Optimal Procurement and Investment in New Technologies under Uncertainty*

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Abstract

We study a buyer's optimal investment strategy for new technologies when costs evolve stochastically and are private information to the suppliers. In a continuous time setting, we show how the asymmetric information on the stochastic variables leads to delays in investment compared to the real option benchmark. We also suggest a payment structure that implements the buyer's optimal investment timing as a Vickrey-type auction.

Key words: Adverse Selection, Dynamic contracting, Irreversible investment, Continuous time, Optimal stopping, Procurement, Principal-Agent model, Real option.

JEL Classification: C73, D82, D86

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1 Introduction

In this paper we develop a framework for determining the optimal investment strategy under uncertainty in the presence of asymmetric information about the stochastic parameter driving that investment's value. This is for instance an important question in deciding upon the optimal investment by regulated firms.[1] Examples of this are the regulation of investments in next generation networks in the telecommunication sector when costs evolve in an uncertain way, and decisions to upgrade the electricity grid in the face of uncertain demand growth. In the telecommunications case, the firm has private information on the evolution of investment costs which may be stochastic but are expected to fall over time as the new technology becomes more mature. If the firm has better information than the regulator about the evolution of those costs, how should the regulator incentivize optimal timing of investment in this new technology? We illustrate the trade-off between giving incentives to reveal the actual cost of investment and the gains from delaying investment to reduce information rents.

Our framework also allows us to analyze questions regarding procurement auctions where, not only investment timing, but also the optimal choice of supplier might depend on the future cost evolution of all suppliers participating in the procurement process. An example is a government's decision to invest in military equipment, such as a new fighter plane, where the choice could be between a newly designed product, the final costs of which are currently still highly uncertain, versus an off-the-shelf proven technology for which costs are relatively stable but the benefits may be lower. Ideally, a procurement process should delay the final choice until future uncertainty is partly resolved, while at the same time giving incentives to those firms in the race to reveal their information on the evolution of those uncertain cost or benefit estimates.

Another practical example of such a phased procurement processes is the so-called pre-commercial procurement, in which public buyers contract with multiple sellers for goods that are not yet available in the market, and which still require R&D input. The European Commission (2007) describes how public buyers can engage with multiple potential providers to promote the necessary R&D, postponing the ultimate choice among those providers in the race until a later stage, when more information on the different technologies is available. Current experiments with such pre-commercial procurement processes range from procurement of new traffic management systems to innovative solutions in health care.[2]

We model such regulation and procurement questions as an agency problem in a real options

[1] See e.g. Guthrie (2006), Vogelsang (2012).

[2] See `cordis.europa.eu/fp7/ict/pcp/key-docs_en.html` for example projects.

framework. We work in a continuous time setting, in which costs are stochastic but private information of the suppliers, who are the agents. We show how the buyer – the principal – can design a set of fees and penalties, to be paid to or by the suppliers, that incentivize the agents to reveal the evolution of their costs and to either invest or abandon the process when pre-determined cost thresholds are reached. In the case of multiple agents, these fees include payments before any investment has occurred, a fee upon investment, and a cancellation fee when a rival agent is called upon to invest instead.

We solve the model by noting that each agent solves a real option problem that is governed by the fees system designed by the principal. Incentive compatibility then requires that the agent's privately optimal investment thresholds should coincide with the principal's intended choice. The principal's task is thus to reverse engineer the agent's real option problem by designing a set of fees that generate those threshold values for the agent's real option problem. Incentive compatible investment decisions imply that the agent's value satisfies smooth-pasting conditions at these cost threshold values. We show that the smooth-pasting conditions at the thresholds translate into a condition on the agent's value function everywhere in the domain, and use that to determine ex ante fees that induce the agents to reveal their initial cost levels as well.

Our main insight is that the principal's problem of finding the optimal investment thresholds reduces to a standard real option problem, with stochastic costs replaced by their virtual counterparts, which depend on the cost levels at time zero. The fact that the map from real costs to virtual costs only depends on the initial costs, and not on the later cost evolution, is consistent with results in (discrete) dynamic mechanism design that state that only information that is asymmetric at the time of contracting creates distortions in efficiency.

Fees that implement the optimal investment behaviour are not unique. Generically, they involve not only rewards for making the investment, but also some combination of rewards for abandonment and penalties for postponing a decision. We show that one way of choosing these is in a Vickrey type of mechanism in which the fees of investing or abandoning reflect the virtual values of choosing a rival's project.

A benefit of our continuous time approach is its tractability: the optimal investment decision takes the form of a (codimension one) threshold in the cost-space. While finding that threshold can appear a daunting task in a discrete environment, this problem is well understood from real option theory, and tractable analytically or amenable to standard numerical methods for analysis of simple partial differential equations with free boundaries.

While we only focus on a simple type of real options problem, a further advantage of our approach is that it is directly applicable to an existing literature of real options models, with

its associated toolbox of solution methods (see e.g. Dixit and Pindyck, 1994).

By introducing private information on the stochastic parameter into the real option framework, we are related to the literature on dynamic mechanism design, where an agent's type evolves stochastically. This literature is in general cast in a discrete-time framework in which the early literature focused on two-period information revelation problems (Baron and Besanko, 1984; Besanko, 1985; Courty and Li, 2000). Recent work by Pavan, Segal and Toikka (2014) has extended these results to many periods. See Bergemann and Said (2011) for a survey of this literature. Note that although most of this literature has focused on discrete time, three notable exceptions are DeMarzo and Sannikov (2006) and Sannikov (2008) who analyze moral hazard in a continuous time environment, and Williams (2011) who studies adverse selection in a non storable goods consumption model in continuous time. A small and quite recent strand of the literature studies continuous time and adverse selection in a similar sprite to this paper. In particular Cvitanić and Zhang (2007) study a continuous time model with adverse selection. However they focus on a situation where the private information is related to the drift of the underlying process, not the realized values of the process. Kruse and Strack (2013) show how optimal stopping rules can be solved as a mechanism design problem. Bergemann and Strack (2014), which appeared concurrently with, and independently from, our paper, is probably the closest paper to ours. Like us, they study optimal contracting under asymmetric information on the agent's *stochastic type*.[3] Their focus is on a monopolist selling a non-durable good to consumers with private information that evolves stochastically over time. However their analysis applies to situations with non-durable goods in a time-separable environment, whereas we focus on durable goods in an environment which is not time-separable. Allowing for such time-inseparability is essential to address real option problems which revolve around the irreversibility of decisions.

In line with the literature on (discrete-time) dynamic mechanism design (e.g. Baron and Besanko, 1984; Besanko, 1985; Eső and Szentes, 2013; Pavan, Segal and Toikka, 2014), only information that is ex ante known to the agent commands such a rent. In particular, the buyer can extract information on future cost shocks without leaving any rent to the suppliers. As indicated, this result carries over to our continuous-time environment as well. In fact, to reduce information rents associated with the ex ante information, the buyer distorts the investment strategy away from the real-option result, leading to delayed investment and distorted project choice compared to that benchmark.

The continuous-time setting that we use to formulate and analyze our model is common in

[3]They also look at the alternative problem of asymmetric information on the (fixed) process parameters.

the real option literature (see Dixit and Pindyck, 1994). One of the main contributions of this literature is to show that a decision maker can benefit from postponing an irreversible decision until more information on investment costs (or some other stochastic variable governing the value of the investment) is available. As in Arve (2013), our model augments this argument by adding private information about those investment costs. However, Arve (2013) only considers a discrete two-period problem and does not fully generalize the environment to continuous time as in the more standard real option literature. A few papers have looked into real options problems when there is asymmetric information on another, non-stochastic, parameter. Broer and Zwart (2013) consider optimal regulation of investments when costs are private information to the regulated firm, and demand is stochastic but public information (see also Board, 2007, who analyzes a similar question in discrete time). In that case, optimal regulation involves distortion of investment timing.

In our model we also show how the optimal mechanism can be implemented in a Vickrey-type auction, with fees that cause suppliers to internalize the utility of both buyer and rival suppliers. Here our work relates to the work on dynamic pivot mechanisms (Bergemann and Välimäki, 2010) in a social choice problem, and applications to auctions in Board (2007) and Kakade, Lobel and Nazerzadeh (2013). Again, these works examine discrete time problems, while we consider a continuous time application.

The paper proceeds as follows. Section 2 introduces the model. In Section 3 we present the benchmark one-agent problem where the issue is only when to invest in a new technology. Section 4 extends the model to a situation with competing technologies and multiple agents. Section 5 briefly concludes. Proofs that do not follow directly from the discussion in the main text are relegated to the Appendix.

2 The model

We consider an investor (the principal) who needs to make a decision regarding investment in a project i that yields benefit v_i. We will first consider a single project, and then extend the analysis to a choice between two alternative investments, $i = 1, 2$, only one of which will be carried out. The investment costs are stochastic. The investment decision is irreversible and the investor can choose both the timing of the investment and the project to adopt. We work in continuous time, with $t = 0$ as the initial date.

We denote by C_i the costs of providing the project. We assume that each C_i evolves independently and stochastically according to a geometric Brownian motion with drift μ_i and

volatility σ_i,

$$dC_i = \mu_i C_i dt + \sigma_i C_i dz_i, \tag{1}$$

where dz_i is the standard Brownian motion increment satisfying $E(dz_i) = 0$ and $E(dz_i^2) = dt, E(dz_i dz_j) = 0$.

We assume that investment in a given project i can only be carried out by firm i (agent i). The firms providing the technology have private information about the respective costs of investing in their technology. This means that firm i has private information both on the initial costs, denoted C_i^0, as well as future (stochastic) costs at subsequent times, C_i^t. The initial cost, C_i^0, is distributed on $[C_i^L, C_i^H]$ according to the cumulative distribution function $F_i(\cdot)$. Let the associated density be $f_i \equiv F_i'$. We assume the hazard rates of the distributions of $\log(C_i^0)$ to be monotone, and initial costs of different agents to be independent.

The process parameters μ_i, σ_i are public information. The interest rate is r and is common to all players.

At $t = 0$ the principal can offer contracts to the agents. The investment itself is contractible. The contracts may involve payments upfront (denoted by ϕ_i^0), payment flows p_i^t at times $t > 0$ before any investments occur, a payment to firm i upon investment by i, $\bar{\phi}_i$, as well as a payment $\underline{\phi}_i$ to firm i upon investment in the competing technology $j \neq i$.

We consider the case in which the principal offers a menu of contracts where each contract consists of the payments described in the previous paragraph and an investment timing so as to maximize his expected surplus W, net of the payments to the agents. The firms, in turn, will maximize their (expected) profits (payments minus investment costs), and we assume that they have no outside option, i.e., they make zero profit if they do not participate.

Remark: In many of the applications we have in mind, the (gross) benefit from investment may not be constant but also evolve stochastically. To keep the model as tractable and simple as possible we abstract from this. However, as long as this process is public information, we can easily generalize our model to allow for this.

3 Benchmark: Investment in a new technology

In this section we look at the investment decision when there is a single investment project under consideration. We therefore consider a principal contracting over investment and investment timing with an agent who is to supply the investment, but who has private information on the evolution of the investment cost. Since there is a single agent, we omit the label $i = 1$ from value v, cost C and the contract fees ϕ, p.

3.1 First-best investment timing

In this subsection we abstract from the private information on the cost of the new technology and derive the optimal investment policy using standard real option theory (Dixit and Pindyck, 1994). There is a benefit from delaying investment, as this allows the investor to condition his decision on more information. However, the associated option value of delay is traded off against postponed benefits (because of the discount rate r). The optimal investment timing depends on the evolution of costs. Standard arguments make sure that the optimal investment strategy can be expressed in terms of a cost threshold, $\bar{C}$, where investment occurs as soon as the stochastic costs C hit that threshold. Let T be the stochastic stopping time, i.e. the time lapse until costs first hit the threshold when starting at some initial cost $C \geq \bar{C}$. We can then express the value of the investment opportunity, when current costs are $C \geq \bar{C}$, as

$$W(C) = \max_{\bar{C}} W(C, \bar{C}) \equiv \max_{\bar{C}} E\left(e^{-rT} \mid C\right)\left(v - \bar{C}\right).$$

It is clear that $W(C)$ satisfies the value-matching criterion, $W(\bar{C}) = v - \bar{C}$. Moreover, at the optimal $\bar{C}$, an application of the envelope theorem also shows that $W(C)$ satisfies the *smooth-pasting condition* at the threshold: at $\bar{C}$, the derivative to C of the option value $W(C, \bar{C})$ equals the derivative of the ex post value at that point, $v - C$. So in this case

$$W_C(\bar{C}, \bar{C}) = -1.$$

When C is a geometric Brownian motion, we have

$$E\left(e^{-rT} \mid C\right) = \left(\frac{C}{\bar{C}}\right)^{\lambda_-}, \tag{2}$$

with λ_- the negative root of the characteristic equation $r = \mu\lambda + \frac{1}{2}\sigma^2\lambda(\lambda - 1)$. Thus, using the smooth-pasting condition to solve for the threshold we get

$$\bar{C} = \frac{\lambda_-}{\lambda_- - 1} v. \tag{3}$$

First-best investment occurs at the moment when the cost function hits $\bar{C}$ for the first time. Since $0 < \frac{\lambda_-}{\lambda_- - 1} < 1$, notice that investment is optimally delayed beyond the moment when benefits equal costs. This reflects the well-known option value of waiting to invest for discrete irreversible investments (McDonald and Siegel, 1986).

3.2 Asymmetric information

The investment threshold in (3) assumes that the principal can perfectly observe the cost and its evolution although it is the firm who is ultimately making the investment. In general, anyone external to a private firm cannot perfectly observe the costs that this firm incurs.[4] In this section we take into account this non-observability of the firm's cost and explore how private information on this parameter affects the payments and investment timing.

Our strategy for solving for the optimal contract will be as follows. First we consider how to incentivize the agent to reveal truthfully that a given investment threshold $\bar{C}$ has been crossed. Achieving such ex post incentive compatibility requires some rents to be left to the agent. We then show how, for given threshold $\bar{C}$, the agent's continuation rents $V(C, \bar{C})$ depend on his current costs C. Given those ex post rents, we analyze ex ante incentive compatibility: what rents need to be left to the agent for him to reveal truthfully his initial costs, C^0. Finally, we optimize the principal's surplus over the choice of threshold $\bar{C}$. In particular we will see how information rents cause the threshold to deviate from the benchmark real option result.

As a preliminary observation, note that we can restrict attention to incentive schemes consisting of

1. an initial payment ϕ^0;
2. a flow of constant payments p during the period;
3. a payment $\bar{\phi}$ upon investing, i.e when the investment threshold $\bar{C}$ is reached.

Any payments after the moment of investment can be absorbed in $\bar{\phi}$. Note that the flow of payments p between contracting date and investment cannot depend on C, as no action is taken before investment that can separate the different intermediate types.

We therefore study contracts consisting of a set of payments $(\phi^0, p, \bar{\phi})$. In the optimum the principal will offer a menu of such contracts, one for each value of the initial cost C^0.

Ex post incentive compatibility

We first take as given the investment threshold $\bar{C}$ and ask how the principal can induce the agent to invest exactly when his privately observable costs C hits that threshold.

We denote by $V(C, \bar{C})$ the value of the agent's expected future rents given current costs C, and investment threshold $\bar{C}$ that is induced by a set of contractual payments, a flow p before

[4] And even if the costs can be deduced from the firm's accounts, "creative accounting" and overbilling might lead the observed costs to be inflated compared to what the actual costs of the firm should be.

investment and payment $\bar{\phi}$ upon investment. In the regime with $C > \bar{C}$, that value satisfies a Hamilton-Jacobi-Bellman equation with source p,

$$-rV + \mu C V_C + \tfrac{1}{2}\sigma^2 C^2 V_{CC} + p = 0, \tag{4}$$

while at the boundary $C = \bar{C}$, we have continuity,

$$V(\bar{C}, \bar{C}) = \bar{\phi} - \bar{C}.$$

The fact that it is incentive compatible for the agent to invest at $\bar{C}$, finally, is reflected in a smooth-pasting condition at $\bar{C}$:

$$V_C(\bar{C}, \bar{C}) = -1.$$

From these conditions, we can actually characterize the value V of any combination of fees p, $\bar{\phi}$ that incentivize investment at $\bar{C}$ up to a constant, as the following lemma shows.

Lemma 1 (Ex post incentive compatibility)**.** *If it is privately optimal for the firm to invest when costs hit threshold $\bar{C}$, then the firm's current value of expected payments from fees p and $\bar{\phi}$ satisfies*

$$\frac{\partial V}{\partial C} = -\frac{\bar{C}}{C} E\left[e^{-rT} \mid C\right] \tag{5}$$

for all $C \geq \bar{C}$.

From Lemma 1 we can express the value $V(C, \bar{C})$ by integrating,

$$V(C, \bar{C}) = -\int_{\tilde{C}}^{C} \bar{C} E\left[e^{-rT} \mid s\right] \frac{ds}{s} \tag{6}$$

for any integration constant $\tilde{C}$. Each choice of $\tilde{C}$ corresponds to a possible combination of fees p and $\bar{\phi}$ that are compatible with investment threshold $\bar{C}$. The fee $\bar{\phi}$ can be computed from the value-matching condition,

$$\bar{\phi} - \bar{C} = V(\bar{C}, \bar{C}) = -\int_{\tilde{C}}^{\bar{C}} \bar{C} E\left[e^{-rT} \mid s\right] \frac{ds}{s}.$$

The flow of payments p in turn follows from inserting V into the HJB equation in (4).

As an example, if we choose the integration constant $\tilde{C}$ equal to the costs of investment, $\bar{C}$, then clearly, $V(\bar{C}, \bar{C}) = 0$ and hence $\bar{\phi} = \bar{C}$: the fixed fee upon investment exactly covers the investment costs. The required fee while waiting, p, can in this case be computed from the

analytic expression for V: plugging in the expression for $E(e^{-rT} \mid C)$ from equation (2), we find for $\tilde{C} = \bar{C}$

$$V(C, \bar{C}) = \frac{\bar{C}}{\lambda_-} \left(1 - \left(\frac{C}{\bar{C}} \right)^{\lambda_-} \right).$$

Substituting in the HJB differential equation (4) then yields

$$p = \frac{r\bar{C}}{\lambda_-} < 0.$$

If the agent only receives a payment $\bar{\phi} = \bar{C}$ equal to his costs at investment, he would prefer to delay until costs are even lower. A tax (negative payments p) before his investment is needed to keep him from doing that.

Another instructive example is obtained by choosing $\tilde{C} = \infty$. In that case, we find $p = 0$ and $\bar{\phi} = \frac{\lambda_- - 1}{\lambda_-} \bar{C} (> \bar{C})$. This is in accord with the standard real option analysis from section 3.1 where in the absence of any flow of payments before investment, a profit maximizing investor will choose his investment threshold as $\bar{C} = \frac{\lambda_-}{\lambda_- - 1} \bar{\phi}$.

Though any of these combinations of fees p and $\bar{\phi}$ induce the same investment choice for the agent, they differ in the expected rents $V(C^0, \bar{C})$ they leave to the agent upon accepting the contract. In particular, in the first example these rents are negative, and the agent must be compensated ex ante to satisfy his participation constraint. We now turn to the principal's ex ante problem.

The ex ante problem

At the initial stage, the principal offers a contract to the agent consisting of an ex-ante payment ϕ^0, as well as a combination of ex-post fees p and $\bar{\phi}$ that jointly determine the investment threshold $\bar{C}$. The question for the principal here is how to design that initial contract $(\phi^0, p, \bar{\phi})$ so as to optimize his expected profits.

Following the standard mechanism design approach (see e.g. Laffont and Martimort, 2002) we focus on menus of contracts that are ex ante incentive compatible (i.e. the agent reveals his initial cost level C^0), and that respect each agent's participation constraint, i.e. the agent's expected rents $U(C^0)$ are non-negative for any initial $C^0 \in [C_L, C_H]$.

The agent maximizes his ex ante utility $U(C^0)$ over the different choices from the menu of contracts $(\phi^0, p, \bar{\phi})$. Thanks to the Revelation Principle (Dasgupta and Maskin, 1979; Myerson, 1979) we can without loss of generality restrict attention to direct mechanisms, in which a choice from the menu of contracts is $(\phi^0, p, \bar{\phi})(\hat{C}^0)$ where $\hat{C}^0$ is the firm's announcement of C^0.

This menu is (ex ante) incentive compatible if the utility-maximizing choice for an agent with time-zero costs C^0 is to choose $\hat{C}^0 = C^0$.

The firm's total expected payoff when it accepts a contract $(\phi^0, p, \bar{\phi})(\hat{C}^0)$ is given by

$$U(C^0, \hat{C}^0) = \phi^0(\hat{C}^0) + E\left(\int_0^T e^{-rt} p(\hat{C}^0)dt + e^{-rT}(\bar{\phi}(\hat{C}^0) - \bar{C}(\hat{C}^0)) \mid C^0\right),$$

where $\bar{C}(\hat{C}^0)$ is the agent's choice of investment threshold given the choice of contract, and T the time until his costs hit that threshold. In the previous paragraph we saw that the second term, the expected ex post rents, are determined, up to a constant, by the investment threshold $\bar{C}$: they are given by $V(C, \bar{C})$ for some choice of integration constant $\tilde{C}$.

Ex ante incentive compatibility now implies that the maximum of $U(C^0, \hat{C}^0)$ is attained at $\hat{C}^0 = C^0$. Denote the associated utility by

$$U(C^0) \equiv U(C^0, C^0).$$

We then have the following lemma.

Lemma 2 (Ex ante incentive compatibility)**.** *Ex ante incentive compatibility requires that contracts $(\phi^0, p, \bar{\phi})(C^0)$ are such that*

$$U'(C^0) = -E\left(\frac{\bar{C}(C^0)}{C^0} e^{-rT} \mid C^0\right), \tag{7}$$

and

$$\frac{d\bar{C}(C^0)}{dC^0} < 0. \tag{8}$$

By the agent's participation constraint, $U(C^0) \geq 0$ for any C^0. Since $U'(C^0) \leq 0$, It is optimal to set the highest initial cost type's utility equal to zero, i.e. $U(C^H) = 0$. Using this in the integration of the incentive compatibility condition in equation (7), an agent with cost C^0 will then have non-negative payoff equal to

$$U(C^0) = \int_{C^0}^{C^H} E\left(\frac{\bar{C}(s)}{s} e^{-rT} \mid s\right) ds. \tag{9}$$

We now turn to the principal's profit maximization. The principal designs a menu of contracts $(\phi^0, p, \bar{\phi})(C^0)$ to maximize his expected surplus W,

$$W = E_{C^0}\left(E\left(e^{-rT}(v - \bar{C}(C^0)) \mid C^0\right) - U(C^0)\right) \tag{10}$$

subject to the incentive compatibility and participation constraints reflected in (9). The first expectation is with respect to the value of the initial type C^0, distributed according to $F(C^0)$. Clearly, only the choice of $\bar{C}(C^0)$ is going to matter. As we saw, given a particular choice of $\bar{C}$, the principal can choose ex post fees p and $\bar{\phi}$ to make sure that the agent will truthfully reveal that $\bar{C}$ has been reached. There are many combinations of p and $\bar{\phi}$ that do the trick, each associated with a different choice of integration parameter $\tilde{C}$ in equation (6). Each different choice will result in a different value of the total ex post surplus for the agent, V. Given an incentive compatible choice of $p, \bar{\phi}$, and associated expected surplus V, the principal then chooses ex ante fee ϕ^0 to make sure total expected utility equals U.

By standard arguments, the solution to this optimization, subject to incentive compatibility and participation constraints, is the following.

Proposition 1. *Investment under asymmetric information is further delayed compared to the benchmark symmetric information case and only takes place when costs hit $\bar{C}$ that solves*

$$W(C_0) = \max_{\bar{C}} E\left[e^{-rT(\bar{C})}\right]\left(v - \alpha(C^0)\bar{C}\right) \tag{11}$$

with

$$\alpha(C^0) = 1 + \frac{1}{C^0}\frac{F}{f}(C^0).$$

The optimal threshold $\bar{C}(C^0)$ satisfies

$$\bar{C}(C^0) = \frac{\lambda_-}{\lambda_- - 1}\frac{v}{\alpha(C^0)}. \tag{12}$$

Compared to standard adverse selection models, note the multiplicative nature of virtual costs, with virtual costs at investment, $\bar{C}$, inflated by a factor $\alpha(C^0)$ to be contrasted with the more standard shift with F/f. This is a result of the multiplicative nature of the geometric Brownian motion process: changes in initial reported costs at time zero feed through into later cost levels multiplicatively. As we note in section 3.4, this is related to the notion of impulse-responses discussed in Pavan, Segal and Toikka (2014).

Except for the lowest type, $C^0 = C^L$, investment is delayed (the threshold is lower) compared to the symmetric information real options benchmark in equation (3). This distortion is introduced to reduce the information rents left to the agents. For the efficient type, one way of implementing the optimal contract is through a sell-out contract: in that case, the pay-off $\bar{\phi} = v$, and flow payments $p = 0$. This implementation corresponds to the choice of integration constant $\tilde{C} = \infty$ as discussed above. For less efficient types, this choice leads to a partial sell-

out contract: the payment at investment is the value v discounted by the virtual cost factor $\alpha(C^0)$. As a result, investment is distorted away from the real option benchmark.

Note, that the distortion in equation (11) only reflects information at time $t = 0$. Even though only the agent is informed about the evolution of his costs after $t = 0$, this informational advantage does not yield any additional rents. Essentially, the principal can extract this ex post information at no costs. The reason is, that at the moment of contracting, the future cost shocks are still unknown to both principal and agent (as in Baron and Besanko, 1984; Besanko, 1985; Eső and Szentes, 2013; Pavan, Segal and Toikka, 2014).

3.3 General diffusions

We can straightforwardly generalize the analysis to arbitrary stationary diffusion processes for the stochastic parameter C. Consider, to that effect, an investment project where C follows the diffusion process

$$dC = \mu(C)dt + \sigma(C)dz.$$

(Thus far, we assumed $\mu(C) = \mu C$ and $\sigma(C) = \sigma C$.)

We first turn to ex post incentive compatibility. Lemma 1 generalizes to

Lemma 3 (Ex post incentive compatibility, general case)**.** *If it is privately optimal for the firm to invest when the stochastic parameter C hits the threshold $\bar{C}$, then the firm's current value of expected payments from fees p and $\bar{\phi}$ satisfies*

$$\frac{\partial V}{\partial C_0} = -E\left[\frac{\sigma(\bar{C})}{\sigma(C_0)} e^{-\int_0^T \left(r - \sigma(C)\frac{d}{dC}\left[\frac{\mu(C)}{\sigma(C)} - \frac{1}{2}\sigma'(C)\right]\right)dt} \mid C_0\right] \tag{13}$$

for all $C_0 \geq \bar{C}$.

We can then use this lemma in a straightforward way to write down the generalized form of proposition 1,

Proposition 2. *The optimal threshold $\bar{C}(C_0)$ maximises*

$$W(C_0) = \max_{\bar{C}} E\left[e^{-rT(\bar{C})}\left(v - \bar{C} - \frac{F}{f}(C_0)\frac{\sigma(\bar{C})}{\sigma(C_0)} e^{\int_0^T \left(\sigma(C)\frac{d}{dC}\left[\frac{\mu(C)}{\sigma(C)} - \frac{1}{2}\sigma'(C)\right]\right)dt}\right) \mid C_0\right] \tag{14}$$

provided that this expression $\bar{C}(C_0)$ is monotone in C_0.

The proof follows from the lemma, following the same logic as in the special case of geometric Brownian motion.

For our geometric Brownian motion example, $\mu(C)/\sigma(C)$ and $\sigma'(C)$ are both independent of C, and hence the exponent vanishes and we get virtual costs $\bar{C}\left(1+\frac{F}{f}\frac{1}{C_0}\right)$. Other examples that lead to simple expressions are arithmetic Brownian motion (with μ and σ constants), and resulting in conventional virtual costs $\bar{C}+\frac{F}{f}$; and the mean reverting Ornstein-Uhlenbeck process, with $\mu(C)=m(\mu-C)$ and σ constant, leading to virtual costs $\bar{C}+\frac{F}{f}e^{-mT(\bar{C})}$.

It is easy to see that we can again implement the optimal mechanism with a menu consisting only of a fee upon initiation of the contract and one upon investment, i.e. $p=0$. For the most efficient ex ante type, the virtual costs equals real costs and $\bar{C}$ just optimizes $W(C_L)=e^{-rT}\left(v-\bar{C}\right)$. Hence, setting $\bar{\phi}=v$ trivially aligns the agent's incentives with those of the principal, and a sell-out contract does the job. For higher ex ante types C_0, the optimal investment threshold will be lower, and implicitly defined as $\bar{\phi}(\bar{C})$, the fee that satisfies

$$\bar{C}=\arg\max_{C} E\left(e^{-rT(C)}(\bar{\phi}-C)\right).$$

As is standard, ex post rents are then extracted ex ante through the initial fee ϕ^0 to make sure total ex ante surplus satisfies the ex ante incentive compatibility and participation constraints.

3.4 Relation to the dynamic mechanism design literature

Here we briefly comment on the discrete-time analogue of our continuous-time problem. Such discrete-time models are more common in the mechanism design and contract theory literature.[5]

In a discrete-time version of the geometric Brownian motion model, let us consider a setting where the private information C^t regarding the investment cost follows a multiplicative random walk process,

$$C^t=\epsilon_t C^{t-1}, \forall t\in\mathbb{N}, \tag{15}$$

where $\epsilon_t>0$ is distributed according to a probability distribution function $G(\epsilon_t)$. For a log-normal distribution, this generates discrete geometrical Brownian motion for C. This distribution function and its support are common knowledge, only the firm observes the realization of the process ϵ_t. Furthermore, the initial cost of the investment, is denoted C^0 and is also the firm's private information. It is common knowledge that this cost is distributed according to $F(C^0)$ on $[C_L,C_H]$ and to ensure monotonicity, a monotone hazard condition is imposed on the distribution of $\log C^0$. In the end, the private information at any period t can be expressed as

[5]See Laffont and Martimort (2002) for an introduction to this literature.

a function of the initial information C^0 and shocks ϵ_k up until period t.

$$C^t = C^0 \Pi_{k=1}^{t} \epsilon_k \tag{16}$$

The procurement agency's problem will be to decide upon the optimal investment threshold $\bar{C}$ and structure the payments so that costs are covered and so that the firm has an incentive to reveal his cost at each time t.

For ease of notation, define the entire cost history up to period k as $C^{(k)} \equiv (C^0, C^1, \ldots, C^k)$ and the future realizations of cost shocks from time k until time t as $\epsilon_k^t \equiv (\epsilon_k, \ldots, \epsilon_t)$.

In discrete time, the expected (total) utility of the firm in period t when investment takes place in period $T(\hat{C}^t \leq \bar{C})$ when reported costs $\hat{C}^t$ first pass the chosen threshold $\bar{C}$ can be expressed as

$$U_t(C_t) = E_{\epsilon_{t+1}^{\infty}} \left[\sum_{k=t}^{T(\hat{C}^t \leq \bar{C})} e^{-r(k-t)} p_k(\hat{C}^k) + e^{-r(T-t)} \left(\bar{\phi} - C^{T(\hat{C}^t \leq \bar{C})} \right) \right], \tag{17}$$

where reports $\hat{C}^t$ are chosen, conditional on shocks ϵ up to that time, to optimize this utility.

Now, recognizing that C^T depends on C^t, $t < T$ multiplicatively, as in equation (16), we can derive the standard derivative condition for incentive compatibility in t (using the envelope theorem),[6]

$$\dot{U}_t(C_t) = -E_{\epsilon_{t+1}^{\infty}} \left[e^{-r(T(\bar{C})-t)} \frac{\bar{C}}{C^t} \right], \tag{18}$$

which can be compared to our continuous-time expression equation (7).

Using standard techniques (Laffont and Martimort, 2002; Besanko, 1985), the principal's *relaxed* optimization problem becomes

$$\max_T E_{C^0, \epsilon_1^{\infty}} e^{-rT} \left[v - C_T \left(1 + \frac{1}{C^0} \frac{F_0}{f_0}(C^0) \right) \right]. \tag{19}$$

We can compare this to the result in our continuous-time analysis, proposition 1. Again it is clear that the principal takes into account the cost of private information at the time of contracting. However, it can also be shown that information that is revealed after the contract has been signed, only affects the expected investment cost, but leads to no further rents to the firm (as in Baron and Besanko, 1984; Besanko, 1985; Pavan, Segal and Toikka, 2014).

[6]The idea that incentive compatibility at time t also involves taking into account the marginal effect of current types on future ones is developed in generality in Pavan, Segal and Toikka (2014), in terms of impulse response functions.

4 Project choice: Competing Technologies

We now turn to the situation with multiple new technologies. Not only does the principal want to decide when to invest, but also which project to buy, and which technology to abandon. This setting includes the case with one firm and a known outside option for the principal. An example would be the choice between an uncertain innovative project, and an off-the-shelf alternative. Here the decision is either to invest in the agent's project, or to terminate the contract with the agent if costs evolve adversely, and choose the outside option instead (at any time from the initial contract signature and beyond).

We will first analyse the general two-agent model in which both agents have private, stochastic costs. The principal wants to design a mechanism that allows to postpone the decision on project choice until more information on both projects' costs has become available. As a benchmark, we again consider the real options solution without information asymmetry. We then solve for the case with asymmetric information. As a special case we consider the situation where only one project has stochastic costs, while the other may represent an off-the-shelf option that is fully developed.

4.1 Full information

Consider the case in which both competing technologies' costs, C_1 and C_2, follow geometric Brownian motions,

$$dC_i = \mu_i C_i dt + \sigma_i C_i dz_i$$

with drifts μ_i, volatilities σ_i, and with dz_i independent Brownian motions.[7]

In a setting with symmetric information, we have a standard real options investment model. This model is now two-dimensional, and only solvable numerically. Formally, the investor's optimization problem is

$$W(C_1, C_2) = \max_T E\left(e^{-rT}\left[\max\left(v_1 - C_1(T), v_2 - C_2(T)\right)\right]\right)$$

where optimization is over the stopping time T, which will be defined as the first time the process (C_1, C_2) hits some boundary, denoted ∂C, in the (C_1, C_2)-plane. Equivalently, therefore, we have to find the optimal boundary ∂C. This boundary will consist of two branches: a boundary component ∂C_1 which, when hit, will lead to investment in project 1 and abandonment of

[7] Assuming independence between the agents' information, on initial costs C_i^0, as well as cost updates dz_i, will allow us to preclude surplus extracting mechanisms of the type considered in Cremer and McLean (1988).

project 2. And, similarly, ∂C_2 for the converse. Figure 4.1 illustrates such a boundary. For the particular development of costs in the figure, when the cost process hits ∂C_2, investment in project 2 is carried out and project 1 is abandoned.

To solve for the principal's value W and his decisions in practice, one again analyzes the partial differential equation that W satisfies before the investment is made, and determining the appropriate optimal boundary by imposing smooth-pasting conditions.

For (C_1, C_2) high enough, there will be no immediate investment, and the investor's value function $W(C_1, C_2)$ satisfies

$$0 = -rW + \mu_1 C_1 W_1 + \mu_2 C_2 W_2 + \tfrac{1}{2}\sigma_1^2 C_1^2 W_{11} + \tfrac{1}{2}\sigma_2^2 C_2^2 W_{22}$$

with $W_i = \frac{\partial W}{\partial C_i}, W_{ij} = \frac{\partial^2 W}{\partial C_i \partial C_j}$. The investment threshold ∂C is the boundary of the domain where this PDE is a valid description of the value process. On this curve, we have boundary conditions

$$W(\partial C_i) = v_i - C_i(\partial C_i),$$

and smooth-pasting conditions

$$W_i(\partial C_i) = -1, W_i(\partial C_j) = 0.$$

The solution of this PDE with free boundary includes the shape and position of the optimal investment threshold.[8]

4.2 Asymmetric information: An auction

We now turn to the case where both C_1 and C_2 are private information to agents 1 and 2, respectively. We solve for the optimal auction by generalizing the analysis from section 3.2. Again, we first focus on a mechanism that incentivizes both agents to report their true costs, $\hat{C}_i = C_i$, at any moment, given a particular choice of the investment boundary ∂C by the principal. We then analyze how information rents lead to distortions in boundary choice in the ex ante menu of contracts.

[8]For completeness, the boundary conditions at infinity will be that as $C_1 \to \infty$, $W(C_1, C_2)$ will converge to the single project option value for project 2, a function of C_2; and vice versa.

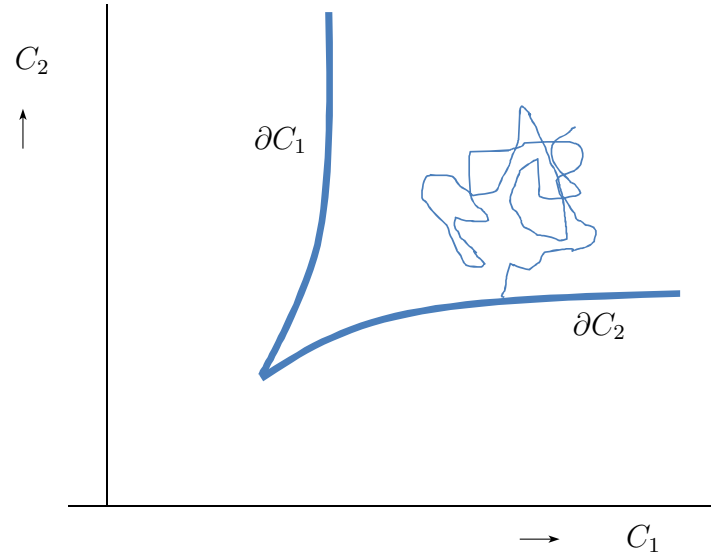


Figure 1: Example of investment boundaries and a potential path for costs C_1, C_2

Ex post incentive compatibility

Consider a fixed choice of boundary, ∂C. We look at mechanisms of ex post payments consisting of

1. An investment fee $\bar{\phi}_i(\hat{C}_1, \hat{C}_2)$ to agent i when $(\hat{C}_1, \hat{C}_2) \in \partial C_i$ and investment in project i occurs.

2. A termination fee $\underline{\phi}_i(\hat{C}_1, \hat{C}_2)$ when $(\hat{C}_1, \hat{C}_2) \in \partial C_j$, $j \neq i$, and investment in project j occurs, i's contract is terminated.

3. A flow of fees $p_i(\hat{C}_j)$, $j \neq i$, as long as no action is taken.

Note that in this case payments may depend on the other agent's reported costs, $\hat{C}_{j\neq i}$. In that way, for instance the investment fee $\bar{\phi}$ can depend on the exact location $(C_1, C_2) \in \partial C_i$ at which the threshold is hit.

With these fees and the boundary ∂C, and assuming agent 2 reports the truth, agent 1's value function $V^1(C_1, C_2)$ in the interior domain satisfies

$$0 = -rV^1 + p_1(C_2) + \mu_1 C_1 V_1^1 + \mu_2 C_2 V_2^1 + \tfrac{1}{2}\sigma_1^2 C_1^2 V_{11}^1 + \tfrac{1}{2}\sigma_2^2 C_2^2 V_{22}^1, \tag{20}$$

with subscripts on V denoting partial derivatives. For ∂C to be the privately optimal boundary for agent 1, we furthermore need value matching at the two boundary components, i.e.,

$$V^1(C_1, C_2) = \bar{\phi}^1(C_2) - C_1 \qquad \text{for } (C_1, C_2) \in \partial C_1,$$
$$V^1(C_1, C_2) = \underline{\phi}^1(C_2) \qquad \text{for } (C_1, C_2) \in \partial C_2,$$

and smooth-pasting,

$$V_1^1(C_1, C_2) = -1 \text{ for } (C_1, C_2) \in \partial C_1, \quad V_1^1 = 0 \text{ for } (C_1, C_2) \in \partial C_2.$$

Similar expressions hold for agent 2. Now, we can generalize the one-agent result, Lemma 1, to this two-dimensional situation.

Lemma 4 (Ex post incentive compatibility: Two agents)**.** *Consider a choice of investment threshold ∂C, with components ∂C_1 and ∂C_2. If the payment scheme, given this threshold, is ex post incentive compatible for agent i, then*

$$\frac{\partial V^i}{\partial C_i} = -E\left[\frac{C_i^{T_i}}{C_i} e^{-rT_i} | C_1, C_2\right]$$

with T_i the time until boundary component ∂C_i is hit, or $T_i = \infty$ if $\partial C_{j \neq i}$ is hit first. Also, $C_i^{T_i}$ denotes the cost C_i at the moment of hitting ∂C_i.

Lemma 4 characterizes the partial derivative of V^i. For any V^i satisfying this condition, we can again compute the required set of fees $p, \bar{\phi}$ and $\underline{\phi}$, which will generally depend on C_j, $j \neq i$, by looking at the values on the boundary, and at the HJB equation (20). Shortly, we will describe one particularly simple choice for these fees. First, however, we turn to the ex ante problem. As in section 3.2, for finding the optimal menu of contracts, we only need to focus on the derivative of V^i, which is given in Lemma 4.

The ex ante optimization

The initial auction once again involves a menu of contracts $(\phi_i^0, p_i, \bar{\phi}_i, \underline{\phi}_i)$, one for each reported set of time zero costs, (C_1^0, C_2^0). The ex post fees will fix the boundary ∂C. The ex ante fees ϕ_i^0 control the rents left to each agent.

Ex ante incentive compatibility requires that for agent i, reporting his true initial costs C_i^0 is optimal if agent j does so too. Assuming truthful revelation of C_2^0 by agent 2, agent 1's time

zero expected utility depends on his initial costs C_1^0, and his reported initial costs $\hat{C}_1^0$

$$U^1(C_1^0, C_2^0, \hat{C}_1^0) = \phi_1^0(\hat{C}_1^0, C_2^0) + V^1(C_1^0, C_2^0, \partial C(\hat{C}_1^0, C_2^0)),$$

with $\partial C(\hat{C}_1^0, C_2^0)$ the boundary corresponding to contract choice $(\hat{C}_1^0, C_2^0)$, and V^1 the expected value of the ex post fees and investment costs, given this boundary and initial condition (C_1^0, C_2^0).

As before (Lemma 2), ex ante incentive compatibility implies a condition on the derivative of $U^1(C_1^0, C_2^0) \equiv U^1(C_1^0, C_2^0, \hat{C}_1^0 = C_1^0)$.

Lemma 5. *If the menu $(\phi_i^0, p_i, \bar{\phi}_i, \underline{\phi}_i)(C_1^0, C_2^0)$ is incentive compatible, then resulting utilities satisfy*

$$\frac{\partial U^i(C_1^0, C_2^0)}{\partial C_i^0} = \frac{\partial V^i}{\partial C_i^0}.$$

Also, the choice of boundary, ∂C, which determines stochastic hitting times T_i, is such that

$$E\left[\frac{C_i^{T_i}}{C_i} e^{-rT_i} | C_1, C_2\right]$$

is decreasing in initial costs C_i^0.

The proof is analogous to that of lemma 2.

With the expressions for $\frac{\partial V^i}{\partial C_i^0}$ from Lemma 4, we can now straightforwardly analyze the menu of contracts that optimizes the principal's utility.

The principal designs the contract menu to optimize his expected utility, respecting both the incentive compatibility constraints from Lemma 5 and the agents' participation constraints, which take the form $U^i(C_H^i) = 0$. Writing

$$U^1(C_1^0, C_2^0) = -\int_{C_1^0}^{C_H} \frac{\partial U^1(s, C_2^0)}{\partial s} ds, \tag{21}$$

we see that the principal's profits depend on the precise choice of ex-post fees $(p_i, \bar{\phi}_i, \underline{\phi}_i)$ only through the investment boundary ∂C. It is therefore sufficient to consider the principal's optimization of expected utility over the choice of these boundaries, along with initial fees ϕ_0:

$$\begin{aligned} W = E_{C_{1,2}^0} \max_{\partial C(C_{1,2}^0), \phi_{1,2}^0(C_{1,2}^0)} E\big[&e^{-rT_1}(v_1 - C_1(T_1)) + e^{-rT_2}(v_2 - C_2(T_2)) \\ &- U^1(C_1^0, C_2^0) - U^2(C_1^0, C_2^0) \mid C_1^0, C_2^0\big] \end{aligned}$$

Using reasoning as in proposition 1 we then straightforwardly get the following result for

the principal's optimal choice of boundary given reported initial costs $C^0_{1,2}$.

Proposition 3 (Optimal auction)**.** *For each value of reported initial costs (C^0_1, C^0_2), the optimal choice of investment boundary ∂C maximizes*

$$W(C^1_0, C^2_0) = \max_{\partial C} E\left[e^{-rT_1}\left(v^1 - \alpha_1 C_1(T_1)\right) + e^{-rT_2}\left(v^2 - \alpha_2 C_2(T_2)\right)|C^0_1, C^0_2\right] \tag{22}$$

with

$$\alpha_i = \left(1 + \frac{F(C^0_i)}{f(C^0_i)}\frac{1}{C^0_i}\right). \tag{23}$$

The associated fees $(\phi^0_i, p_i, \bar{\phi}_i, \underline{\phi}_i)$ are chosen so as to satisfy ex post incentive compatibility for that chosen boundary, with ϕ^0_i adjusted so as to make sure the agent's ex ante utility equals $U^i(C^0_1, C^0_2)$.

There is again a continuum of choices of fees for each initial cost pair $C^0_{1,2}$ that implement the same decision rule ∂C and give ex ante utilities U^i. Upon inspecting equation (22) it is clear that one particularly convenient choice consists of ex post fees $(p_i, \bar{\phi}_i, \underline{\phi}_i)$ that directly internalize the principal's objective into the agents' pay-offs à la Vickrey.

Proposition 4 (Vickrey implementation)**.** *The optimal auction can be implemented by a fee upon investment, $\bar{\phi}_i$, and a termination fee $\underline{\phi}_i$, satisfying*

$$\bar{\phi}^i = \frac{v^i}{\alpha^i} \quad \underline{\phi}^i = \frac{v^j - \alpha^j \bar{C}_j}{\alpha^i}$$

where $\bar{C}^j(C_i, C_j)$ is the value of C_j when investment in project j is undertaken. There is no flow fee $p = 0$. The required initial payment in this case is

$$\phi^0_i(C^0_i, C^0_j) = \int_{C^0_i}^{C_H} E\left(\frac{C^T_i}{s_i}e^{-rT_i} \mid s_1, C^0_j\right) ds_i - \frac{1}{\alpha_i} W(C^0_i, C^0_j)$$

for agent $i \in \{1, 2\}$, $j \in \{1, 2\}$ and $j \neq i$. Here $W(C^0_i, C^0_j)$ is the principal's value, as in Proposition 3.

This mechanism relates to pivotal mechanisms as discussed by Bergemann and Välimäki (2010), in a public choice context, as a dynamic realization of the AGV mechanism. Board (2007) and Kakade, Lobel and Nazerzadeh (2013) apply similar ideas in an auction context in discrete time. The payments ensure that both agents internalize the effects of their actions both on the principal and on their rivals. For the most efficient types, $\alpha^i = 1$, and this internalization is complete.

To ensure ex ante incentive compatibility, the rents U^i that need to be left to efficient types depend on decision rules for higher cost-types. The contracts to higher cost-types are distorted to reduce these rents. For these agents, $\alpha^i > 1$, and hence they internalize only part of the principal's value. This delays investment compared to the first-best, but allows for lower rents to be left to the agents.

4.3 Application: Old versus new technology

We now focus on a special case, where the choice for the principal is between one new, uncertain project, and an old proven technology the costs of which are certain. One might think of a telecom firm that either has to invest in a new, next generation network, or to invest in maintaining its existing network. Or consider a government's choice between either buying a new jet fighter, the ultimate costs of which are still uncertain, or on the other hand upgrading its old fighters at predictable costs instead. Those costs of the outside option might be known to the principal, or there may be asymmetric information surrounding these. But the main point here is that they are non-stochastic.

In terms of the model, the old technology (agent 2) is non-stochastic, so C_2 is constant, while the new technology, costs C_1, is stochastic. We assume that C_2, though constant, is private information to player 2. We can then also look at the limit where this information asymmetry vanishes.

Applying the general two-agent framework above, in this case, the partial differential equations collapse into ordinary differential equations, since C_2 is no longer dynamic. For any given initial constant C_2, only two points on the boundary ∂C matter: $\underline{C}_1$, the upper threshold C_1 will have to pass for the new technology to be abandoned in favour of the old, certain one; and $\bar{C}_1$, the lower threshold at which the new technology 1 becomes so attractive that it will be bought at the expense of the outside option 2.

Let us first consider the old, non-stochastic technology, provided by agent 2. Applying Lemma 4, we have that agent 2's ex post value V^2 satisfies

$$\frac{\partial V^2}{\partial C_2} = -E\left(e^{-r\underline{T}} \mid C_1\right)$$

where $T_2 = \underline{T}$ is shorthand for the first time the $\underline{C}_2$ threshold is hit, and investment in agent 2's technology is undertaken. $\underline{T}$ is taken to be ∞ when instead $\bar{C}_1$ is crossed first and project 2 is abandoned. If we take the thresholds as given, the expectation does not depend on the level of C_2 any more.

By integration, we now have

$$V^2(C_1, C_2) = (\tilde{C}_2 - C_2)E\left(e^{-r\underline{T}} \mid C_1\right),$$

with $\tilde{C}_2$ an integration constant. If we choose $\tilde{C}_2$ to be independent of C_1, we find that the premium paid to the old technology firm when its contract is abandoned, $\underline{\phi}_2$ is zero,

$$\underline{\phi}_2 = V^2(\underline{C}_1, C_2) = 0$$

since $\underline{T} = \infty$ in this case. Moreover, since $E\left(e^{-r\underline{T}} \mid C_1\right)$ satisfies the HJB equation without source-term (which only involves C_1 derivatives now since C_2 is non-stochastic), we find that the flow fees for agent 2, $p_2 = 0$ as well. Hence, for C_2's problem we only have a potentially non-zero fee $\bar{\phi}_2$ when project 2 is invested in. This fee, moreover, may be set to the ex ante revealed costs C_2 by an appropriate choice of the ex ante fee ϕ_2^0. The ex post rents for agent 2 are then zero.

The irrelevance of ex post fees for agent 2 is intuitive: once agent 2 has given up his initial costs C_2, there is no more information to extract as his costs are constant. After the initial auction, agent 2 is a passive player.

Now turn to agent 1 who has stochastic costs C_1. For any fixed C_2, agent 1's problem is as in our benchmark, one-dimensional case studied in section 3. The only twist here is that we have added abandonment of his project 1 if costs grow so high (to a level of $\underline{C}_1$) that choosing for the non-stochastic technology 2 becomes optimal.

The principal's problem is therefore to determine, for each report of initial costs, the set of levels $\bar{C}_1, \underline{C}_1$ that trigger investment in project 1, or termination of his contract. To ensure incentive compatibility, the required optimization then is

$$W(C_1^0, C_2) = \max_{\bar{C}_1, \underline{C}_1} E\left[e^{-r\bar{T}}\left(v^1 - \alpha_1 \bar{C}_1\right) + e^{-r\underline{T}}\left(v^2 - \alpha_2 C_2\right) | C_1^0, C_2\right].$$

This is a standard problem in the real options literature, where investment in alternative projects is considered, one that becomes attractive when the stochastic parameter (C_1) drops, the other when it rises (see e.g. Décamps, Mariotti and Villeneuve, 2006). The only difference here is the inclusion of the "virtual costs" mark-up factor α_i, that reflects the impact of information rents.

The principal can then implement the strategy resulting in investment and abandonment thresholds $\bar{C}_1(C_1^0, C_2), \underline{C}_1(C_1^0, C_2)$ by auctioning two contracts, one for each agent. The new

project, developed by agent 1, gets three fees that depend on bids (C_1^0, C_2) as in Proposition 4: an ex ante fee ϕ_1^0, a fee upon investment $\bar{\phi}_1 = \frac{v_1}{\alpha_1}$, and a severance fee for terminating the project $\underline{\phi}_1 = \frac{v^2 - \alpha_2 C_2}{\alpha_1}$. The old technology, agent 2, only gets an ex ante fee, and repayment of costs C_2 when the new project is abandoned and agent 2 is called upon to invest. In that case, ϕ_2^0 equals agent 2's rents,

$$\phi_2^0(C_1^0, C_2) = U^2(C_1^0, C_2).$$

In the special case where the costs of the outside option C_2 are public information, we have $\alpha_2 = 1$. Agent 2's rents are zero, and the problem becomes genuinely one-dimensional.

5 Conclusion

In this paper we have studied the optimal contract for investments in new and uncertain technologies, when stochastic costs are privately observed by the agents. An optimal procurement contract should allow to postpone the investment decision until future uncertainty on cost evolution is partly resolved. At the same time, contracts for all participating firms should be structured such that agents have incentives to truthfully reveal those cost evolutions. We have considered both the case of optimal investment timing for one technology (without competing technologies) and how to optimally decide between several competing technologies, each developed by a separate agent. We show how private information affects the ex post payments to the firms as well as how this private information delays investment compared to a real options benchmark, in order to reduce the firms' information rents. Ultimately, the principal's optimization problem turns out to be a straightforward variation on the real options symmetric information benchmark, with stochastic parameters replaced by an appropriate, virtual, counterpart. To provide proper incentives, firms need to be awarded not only fees at investment, but also severance payments when a rival technology is finally chosen. Ex ante payments can be used to extract some of these ex post information rents. Applications of our theory include investments in technologies that are still in a pre-commercial stage and have costs that are still evolving; examples include regulated investments in new telecommunications networks and procurement of high tech goods such as weapons systems. The methodology we presented lends itself to straightforward application to more general real options problems such as, for instance, those presented in the well-known textbook by Dixit and Pindyck. Examples would be analysis of start-up or lay-up costs, or sequential investments. Application of the methods used in this paper allows us to describe the optimal transfer policy and associated payments.

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A Appendix

Proof of Lemma 1:

Suppose that, given p and $\bar{\phi}$, the firm's privately optimal investment threshold is $\bar{C}$. In that case, in the domain $C \geq \bar{C}$, $V(C)$ satisfies the HJB equation

$$-rV + \mu C V_C + \tfrac{1}{2}\sigma^2 C^2 V_{CC} + p = 0,$$

(with source term p) while on the boundary, $C = \bar{C}$, we have value matching

$$V(\bar{C}) = \bar{\phi} - \bar{C}$$

and smooth pasting,

$$V_C(\bar{C}) = -1.$$

Now, since p is constant, taking the derivative of the HJB equation, and multiplying by C, we obtain

$$-r(CV_C) + \mu C \frac{\partial}{\partial C}(CV_C) + \tfrac{1}{2}\sigma^2 C^2 \tfrac{\partial^2}{\partial C^2}(CV_C) = 0,$$

so CV_C satisfies a similar, but sourceless, HJB equation.

From the smooth-pasting condition, we have that CV_C satisfies boundary condition

$$\bar{C}V_C(\bar{C}) = -\bar{C}.$$

Imposing finally that $V_C \to 0$ as $C \to \infty$, the unique solution to this differential equation is

$$CV_C = -\bar{C}E\left[e^{-rT} \mid C\right]$$

for $C \geq \bar{C}$ and with T the first time the cost hits the threshold $\bar{C}$.[9] *Q.E.D.*

Proof of Lemma 2:

A firm with initial costs C^0, announcing costs $\hat{C}^0$, has total expected rents

$$U(C^0, \hat{C}^0) = \phi^0(\hat{C}^0) + V(C^0, \bar{C}(\hat{C}^0)),$$

Incentive compatibility means that this expression is maximized at $\hat{C}^0 = C^0$. But then, by the

[9] That this expression satisfies the differential equation follows from expanding the equality $E\left(e^{-rT} \mid C\right) = e^{-rdt}E\left(e^{-rT} \mid C + dC\right)$. It is straightforward that it satisfies the boundary conditions as well.

envelope theorem, we have

$$\left.\frac{dU(C^0,\hat{C}^0)}{dC^0}\right|_{\hat{C}^0=C^0} = \left.\frac{\partial U(C^0,\hat{C}^0)}{\partial C^0}\right|_{\hat{C}^0=C^0} = \frac{\partial V}{\partial C^0} = -E\left(\frac{\bar{C}(C^0)}{C^0}e^{-rT} \mid C^0\right).$$

With the definition

$$U(C^0) = U(C^0, C^0)$$

this proves (7). To prove (8), note that

$$U(C^0,C^0) \geq U(C^0,\hat{C}^0) = U(\hat{C}^0,\hat{C}^0) - \int_{\hat{C}^0}^{C^0} E\left(\frac{\bar{C}(\hat{C}^0)}{s}e^{-rT} \mid s\right) ds$$

and, by reversing the two arguments,

$$U(\hat{C}^0,\hat{C}^0) \geq U(\hat{C}^0,C^0) = U(C^0,C^0) + \int_{\hat{C}^0}^{C^0} E\left(\frac{\bar{C}(C^0)}{s}e^{-rT} \mid s\right) ds.$$

Combining these two inequalities, we get

$$\int_{\hat{C}^0}^{C^0} E\left(\frac{\bar{C}(C^0)}{s}e^{-rT} \mid s\right) ds \leq \int_{\hat{C}^0}^{C^0} E\left(\frac{\bar{C}(\hat{C}^0)}{s}e^{-rT} \mid s\right) ds$$

Hence, if $C^0 > \hat{C}^0$, then the integrand on the left-hand side must be lower than the one on the right-hand side, and so $\bar{C}(C^0) < \bar{C}(\hat{C}^0)$. *Q.E.D.*

Proof of Proposition 1:

The principal's problem is to choose a C^0-dependent threshold $\bar{C}(C^0)$ that optimizes his expected surplus W in equation (10), with the expectation taken over the distribution F of initial costs C^0. Substituting for $U(C^0)$ from (9), which incorporates incentive compatibility and participation constraints, we get the principal's problem

$$\max_{\bar{C}(C^0)} \int_{C_L}^{C_H} \left(E\left(e^{-rT(\bar{C}(C^0))} \mid C^0\right)(v - \bar{C}(C^0)) - \int_{C^0}^{C^H} E\left(\frac{\bar{C}(s)}{s}e^{-rT} \mid s\right) ds \right) dF.$$

Doing a partial integration on the final term, we get an integrand given in the expression in the proposition. Since $\frac{1}{C^0}\frac{F}{f}(C^0)$ is monotone, we also have monotonicity of the threshold $\bar{C}$, equation (8). The derivation of the optimal threshold level then proceeds as in the symmetric information case.

Proof of Lemma 3:

It is convenient to change variables to $X(C)$ satisfying $\frac{dX}{dC} = \sigma^{-1}(C)$. By Ito's lemma, X satisfies the SDE

$$dX = (\frac{\mu(C)}{\sigma(C)} - \tfrac{1}{2}\sigma'(C))dt + dz.$$

In terms of X, the agent's value function solves the HJB equation

$$rV = \tilde{\mu}V_X + \tfrac{1}{2}V_{XX} + p,$$

with $\tilde{\mu}(X)$ the drift coefficient in dX as given above. Taking the derivative, V_X in turn satisfies

$$(r - \tilde{\mu}_X)V_X = \tilde{\mu}(V_X)_X + \tfrac{1}{2}(V_X)_{XX}.$$

At the boundary $\bar{X} = X(\bar{C})$, we have

$$V_X(\bar{X}) = \sigma(\bar{C})V_C = -\sigma(\bar{C}).$$

The result then follows from the boundary condition and the HJB equation for V_X, by the Feynman-Kac equation. *Q.E.D.*

Proof of Lemma 4:

Taking the derivative to C_1 of the HJB equation for V^1, equation (20), and multiplying by C_1, we find that $C_1V_1^1$ satisfies a similar HJB equation, without source $p_1(C_2)$

$$0 = -r(C_1V_1^1) + \mu_1C_1(C_1V_1^1)_1 + \mu_2C_2(C_1V_1^1)_2 + \tfrac{1}{2}\sigma_1^2C_1^2(C_1V_1^1)_{11} + \tfrac{1}{2}\sigma_2^2C_2^2(C_1V_1^1)_{22}.$$

From the smooth-pasting conditions we obtain the value of $C_1V_1^1$ on the boundary ∂C: $C_1V_1^1 = -C_1$ on ∂C_1, and $C_1V_1^1 = 0$ on ∂C_2.

Combining the HJB equation with the boundary conditions on $C_1V_1^1$, by the Feynman-Kac formula, we have

$$C_1V_1^1 = E\left[-C_1^{T_1}e^{-rT_1} + 0 \cdot e^{-rT_2}|C_1, C_2\right],$$

and the lemma follows. *Q.E.D.*

Proof of Proposition 4:

With payments as in the proposition, agent 1's ex post utility becomes

$$V^1(C_1, C_2) = \max_{T_1,T_2} E\left[e^{-rT_1}\left(\frac{v_1}{\alpha_1} - C_1(T_1)\right) + e^{-rT_2}\left(\frac{v_2}{\alpha_1} - \frac{\alpha_2}{\alpha_1}C_2(T_2)\right)\right],$$

which up to the overall factor $1/\alpha_1$ coincides with the principal's optimization. The same holds for agent 2, and all agents optimize their profits by following the same investment strategy (i.e., choice of stopping times $T_{1,2}$ plus investment by agent 1 versus abandonment in favour of agent 2) as the principal would have.

Agent i's ex-post utility equals $1/\alpha_i$ times the principal's utility W. The prescribed ex ante payment ϕ_i^0 makes sure total expected utility U_i equals that prescribed in equation (21), consistent with incentive compatibility and the participation constraint. *Q.E.D.*