case of asymmetric information. Consider a single price-quality combination (s, p) that is bought by all consumers. Then $s = s_1^{FB,R}$, $p = p_1^{FB,R}$.
Proof:
The monopolist maximizes:

$$\max_{p,s}\; p - C(s)$$
$$U(\theta_1, s) - p \ge -\beta_s\delta U(\theta_1, s) + \beta_p p$$
$$U(\theta_2, s) - p \ge -\beta_s\delta U(\theta_2, s) + \beta_p p$$

The constraints can be rewritten as:

$$p \leq \frac{1+\beta_s\delta}{1+\beta_p}U(\theta_1, s)$$

$$p \leq \frac{1+\beta_s\delta}{1+\beta_p}U(\theta_2, s)$$

Given (A1), the binding constraint is the first one. It follows that the monopolist makes sure that the first constraint is met with equality, and maximizes $\max_s \frac{1+\beta_s\delta}{1+\beta_p}U(\theta_1, s) - C(s)$. This leads to the low price and low quality offered in the first best.
QED

We finally consider the monopolist's optimal price and quality when only offering high quality and a high price to the high-valuation consumers, where the low-valuation consumers do not buy the good. This time, the monopolist sets the same price and quality as offered to the high-valuation consumer in the first best.

Proposition 4. Consider the case of asymmetric information. Consider a single price-quality combination (s, p) that is bought only by high-valuation consumers. Then $s = s_2^{FB,R}$, $p = p_2^{FB,R}$.
Proof:

$$\max_{p,s} \lambda[p - C(s)]$$

$$0 \geq \beta_s U(\theta_1, s) - \beta_p \delta p$$

$$U(\theta_2, s) - p \geq -\beta_s \delta U(\theta_2, s) + \beta_p p$$

The constraints can be rewritten as:

$$p \geq \frac{\beta_s}{\beta_p\delta}U(\theta_1, s)$$

$$p \leq \frac{1+\beta_s\delta}{1+\beta_p}U(\theta_2, s)$$

As the monopolist wants the price to be as high as possible for any given quality, it follows that $p = \frac{1+\beta_s\delta}{1+\beta_p}U(\theta_2, s)$, so that the monopolist sets $\frac{1+\beta_s\delta}{1+\beta_p}U_s(\theta_2, s) = C'(s)$. It follows that the monopolist offers the same quality at the same price as to the high-valuation type.
QED

We are now ready to determine the conditions under which the consumer prefers to offer two price-quality combinations rather than one. We limit ourselves to comparing the case where the case of menu pricing to the case of the screening of high-valuation consumers. In Proposition 5, we show that, when the monopolist's marginal costs are not too high and the difference between the marginal utility of high-valuation consumers and low-valuation consumers is not too high, it may be that menu pricing only is desirable if consumers have reference-dependent preferences.

Proposition 5. Consider two identical monopolists *A* and *B*, which face identical consumers except for the fact that the consumers of monopolist *B* have gain-loss utility on top of their intrinsic utility (have reference-dependent preferences). Then, if the monopolists' marginal costs are not too high, and if the difference in marginal utility between high-valuation and low-valuation consumers is not too high, monopolist *B* is more likely to prefer menu pricing to screening of high valuation consumers than monopolist *A*.

Proof:

By Proposition 2, given that high quality is the same as in the first best, monopolist *B*'s expected profits with menu-pricing equal:

$$(1-\lambda)\left[\frac{1+\beta_s\delta}{1+\beta_p}U(\theta_1,s_1^{SB,R})-C(s_1^{SB,R})\right]+ \\ \lambda\left[\frac{1+\beta_s\delta}{1+\beta_p}U(\theta_2,s_2^{FB,R})-\frac{1+\beta_s\delta}{1+\beta_p}\left[U(\theta_2,s_1^{SB,R})-U(\theta_2,s_1^{SB,R})\right]-C(s_2^{FB,R})\right] \quad (33)$$

By Proposition 4, given that quality is the same as offered to the high-valuation consumer in the first best, monopolist *B*'s expected profits when screening high-valuation consumers equal:

$$\lambda\left[\frac{1+\beta_s\delta}{1+\beta_p}U(\theta_2,s_2^{FB,R})-C(s_2^{FB,R})\right] \quad (34)$$

Using (33) and (34), it can be calculated that monopolist *B* prefers menu pricing iff

$$\lambda<\frac{\frac{1+\beta_s\delta}{1+\beta_p}U(\theta_1,s_1^{SB,R})-C(s_1^{SB,R})}{\frac{1+\beta_s\delta}{1+\beta_p}U(\theta_2,s_1^{SB,R})-C(s_1^{SB,R})}=\lambda^* \quad (35)$$

For monopolist *A*, expression (35) is identical, except that $\beta_s=\beta_p=0$, and we have everywhere $s_1^{SB,NR}$ instead of $s_1^{SB,R}$. The effect of monopolist *A* facing consumers with reference-dependent preferences can be evaluated by setting $\frac{1+\beta_s\delta}{1+\beta_p}=\alpha$, specifying a function $s_1(\alpha)$, and taking the derivative of λ^* with respect to α, where higher α represents the switch to reference-dependent preferences. It follows that

$$\operatorname{sgn}\frac{\partial\lambda^*}{\partial\alpha} =$$

$$\operatorname{sgn}\left\{\begin{matrix}[U[\theta_1,s_1(\alpha)]-C'(s_1(\alpha))+U'[\theta_1,s_1(\alpha)]s_1'(\alpha)]\,[\alpha U[\theta_2,s_1(\alpha)]-C(s_1(\alpha))]- \\ [U[\theta_2,s_1(\alpha)]-C'(s_1(\alpha))+U'[\theta_2,s_1(\alpha)]s_1'(\alpha)][\alpha U[\theta_1,s_1(\alpha)]-C(s_1(\alpha))]\end{matrix}\right\} =$$

$$\operatorname{sgn}\left\{\begin{matrix}\underbrace{\{U'[\theta_1,s_1(\alpha)]U[\theta_2,s_1(\alpha)]-U'[\theta_2,s_1(\alpha)]U[\theta_1,s_1(\alpha)]\}}_{?}\underbrace{s_1'(\alpha)\alpha}_{>0}+ \\ \underbrace{\{U'[\theta_2,s_1(\alpha)]-U'[\theta_1,s_1(\alpha)]\}s_1'(\alpha)C(s_1(\alpha))}_{>0}+ \\ \underbrace{\{U[\theta_2,s_1(\alpha)]-U[\theta_1,s_1(\alpha)]\}}_{>0}\underbrace{[C(s_1(\alpha))-C'(s_1(\alpha))\alpha]}_{?}\end{matrix}\right\} \tag{36}$$

It follows from (36) that if $\frac{U[\theta_2,s_1(\alpha)]}{U[\theta_1,s_1(\alpha)]} > \frac{U'[\theta_2,s_1(\alpha)]}{U'[\theta_1,s_1(\alpha)]}$, $\frac{C(s_1(\alpha))}{\alpha} > C'(s_1(\alpha))$, then $\operatorname{sgn}\frac{\partial\lambda^*}{\partial\alpha} > 0$.

QED

As a caveat, we repeat that an essential feature of our model is the assumption that the consumers expect to buy the good, and moreover that rich consumers expect to receive high quality, and poor consumers expect to receive low quality. Our results are changed if consumers may also expect not to buy the good (see Section 3), or expect to buy the other variety of the good. In Köszegi and Rabin's (2006) model, consumers may ex ante consider their overall utility based on the different expectations that they may have, where they then pick the expectations that lead to the highest overall utility. What is the effect of this on our analysis? Our analysis would seem to be maintained if there are no personal equilibria where the rich consumer picks the low quality variety and/or the poor consumer picks the high quality variety. This is the case iff

$$U(\theta_1,s_1)-p_1-\beta_s\delta[U(\theta_1,s_2)-U(\theta_1,s_1)]+\beta_p(p_2-p_1)\geq U(\theta_1,s_2)-p_2 \tag{37}$$

$$U(\theta_2,s_2)-p_2+\beta_s[U(\theta_2,s_2)-U(\theta_2,s_1)]-\beta_p\delta(p_2-p_1)\geq U(\theta_2,s_1)-p_1 \tag{38}$$

These conditions can be summarized as:

$$\frac{1+\beta_s}{1+\beta_p\delta}[U(\theta_2,s_2)-U(\theta_2,s_1)]\geq p_2-p_1\geq\frac{1+\beta_s\delta}{1+\beta_p}[U(\theta_1,s_2)-U(\theta_1,s_1)] \tag{39}$$

As soon as consumers are loss averse ($\delta>1$), condition (39) makes constraints (17) and (18) slack. Moreover, the result in Proposition 2 that the single-crossing condition is not needed to make menu pricing possible is not maintained, as (39) on the contrary makes the single-crossing condition not sufficient for menu pricing to be possible. Alternatively, constraints (37) and (38) are not valid, and there are multiple personal equilibria. In this case, Köszegi and Rabin (2006) assume that the consumer is able to choose her preferred personal equilibrium. The condition for the rich consumer's preferred personal equilibrium to be that she chooses the high-quality variety, and for the poor consumers that she chooses the low-quality variety, leads to incentive constraints that are indistinguishable from the self-selection constraints of consumers that do not have reference-dependent preferences, however. Along with the condition that the consumers also should prefer a personal equilibrium where they buy a variety of the good to a personal equilibrium where they do not (see Section 3), one obtains a model that is indistinguishable from the one without reference-dependent preferences.

4. Discussion

It is undeniable that in the real-world, any price menu contains less prices than predicted by theoretical models with a continuum of consumer types. One does not need reference-dependent preferences to explain this fact. It may both be explained by firms' costs of maintaining a large price list and/or consumers' difficulty in processing an extensive price list. From this perspective, it is on the contrary the existence of price discrimination that comes as a surprise. This paper explains price discrimination by the fact that consumers with reference-dependent preferences both make price discrimination more feasible, and possibly also more desirable to firms.

References

Armstrong, M. and S. Huck (2010) Behavior economics as applied to firms: a primer. *mimeo*.

Hahn, J.-H., Kim, J., Kim, S.-H. and Lee, J. (2010) Screening loss averse consumers. Working paper, Seoul National University.

Heidhues, P. and Köszegi, B. (2005) The impact of consumer loss aversion on pricing, *mimeo*.

Heidhues, P. and B. Köszegi (2008) Competition and price variation when consumers are loss averse. *American Economic Review* 98. 1245-1268.

Köszegi, B. and Rabin, M. (2006) A model of reference-dependent preferences. *Quarterly Journal of Economics* 121, 1133-1165.

Mussa, M. and S. Rosen (1978) Monopoly and product quality. *Journal of Economic Theory* 18, 301-317.