

# Credit Market Competition and Liquidity Crises

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## **Abstract**

We develop a two-period model where banks invest in liquid reserves and loans, and are subject to aggregate liquidity shocks. When banks face a shortage of liquidity, they can sell loans on the interbank market. Two types of equilibria emerge. In the no default equilibrium, banks keep enough reserves and remain solvent. In the mixed equilibrium, some banks default with positive probability. The former equilibrium exists when credit market competition is intense, while the latter emerges when banks exercise market power. Thus, competition is beneficial to financial stability. The structure of liquidity shocks affects the severity and the occurrence of crises, as well as the amount of credit available in the economy.

# 1 Introduction

There is a long and wide standing debate both among academics and policymakers on the nexus between competition and financial stability. The key issue is how competition affects banks' and borrowers' risk taking behavior. One view is that, by reducing banks' franchise value, competition reduces the incentives for banks to behave prudently (see, Keeley, 1990, and the subsequent papers surveyed in Carletti, 2008, and Carletti and Vives, 2009). An opposite view is that low loan rates induce borrowers to take less risk, thus reducing the risk of banks' portfolio. This implies a positive relationship between competition and financial stability if loan returns are perfectly correlated (Boyd and De Nicoló, 2005) or a U-shaped relationship if loan returns are not perfectly correlated (Martinez-Miera and Repullo, 2010). Along with the theoretical literature, the empirical evidence is inconclusive on whether competition is beneficial or detrimental to financial stability. Results differ across samples and time periods and very much depend on the estimates used to measure competition and stability (see the discussion in Carletti, 2010).

The debate considers credit risk as the only source of risk taking by banks. However, in practice, banks are also subject to liquidity risk in their role as liquidity providers (e.g. Diamond and Dybvig, 1983, and subsequent literature). As the recent crisis has shown, the maturity mismatch between assets and liabilities can become a crucial source of risk if banks are unable to raise liquidity on demand. When faced with large liquidity demands, banks with small reserve holdings need to raise additional liquidity by either borrowing or selling illiquid assets at fire sale prices. When asset prices are low, as it happens when market liquidity is scarce, banks may be unable to withstand the liquidity shock and become insolvent. Key to the emergence of liquidity crises are then the amount of liquid assets that banks hold and the total supply of liquidity on the market. The former affects individual banks' need of additional liquidity; the latter determines market liquidity and thus the level of asset prices.

In this context, we develop a novel theory where we link credit market competition

with the emergence of liquidity crises. The degree of competition crucially affects the level of profits that banks can generate from lending and thus the opportunity cost of holding reserves in terms of both foregone lending returns and level of asset prices. The theory provides numerous new insights into the relationship between competition and stability. In contrast to the charter value hypothesis described above, we show that competition is beneficial to financial stability as it induces banks to behave prudently and hold more liquidity. The result is in line with that in Boyd and De Nicoló (2005), although our focus is on liquidity risk rather than credit risk as a source of instability, and thus on how banks' profitability, as determined by the degree of credit market competition, affects liquidity risk management rather than borrowers' risk taking incentives.

We build on a standard two-period banking model as developed in Allen and Gale (2004a, 2004b) and Allen, Carletti and Gale (2009). On the asset side, banks invest in a one-period liquid asset (reserves) or in two-period loans to entrepreneurs. On the liability side, banks raise funds from risk-averse consumers in the form of demandable deposits and face aggregate uncertainty relative to their demand for liquidity at the interim date. There is a good state with a small fraction of early depositors, and a bad state where the fraction of early depositors is larger. Banks can meet their liquidity demands by holding reserves initially or selling loans on a (competitive) interbank market at the interim period, where prices are endogenously determined by the demand and supply of liquidity in each state of the world.

Credit market competition affects banks' liquidity risk management in various ways. Firstly, it determines the cost for banks to hold reserves in terms of foregone return on the loans and the terms of the deposit contracts they offer to depositors. Secondly, by affecting loan profitability and banks' incentives to hold liquid reserves, it affects the supply and demand of liquidity in the interbank market and thus the level of asset prices. This in turn determines banks' ability to withstand the liquidity shocks and thus the emergence of liquidity crises.

We first show that two types of equilibria can emerge, depending on the degree of

competition in the credit market. A no default equilibrium emerges when competition is intense. As loans are not very profitable, the opportunity cost for banks of holding reserves is low. All banks find it optimal to keep enough reserves to repay the early depositors in both states of nature. Asset prices are consistent with this equilibrium in that no bank find it convenient to reduce its reserve holdings and use the interbank market to obtain additional liquidity. As competition decreases, holding reserves becomes increasingly more costly and the no default equilibrium ceases to exist. In the new equilibrium, defined as mixed, banks behave differently despite being ex ante alike. Some banks, which we call risky, invest only in loans and default in the bad state of nature when all consumers withdraw and a bank run occurs. As risky banks sell all their loans, in the bad state asset prices drop significantly and consumers obtain the liquidation proceeds instead of the promised repayments. The remaining banks, defined as safe, hold enough liquidity to always meet their commitments and acquire the loans of the risky banks.

We then show that the degree of competition for which default starts to emerge and the number of defaulting banks crucially depend on the structure of liquidity shocks. When the probability of the bad state of nature occurring is low, default is more unlikely to occur and more banks have incentives to reduce their reserve holdings. Thus, when the economy is characterized by a more stable environment, crises are less frequent but are more severe in that they involve a larger number of banks and emerge in more competitive credit markets. In this situation, crises are also associated with greater credit availability. In contrast, in economies characterized by a high probability of large liquidity shocks, banks prefer to behave prudently. Fewer banks behave risky and default only occurs when banks exercise enough market power. In aggregate the banking system provides also less credit to the economy. These results suggest that credit market competition and the structure of liquidity shocks, which can be interpreted as the level of exogenous risk in the economy, are substitutes in terms of their impact on banks' risk taking behavior.

The key feature of the model is that there is a wedge between the loan return accruing to banks and the return from holding reserves. The magnitude of such a wedge is determined

by the level of competition in the credit market. The less competitive the credit market, the more profitable loans are and the more costly holding reserves is. Any other factor affecting the difference in the profitability of loans and reserves is consistent with our story. For example, banks granting loans to more profitable industries have a higher opportunity cost of holding reserves and are therefore more prone to behave risky. Similarly, highly leveraged banks are able to obtain higher returns from their investments and have therefore lower incentives to insure themselves against liquidity shocks.

Our basic model can be extended in a number of directions. We first consider alternative consumers' utility functions. We show that the main result of the basic model concerning the existence of the no default and the risky equilibria remain valid when consumers have a degree of risk aversion greater than one. The only difference is that beyond a critical level of risk aversion, risky banks start holding a positive amount of reserves as a way to contain the fire sales of their loans when the bad liquidity state occurs. We then consider the situation where deposits are insured. This implies that risky banks pay a premium against a certain level of insurance to their depositors when the bad state realizes. We show that the mixed equilibrium emerges for a larger range of parameters if the premium required for the insurance is lower than the benefit that risky banks enjoy from the insurance in terms of a lower compensation to their depositors for the risk they take. Whether this occurs depends on the structure of liquidity shocks. Finally, we consider the case where banks compete also in the deposit market where they have to split the surplus generated by lending with consumers. Again the main result of our basic model remains valid, and we show that the threshold value of credit market competition below which the mixed equilibrium exists varies non-monotonically with the degree of competition on the deposit market.

The paper has a number of empirical implications. First, it predicts that banks in competitive banking systems behave more prudently than banks in less competitive systems. Second, systems with similar levels of competition are more likely to be unstable when large liquidity shocks are less likely. Third, crises occurring in systems with a low ex-

pectation of large liquidity shocks are more severe in terms of number of defaulting banks but are associated with greater credit availability. Finally, the model predicts that crises emerge in more monopolistic credit markets when banks operate in risky environments and thus have to pay large deposit insurance premia or when they are subject to more intense competition on the deposit market.

The novelty of the paper is to analyze the relationship between competition and liquidity risk, and to look at the implications for the severity of crises and credit availability. In this sense, it is linked to various strands of literature. A few papers have looked at the effect of competition on bank instability in terms of runs (see also Carletti, 2008, and Carletti and Vives, 2009, for a survey). The analysis of Rochet and Vives (2004) and Goldstein and Pauzner (2005) suggests that when banks offer higher repayments to early depositors (as would be the case with more intense competition on the deposit market), bank runs are more likely to occur as a result of coordination failures. Matutes and Vives (1996) show that deposit market competition does not have a clear effect on banks' vulnerability to runs, but higher promised repayments to depositors tend to make banks more unstable. Carletti et al. (2007) analyze the impact of credit market competition on banks' incentives to hold liquidity after a merger. They show that an increase in market power as after a merger among large banks increases banks' liquidity needs and thus the probability of liquidity crises. In contrast to these papers, we focus on the impact of credit market competition and banks' holdings in a framework where runs are due to deterioration of asset prices rather than to depositors' coordination failures.

Our paper shows that competition is beneficial to financial stability but not necessarily to credit availability. Default can entail greater or smaller credit availability depending on the effect it has on banks' total reserve holdings. This idea is related to that in Allen and Gale (1998) that bank runs can be beneficial as they improve risk sharing between early and late depositors. Similarly, Boyd, De Nicoló and Smith (2004) and De Nicoló and Lucchetta (2012) show that competitive banking systems are more exposed to crises than monopolistic ones, but provide better inter-temporal insurance to depositors or more

intermediated funds.

Several recent contributions on financial stability have focused on crises generated from asset price volatility and fire sales losses. Examples are Acharya and Yorulmazer (2008), Acharya, Shin and Yorulmazer (2011) Diamond and Rajan (2011) and, in particular, Allen and Gale (1994, 2004a, 2004b), and Allen and Carletti (2006, 2008). We contribute to this literature by analyzing how competition affects asset prices and thus the emergence of liquidity crises.

Finally, we show that the presence of competitive interbank markets supports the existence of a mixed equilibrium where some banks default in one state of nature and sell their loans to other banks in the interbank market. In this sense, the paper is related to some contributions that focus on the interbank market such as Flannery (1996), Freixas and Jorge (2008) and Acharya, Gromb and Yorulmazer (2011).

The paper proceeds as follows. Section 2 describes the model. Section 3 derives the no default and the mixed equilibria and analyze their properties. Section 4 extends the basic model in various directions, and Section 5 discusses the main implications of the model. Section 6 concludes. All proofs are in the appendix.

## **2 The model**

Consider a three date ( $t = 0, 1, 2$ ) economy with three types of agents: banks, consumers and entrepreneurs. Banks raise funds from consumers in exchange for a deposit contract and provide loans to entrepreneurs.

Each bank raises funds at date 0 from a continuum of mass one of consumers endowed with one unit at date 0 and nothing thereafter. Consumers are all ex ante identical but are either early or late types ex post. The former value consumption only at date 1; the latter value consumption only at date 2. Each consumer has a probability of being an

early type  $\lambda_\theta$  given by

$$\lambda_\theta = \begin{cases} \lambda_L & \text{w. pr. } \pi \\ \lambda_H & \text{w. pr. } (1 - \pi), \end{cases}$$

with  $\lambda_H > \lambda_L$ . From the Law of Large Numbers,  $\lambda_\theta$  represents the fraction of early types at each bank. As there is only aggregate uncertainty, the realization of  $\lambda_\theta$  is the same for all banks. Thus, there are two states of nature,  $L$  and  $H$ , which we refer to as the good and the bad state respectively.

Consumers can either store or deposit their endowment at the bank in exchange for a demand deposit contract promising a (non-contingent) amount  $c_1$  at date 1 or  $c_2$  at date 2. Depositors' utility function  $u(c_t)$  is twice differentiable and satisfies the usual neoclassical assumptions:  $u'(c) > 0$ ,  $u''(c) < 0$  and  $\lim_{c \rightarrow 0} u'(c) = \infty$ .

Each bank invests a fraction  $R$  of its funds in a storage technology defined as reserves and a fraction  $L$  in loans to entrepreneurs in exchange for a per-unit loan rate of  $r$  at date 2. Entrepreneurs invest the amount received by the bank in a (divisible) project yielding  $V > 1$  at date 2 and have an opportunity cost equal to  $v \in (0, \bar{v})$ . Thus, the loan rate has to satisfy

$$V - r \geq v. \tag{1}$$

A higher  $v$  forces the bank to reduce the loan rate  $r$  to ensure that the entrepreneurs will accept the loan. Clearly, (1) will always hold with equality in equilibrium. In this sense, the parameter  $v$  represents a way to split the surplus  $V$  generated by the project between banks and entrepreneurs and can be interpreted as the degree of competition in the credit market. The idea is similar to Salop (1979) model where competition, as measured by the size of the transportation costs, is associated with a lower loan rate and thus lower banks' profits.

Loans can be sold on a (competitive) interbank market at date 1 for a price  $P_\theta$ . Participation in this market is limited in that only banks can buy and sell loans. The price  $P_\theta$  is endogenously determined in equilibrium by the aggregate demand and supply of liquidity

in the market, as explained further below. As there are only two states  $\theta = H, L$ , the price  $P_\theta$  can take at most two values.

The timing of the model is as follows. At date 0, banks choose the deposit contract  $(c_1, c_2)$  and the initial portfolio allocation between reserves and loans in order to maximize their expected profits. At the beginning of date 1 all uncertainty is resolved: consumers learn privately their type and the state  $\theta = H, L$  is realized. Early consumers always demand  $c_1$  at date 1 to meet their consumption needs. In contrast, late consumers can either wait and demand the promised consumption  $c_2$  at date 2, or claim to be early types and demand  $c_1$  at date 1, thus precipitating a run. In the absence of runs, a fraction  $\lambda_\theta$  of consumers are paid  $c_1$  at date 1 and the remaining fraction  $1 - \lambda_\theta$  are paid  $c_2$  at date 2. When a run occurs, the bank has to sell all its loans and it goes bankrupt, and consumers receive a pro rata share of the bank's resources. A run occurs in the model only when the value of the bank's portfolio at date 2 does not suffice to repay at least  $c_1$  to the late consumers. That is, sunspot runs do not occur.

### 3 Equilibrium

Two equilibria arise endogenously in the model. In the first, that we define as no default equilibrium, runs do not occur and all banks remain solvent in both states  $\theta = H, L$ . In the second, defined as mixed equilibrium, some banks experience a run and go bankrupt in some state, while some others always remain solvent. We first characterize the two equilibria. Then, we analyze for which parameter space, and in particular for which level of competition in the credit market, as represented by the size of the parameter  $v$ , the two equilibria exist. We start with the candidate equilibrium in which there is no default.

#### 3.1 The no default equilibrium

The no default equilibrium exists when all consumers withdraw according to their time preferences so that runs do not occur and all banks remain solvent. As they are all ex

ante identical and none defaults, there is no loss of generality in assuming that banks behave alike at the initial date concerning both their portfolio allocation and the terms of the deposit contract. Each bank chooses the deposit contract  $(c_1, c_2)$  and the portfolio allocation  $(R, L)$  simultaneously to maximize its expected profit at  $t = 0$ . The bank's maximization problem is then given by

$$\underset{c_1, c_2, R, L}{Max} \Pi = rL + \pi(R - \lambda_L c_1) + (1 - \pi)(R - \lambda_H c_1) - [\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)]c_2 \quad (2)$$

subject to

$$R + L = 1 \quad (3)$$

$$\lambda_\theta c_1 \leq R \quad (4)$$

$$(1 - \lambda_\theta)c_2 \leq rL + R - \lambda_\theta c_1 \quad (5)$$

$$c_2 \geq c_1 \quad (6)$$

$$E[u(c_1, c_2, \lambda_\theta)] = [\pi\lambda_L + (1 - \pi)\lambda_H]u(c_1) + [\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)]u(c_2) \geq u(1) \quad (7)$$

$$0 \leq R \leq 1, c_1 \geq 0.$$

for any  $\theta = L, H$ . Bank's profit in (2) is given by sum of the returns from the loans  $rL$  and the expected excess of liquidity  $\pi(R - \lambda_L c_1) + (1 - \pi)(R - \lambda_H c_1)$  minus the expected payments  $[\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)]c_2$  to depositors at date 2. Constraint (3) represents the budget constraint at date 0. The next two constraints are the resource constraints at dates 1 and 2. Constraint (4) requires that the bank has enough resources at date 1 to satisfy the demands  $\lambda_\theta c_1$  by the early consumers for any  $\theta = L, H$ . Constraint (5) requires that the resources  $rL + R - \lambda_\theta c_1$  available to the bank at date 2 are enough to repay the promised amount  $(1 - \lambda_\theta)c_2$  to the late consumers. Constraint (6) ensures that at date 0 the late consumers are offered a repayment  $c_2$  at least equal to  $c_1$ . Taken together, (5) and

(6) imply that the deposit contract is incentive compatible both at dates 0 and 1 so that no run occurs. Constraint (7) is consumers' participation constraint at date 0. It requires that  $E[u(c_1, c_2, \lambda_\theta)]$  is at least equal to the utility  $u(1)$  that consumers would obtain from storing. Finally, the last constraint is simply a non-negative requirement for reserves and consumption bundles.

We assume for the moment that depositors have a logarithmic utility function, that is  $u(c_t) = \ln(c_t)$  with  $t = 1, 2$ . This simplifies the analysis and allows us to obtain closed form solutions, without affecting our qualitative results. We consider alternative utility functions in Section 4. We have the following.

**Proposition 1** *There exists a unique (symmetric) no default equilibrium, in which each bank invests an amount*

$$R^{ND} = \lambda_H c_1^{ND} \quad (8)$$

*in reserves and  $L^{ND} = 1 - R^{ND}$  in loans, and it offers consumers a deposit contract*

$$c_1^{ND} = \left( \frac{\pi \lambda_L + (1 - \pi) \lambda_H}{\pi \lambda_L + (r - \pi) \lambda_H} \right)^{\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)} < 1 \quad (9)$$

*and*

$$c_2^{ND} = \left( \frac{\pi \lambda_L + (r - \pi) \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H} \right)^{\pi \lambda_L + (1 - \pi) \lambda_H} > 1. \quad (10)$$

The intuition behind Proposition 1 is simple. In the no default equilibrium all banks behave alike. Each bank holds an amount of reserves just enough to satisfy the highest liquidity demand  $\lambda_H c_1^{ND}$  by early consumers in state  $H$ . The loan rate and the deposit contract maximize the bank's expected profit while satisfying consumers' and entrepreneurs' participation constraint with equality. No bank defaults and depositors always receive the promised repayments

Substituting (9) and (10) into the expression for the bank's expected profit as in (2), we obtain

$$\Pi^{ND} = r - c_2^{ND}. \quad (11)$$

The bank's profit is simply equal to the difference between the return on the loans and the promised repayment  $c_2^{ND}$  to the late consumers. This means that the reserve holdings and the liquidity demand by the early consumers affect the bank's profits only to the extent that they affect  $c_2^{ND}$ .

Since all banks hold enough reserves to self-insure themselves against liquidity shocks and there is only aggregate uncertainty in the model, no loans are traded on the interbank market at date 1. Still, the equilibrium allocation must be supported by a vector of prices that satisfies the market clearing conditions. These require that the total demand for liquidity does not exceed the total supply of liquidity for any state  $\theta$ . Both demand and supply are inelastic at date 1. The demand for liquidity is determined by consumers' preferences. The supply is fixed by the bank's portfolio decisions at date 0. Shocks to the demand cause price volatility across states. Given (8) and  $\lambda_H > \lambda_L$ , there is an excess of liquidity in state  $L$  and date 1. Thus, it must hold that

$$P_L = r \tag{12}$$

for banks to be indifferent between buying loans and storing the excess liquidity between dates 1 and 2. With  $P_L < r$ , loans would dominate storage between dates 1 and 2, while  $P_L > r$  would imply the opposite.

The price  $P_H$  must ensure that banks are willing to hold both reserves and loans between dates 0 and 1. This means that  $P_H$  must satisfy

$$\pi \frac{r}{P_L} + (1 - \pi) \frac{r}{P_H} = r, \tag{13}$$

from which

$$P_H = \frac{P_L(1 - \pi)}{P_L - \pi}. \tag{14}$$

Given  $P_L > 1$ , this implies  $P_H < 1$ . Otherwise loans would dominate reserves at date 0. The equilibrium is thus characterized by price volatility as a consequence of the aggregate

uncertainty of the demand for liquidity and the inelasticity of supply at date 1.

### 3.2 The mixed equilibrium

So far we have considered the equilibrium where no banks default. However, avoiding default is costly as it requires banks to hold a large enough amount of reserves and forego the higher return on the loans. In this section, we characterize the candidate equilibrium when default is optimal.

A bank defaults when its late consumers run at date 1 and the price  $P_\theta$  drops enough to generate insolvency. In equilibrium not all banks can default simultaneously. If all banks made the same investments at date 0 and all defaulted at date 1, there would be no bank willing to buy the loans of the defaulting banks so that  $P_\theta = 0$ . This cannot be an equilibrium since it would be optimal for a bank to remain solvent and buy the loans at the price  $P_\theta = 0$ . This implies that an equilibrium with default must be mixed.

Despite being ex ante identical, banks must differ in terms of initial portfolio allocations and deposit contracts. A fraction  $\rho$  of banks, that we define as safe, invest enough in reserves at date 0 to remain solvent at date 1 in either state  $L$  or  $H$ . The remaining  $1 - \rho$  banks, defined as risky, invest so much in loans that they may not have enough resources to satisfy consumers' liquidity demands at date 1 and thus default. Given that in equilibrium all banks must have the same expected profits to be indifferent between being either of the two types, risky banks remain solvent in state  $L$  and default in state  $H$ . Consumers know the type of banks they deposit their endowment in. Safe and risky banks offer different deposit contracts so that depositors are indifferent between either type of banks.

We start by characterizing the problem for the safe banks. This is similar to the one in the no default equilibrium, with the difference that banks have now the possibility to buy loans on the interbank market at date 1. Given the market prices  $P_L$  and  $P_H$ , each safe bank chooses simultaneously the deposit contract  $(c_1^S, c_2^S)$ , the amount of reserves  $R^S$

and of loans  $L^S$  to solve the following problem:

$$\max_{c_1^S, c_2^S, R^S, L^S} \Pi^S = rL^S + \pi \left( \frac{R^S - \lambda_L c_1^S}{P_L} \right) r + (1 - \pi) \left( \frac{R^S - \lambda_H c_1^S}{P_H} \right) r - [\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)] c_2^S \quad (15)$$

subject to

$$R^S + L^S = 1 \quad (16)$$

$$\lambda_\theta c_1^S \leq R^S \quad (17)$$

$$(1 - \lambda_\theta) c_2^S \leq r \left( L^S + \frac{R^S - \lambda_\theta c_1^S}{P_\theta} \right) \quad (18)$$

$$c_2^S \geq c_1^S \quad (19)$$

$$E[u(c_1^S, c_2^S, \lambda_\theta)] = [\pi \lambda_L + (1 - \pi) \lambda_H] u(c_1^S) + [\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)] u(c_2^S) \geq u(1). \quad (20)$$

$$0 \leq R^S \leq 1, c_1^S \geq 0. \quad (21)$$

The bank's profit in (15) is given by the sum of the returns from the loans  $rL^S$  and from the expected excess of liquidity  $\pi \left( \frac{R^S - \lambda_L c_1^S}{P_L} \right) r$  and  $(1 - \pi) \left( \frac{R^S - \lambda_H c_1^S}{P_H} \right) r$  in states  $L$  and  $H$  minus the expected payments  $[\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)] c_2^S$  to late consumers at date 2. Safe banks use any excess liquidity at date 1 to acquire loans from the risky banks. With probability  $\pi$  the safe bank has  $R^S - \lambda_L c_1^S$  units of excess liquidity and buys  $\frac{R^S - \lambda_L c_1^S}{P_L}$  units of loans from the risky banks yielding a per-unit return of  $r$ . The same happens in state  $H$ . Constraints (16)-(18) are the resource constraints at dates 0 and 1. Their meaning is similar to that in the no default equilibrium. Constraint (19) requires the deposit contract to be incentive compatible at date 0. Finally, constraints (20) and (21) are the usual consumers' participation constraints and the non-negative requirement on reserves and consumption.

Risky banks make positive profits in state  $L$ , while they sell all loans and go bankrupt in state  $H$ . Anticipating this, each risky bank offers the deposit contract  $(c_1^R, c_2^R)$  and

chooses the amounts of reserves  $R^R$  and loans  $L^R$  to solve the following problem:

$$\max_{c_1^R, c_2^R, R^R, L^R} \Pi^R = \pi \left( rL^R - r \left( \frac{\lambda_L c_1^R - R^R}{P_L} \right) - (1 - \lambda_L) c_2^R \right) \quad (22)$$

subject to

$$R^R + L^R = 1$$

$$\lambda_L c_1^R \leq R^R + P_L L^R \quad (23)$$

$$(1 - \lambda_L) c_2^R \leq r \left( L^R - \frac{\lambda_L c_1^R - R^R}{P_L} \right) \quad (24)$$

$$c_2^R \geq c_1^R \quad (25)$$

$$E[u(c_1^R, c_2^R, \lambda_\theta)] = \pi[\lambda_L u(c_1^R) + (1 - \lambda_L)u(c_2^R)] + (1 - \pi)[u(R^R + P_H L^R)] \geq u(1) \quad (26)$$

$$0 \leq R^R \leq 1, c_1^R \geq 0.$$

Risky banks make positive profits only with probability  $\pi$  when state  $L$  occurs. These are equal to the returns from the initial investment in loans  $rL^R$  minus the foregone return  $r$  on the  $(\frac{\lambda_L c_1^R - R^R}{P_L})$  units of loans sold at date 1 to cover the shortage of liquidity  $\lambda_L c_1^R - R^R$  and the expected repayments  $(1 - \lambda_L)c_2^R$  to the late consumers. The first constraint is the usual budget constraint at date 0, which always binds. The second constraint is the resource constraint in state  $L$  at date 1. It states that the maximum amount  $R^R + P_L L^R$  of available resources from reserves and all liquidated loans is enough to satisfy the demands  $\lambda_L c_1^R$  by the early consumers. Constraint (24) ensures that at date 2 the bank has enough resources in state  $L$  to honor the promised repayments  $(1 - \lambda_L)c_2^R$  to the late consumers. These two constraints must hold with strict inequality in order for the risky banks to make positive profits in state  $L$ . As usual, the deposit contract has to be incentive compatible as indicated by (25), and it has to satisfy consumers' participation constraints (26). As the risky banks default in state  $H$ , consumers receive  $(c_1^R, c_2^R)$  only in state  $L$  and the proceeds  $R^R + P_H L^R$  of the bank's liquidated portfolio in state  $H$ . The last constraint is

the usual non-negative requirement.

As mentioned above, in equilibrium banks have to be indifferent between being safe or risky. This requires the expected profits of safe and risky banks to be the same, that is

$$\Pi^S = \Pi^R. \quad (27)$$

It remains to determine the prices  $P_L$ , and  $P_H$  and the fractions  $\rho$  and  $1 - \rho$  of safe and risky banks. The solutions to the banks' maximization problems must be consistent with the market clearing conditions determining  $P_L$  and  $P_H$ .

Consider first state  $L$ . Market clearing requires that at date 1 the demand for liquidity equals the supply of liquidity in aggregate. Thus, it must be the case that

$$(1 - \rho)(\lambda_L c_1^R - R^R) = \rho(R^S - \lambda_L c_1^S). \quad (28)$$

The left hand side represents the aggregate liquidity demand as given by the liquidity shortage  $\lambda_L c_1^R - R^R$  of each of the  $1 - \rho$  risky banks. The right hand side is the aggregate liquidity supply as determined by the excess liquidity  $R^S - \lambda_L c_1^S$  of each of the  $\rho$  safe banks. Condition (28) requires  $P_L \leq r$  so as to guarantee that the safe banks are willing to use their excess liquidity to purchase loans from the risky banks.

Now consider state  $H$ . The risky banks sell all their  $(1 - \rho)L^R$  loans at the price  $P_H$ , while the safe banks have  $\rho(R^S - \lambda_H c_1^S)$  excess of liquidity in total. Market clearing requires the supply and demand to be equal at the price  $P_H$ . Thus, it must be the case that

$$(1 - \rho)P_H L^R = \rho(R^S - \lambda_H c_1^S). \quad (29)$$

Conditions (28) and (29) imply that there is cash-in-the-market pricing in the model. The prices  $P_L$  and  $P_H$  vary endogenously across the two states and depend on the supply and demand of liquidity in the market.

The mixed equilibrium is characterized by the vector  $\{R^S, L^S, c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, P_L, P_H, \rho\}$ .

We have the following result.

**Proposition 2** *The mixed equilibrium is characterized as follows:*

1. *The safe banks invest an amount*

$$R^S = \lambda_H c_1^S + \frac{1-\rho}{\rho} P_H, \quad (30)$$

*in reserves and  $L^S = 1 - R^S$  in loans, and offer consumers a deposit contract  $(c_1^S, c_2^S)$  such that*

$$c_1^S = \left( \frac{P_L \pi \lambda_L + (1-\pi)\lambda_H}{r \pi \lambda_L + (P_L - \pi)\lambda_H} \right)^{\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)} < 1 \quad (31)$$

$$c_2^S = \left( \frac{r \pi \lambda_L + (P_L - \pi)\lambda_H}{P_L \pi \lambda_L + (1-\pi)\lambda_H} \right)^{\pi \lambda_L + (1-\pi)\lambda_H} > 1. \quad (32)$$

2. *The risky banks invest an amount*

$$R^R = 0$$

*in reserves and  $L^R = 1$  in loans, and offer consumers a deposit contract  $(c_1^R, c_2^R)$  such that*

$$c_1^R = \frac{P_L}{r} c_2^R \quad (33)$$

$$c_2^R = \frac{c_2^S - r(1-\pi)}{\pi} > 1. \quad (34)$$

3. *The price  $1 \leq P_L \leq r$  is the solution to (26), while  $P_H$  is still given by (14).*

4. *The fraction of safe banks is*

$$\rho = \frac{\lambda_L c_1^R - P_H}{\lambda_L c_1^R - P_H + (\lambda_H - \lambda_L) c_1^S} < 1. \quad (35)$$

The proposition shows that safe and risky banks behave quite differently. Each safe bank holds an amount  $\frac{1-\rho}{\rho} P_H$  of reserves in excess of the early liquidity demand  $\lambda_H c_1^S$  in state  $H$ , and uses it to purchase the loans  $(1-\rho)P_H$  sold by the risky banks. As in

the no default equilibrium, the safe banks offer  $c_2^S > 1 > c_1^S$  and always remain solvent. Both repayments depend now on the loan return as well as the market prices since the interbank market is active.

The risky banks do not hold any reserves and default at date 1 in state  $H$ . As default is anticipated and  $P_L > 1$ , they find it optimal to invest everything in loans at date 0. At date 1 in state  $L$  the risky banks sell  $\frac{\lambda_L c_1^R}{P_L}$  units of loans to satisfy the liquidity demand  $\lambda_L c_1^R$  of the early depositors but remain solvent. In state  $H$  they liquidate their entire portfolio and default. Depositors at the risky banks receive the promised repayments  $c_1^R$  and  $c_2^R$  in state  $L$  only. These repayments, together with the amount  $P_H$  that consumers receive in state  $H$ , have to satisfy their participation constraint.

The prices  $P_L$  and  $P_H$  satisfy the market clearing conditions in each state. As in the no default equilibrium, the level of  $P_H$  must ensure that the safe banks are willing to hold both reserves and loans between dates 0 and 1 and, again,  $P_L > 1 > P_H$  must hold. As before, the price volatility crucially depends on the aggregate uncertainty of the liquidity demand and the inelasticity of the supply at date 1. The difference is that in the mixed equilibrium the demand for liquidity is no longer driven entirely by consumers' preferences. In state  $H$ , when a run occurs, the total demand for liquidity is  $(1 - \rho)P_H$  as all consumers at the risky banks withdraw and receive the proceeds  $P_H$  of the liquidated portfolio. The larger demand for liquidity relative to state  $L$ , coupled with the inelasticity of the supply, drives down  $P_H$  to a level that is too low for the risky banks to remain solvent. This means that default occurs in the model as a consequence of the endogenous determination of the market prices.

Finally, the proportion  $\rho$  of safe banks is always positive and smaller than one given that  $\lambda_H > \lambda_L$ . Thus, the model generates partial default in that only a group of banks experience a run and go bankrupt. This is due to the endogenization of interbank market prices.

### 3.3 Existence of equilibria

Now that we have characterized the two equilibria of the model, we analyze the parameter space in which they exist. The key is whether behaving risky and defaulting in state  $H$  is optimal. This depends on the profitability of investing in loans relative to holding reserves, and thus on the parameter  $v$  representing the degree of competition in the credit market.

In the no default equilibrium, as characterized in Proposition 1, banks hold an amount of reserves that allows them to remain always solvent but at the cost of foregoing the higher return  $r = V - v$  on the loans. As  $v$  decreases, it may become optimal for a bank to behave risky in order to reduce its reserve holdings and appropriate the higher returns on the loans. Thus, the no default equilibrium exists only if banks do not have an incentive to deviate and choose a different portfolio allocation and deposit contract that result in default in state  $H$ .

The mixed equilibrium, as characterized in Proposition 2, exists if and only if neither safe banks nor risky banks prefer portfolio allocations and deposit contracts that are not consistent with the occurrence of default in state  $H$ . For default to be sustained as an equilibrium, safe banks must be willing to hold excess liquidity at date 0 and use it to buy loans in the interbank market at date 1. This requires that the prices are admissible, i.e., the price  $P_L$  must lie in the interval

$$1 < P_L \leq r. \tag{36}$$

The lower bound is consistent with  $P_H < 1$ , while the upper bound ensures that the safe banks are willing to buy loans at date 1. From (14), the price  $P_H$  is always admissible as it adjusts with  $P_L$  so as to guarantee that safe banks hold reserves at date 0. Thus, only (36) matters for the existence of the mixed equilibrium.

To prove the existence of the two equilibria, we start by analyzing when –i.e., for which level of competition as represented by  $v$ – a bank has an incentive to deviate from the no default equilibrium and choose  $\{c_1, c_2, R, L\}$  so that it defaults in state  $H$ . Then,

we show that the mixed equilibrium starts to exist at the same critical level of  $v$ , denoted as  $v^*$ , at which deviating from the no default equilibrium becomes optimal. This is due to the fact that the bank deviating from the no default equilibrium behaves exactly as a risky bank in the mixed equilibrium when market prices are given by  $P_L = r$  and  $P_H$  is as in (14). We have the following result.

**Proposition 3** *If the probability  $\pi$  of state  $L$  is greater than some cutoff value  $\underline{\pi}$ , there exists a degree of credit market competition  $v^* \in (0, \bar{v})$  such that the mixed equilibrium exists for any  $v \leq v^*$  and the no default equilibrium exists for any  $v \geq v^*$ .*

The proposition shows that the no default equilibrium exists when competition is intense, while the mixed equilibrium exists when competition is less intense. The two equilibria coexist at  $v = v^*$ . This result implies a positive relationship between competition and financial stability. The result resembles that in Boyd and De Nicoló (2005) that a stronger competition in the credit market as represented by lower loan rates leads to a more stable financial system. However, the underlying mechanism is quite different. In Boyd and De Nicoló lower loan rates induce entrepreneurs' to take less risk thus reducing the risk of banks' portfolio. In contrast, in our paper the level of loan rates affects banks' portfolio decision and thus their resiliency to liquidity shocks.

The intuition for the result in the proposition is as follows. The mixed equilibrium is sustainable as long as the safe banks are willing to acquire the loans from the risky banks on the interbank market at date 1, as required by (36). Furthermore, loans have to be profitable enough for the risky banks to be able to make the same expected profits as the safe banks. The profitability of loans is twofold. On the one hand, the loan rate  $r$  determines the loan return to the bank at date 2. On the other hand, interbank market prices determine the returns to the safe banks from acquiring loans at date 1 and the cost to the risky banks from having to sell loans at date 1. The price  $P_\theta$  is a function of the loan rate  $r$ . In particular,  $P_L$  increases with  $r$  as it reflects the fundamental value of loans but less than proportionally as  $r$  grows. In contrast,  $P_H$  decreases with  $P_L$  and thus  $r$  so

that banks are indifferent at date 0 between investing in reserves and loans as required by condition (13). As long as competition is low enough, loans are sufficiently profitable both in terms of date 2 return and price on the interbank market to sustain the mixed equilibrium. As  $v$  reaches  $v^*$  the return on loans is just enough for the risky banks to be indifferent between maintaining no reserves and defaulting in state  $H$  and behave like a safe bank. As  $v > v^*$  being a risky bank is no longer profitable and the only possible equilibrium is the no default equilibrium, where all banks behave safe and the interbank prices satisfy (12) and (13).

### 3.4 Comparative statics with respect to $\pi$

Now that we have characterized the existence of the two equilibria of the model in terms of the parameter  $v$ , we analyze how the structure of the liquidity shock, as represented by the parameter  $\pi$ , affects our results. In particular, we focus on the effect of  $\pi$  on the critical threshold  $v^*$ , the number of non defaulting banks  $\rho$  and credit availability at the threshold  $v^*$ .

We start with the following result.

**Proposition 4** *The threshold  $v^*$  increases with  $\pi$  (i.e.,  $\frac{dv^*}{d\pi} > 0$ ).*

The proposition states that the range of  $v$  in which the mixed equilibrium exists becomes larger as  $\pi$  increases. The reason is that as the good state becomes more likely, behaving risky becomes more profitable for any given  $v$ . This has also a direct implication for the fraction of safe and risky banks in the mixed equilibrium, as the following result illustrates.

**Proposition 5** *The number of safe banks  $\rho^*$  at  $v = v^*$  decreases with  $\pi$  (i.e.,  $\frac{d\rho^*}{d\pi} < 0$ ).*

Since a higher  $\pi$  increases banks' incentives to behave risky, it leads to a higher fraction of risky banks in the economy at any given  $v^*$ . Taken together, these two propositions suggest that as the probability of the good state increases, crises are less likely to occur

but emerge for a wider range of credit market competition and are more severe in terms of number of defaulting banks. In this sense, the structure of the liquidity shock plays an important role in shaping the relationship between competition and stability.

The probability of the state  $L$  occurring also affects credit availability across the two equilibria. To see this, we focus on the level of competition  $v = v^*$  where the no default and the mixed equilibria coexist. At  $v^*$  all banks have the same expected profits ( $\Pi^{ND} = \Pi^S = \Pi^R$ ) and market clearing requires  $P_L = r$  and  $P_H$  as in (14). This implies that all non-defaulting banks –i.e., those in the no default equilibrium and the safe ones in the mixed equilibrium– offer the same deposit contract ( $c_1^{ND} = c_1^S = c_1^{nd}$  and  $c_2^{ND} = c_2^S = c_2^{nd}$ ) while the defaulting banks behave alike either in the mixed equilibrium or deviating from the no default equilibrium ( $c_1^R = c_2^R = c_1^d$ ). With this in mind and after substituting (8) and (30), at  $v = v^*$  the loan supply  $TL$  is then simply given by

$$TL^{ND}|_{v=v^*} = 1 - R^{ND} = 1 - \lambda_H c_1^{nd} \quad (37)$$

in the no default equilibrium and by

$$\begin{aligned} TL^M|_{v=v^*} &= (1 - \rho) + \rho(1 - R^S) \\ &= (1 - \rho)(1 - P_H) + \rho(1 - \lambda_H c_1^{nd}) \end{aligned} \quad (38)$$

in the mixed equilibrium. Then it follows:

**Lemma 1** *At  $v = v^*$ , credit availability is lower in the mixed equilibrium than in the no default equilibrium if*

$$P_H > \lambda_H c_1^{nd}, \quad (39)$$

*and it is higher otherwise.*

The lemma states that the impact of default on credit availability depends on the comparison between the repayment  $P_H$  accruing to all consumers at a risky bank and that

to the early consumers at a non-defaulting bank in state  $H$ . The reason is that the supply of loans at date 0 depends on the amount of liquidity needed at date 1 to repay the early consumers which, in turn, must equal the total reserves that banks hold in aggregate. Since all non-defaulting banks offer the same consumption to their early consumers at  $v = v^*$ , the difference in the loan supply between the no default and the mixed equilibrium depends only on the difference between  $\lambda_H c_1^{nd}$  and  $P_H$ . When  $P_H > \lambda_H c_1^{nd}$ , the system needs more reserves in the mixed equilibrium than in the no default equilibrium to repay all early withdrawing consumers in state  $H$ , thus implying a lower investment in loans in aggregate at date 0.

The sign of inequality (39) depends on the condition (27) that the expected profit of safe and risky banks must be the same in the mixed equilibrium. Rearranging the expressions for  $\Pi^S$  and  $\Pi^R$  as in (15) and (22) after substituting  $R^S$  as in (30) and  $P_L = r$  at  $v = v^*$  gives

$$(1 - \pi)\left(1 - \frac{\lambda_H c_1^{nd}}{P_H}\right)r = \pi\lambda_L c_1^{nd} + [\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)]c_2^{nd} - \pi c_2^d. \quad (40)$$

The left hand side can be interpreted as the difference in the loan returns between a risky and a safe bank in state  $H$ . With probability  $1 - \pi$  the bad state occurs and the risky bank loses the return  $r$  on the loans while the safe bank loses  $r$  on the  $\frac{\lambda_H c_1^{nd}}{P_H}$  units of loans that it holds to meet the commitments to its early consumers. The right hand side represents the difference in the repayments to consumers between a safe and a risky bank other than those at date 1 in state  $H$ . The first two terms are the expected repayments of a safe bank to the early consumers in state  $L$  and to the late types in both states. The last term is the expected repayment of a risky bank to early and late consumers in the good state given that  $c_1^R = c_2^R = c^d$  at  $v = v^*$ . For (40) to hold, if a risky bank suffers a net loss in terms of loan returns relative to a safe bank (i.e., if the LHS is positive), it must benefit in terms of consumers' repayments (i.e., RHS must be positive). It follows that the difference  $P_H - \lambda_H c_1^{nd}$  is positive if the risky bank has a cost advantage relative

to the safe bank and it is negative if, instead, the risky bank has higher net returns on loans.

The probability  $\pi$  of the good state affects all terms in the expression (40) and thus the sign of the difference  $P_H - \lambda_H c_1^{nd}$ , as illustrated in the following proposition.

**Proposition 6** *Define  $\bar{\pi}$  as the cutoff value of the probability of state  $L$  such that  $P_H - \lambda_H c_1^{nd} = 0$  at  $v^* \rightarrow 0$ . Then, if the difference  $P_H - \lambda_H c_1^{nd}$  is decreasing in  $\pi$  (i.e.,  $\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} < 0$ ), there exists a value  $\hat{\pi} \in (\bar{\pi}, 1)$  such that at  $v = v^*$  the mixed equilibrium leads to lower credit availability than the no default equilibrium for  $\pi < \hat{\pi}$ , and to higher credit availability otherwise.*

Insert Figure 1

The result of the proposition is illustrated in Figure 1, which plots the amount of loans granted in the no default and mixed equilibrium as a function of  $v$  for different levels of  $\pi$ . As the figure shows, credit availability is lower in the mixed equilibrium than in the no default equilibrium at  $v = v^*$  when the good state is not very likely (i.e., for  $\pi_0 < \hat{\pi}$ ) because in the former more reserves are needed in aggregate to satisfy the higher consumers' repayments in state  $H$ . By contrast, when the good state is more likely (i.e., for  $\pi_1 > \hat{\pi}$ ) credit availability is higher in the mixed equilibrium where default occurs. The basic intuition behind this result is that the possibility for banks to obtain liquidity in the interbank market and to default in state  $H$  introduces some contingency in the model. Whereas all non defaulting banks always honour their promised non-contingent repayments to consumers, risky banks distribute the proceeds  $P_H$  of their liquidated portfolios to all their consumers in state  $H$ . This makes the total demand for liquidity in state  $H$  elastic since it depends on the market price  $P_H$  and, for  $\pi$  high enough, it allows the economy to save reserves and grant more loans.

The result in Proposition 6 requires that the difference  $P_H - \lambda_H c_1^{nd}$  is decreasing in  $\pi$  to ensure the uniqueness of the cutoff values  $\hat{\pi}$ . Unfortunately, it is not easy to prove the

monotonicity analytically. The consumption  $c_1^{nd}$  increases with  $\pi$ , but  $P_H$  is not monotonic in  $\pi$ . To see this, consider the derivative of  $P_H$  with respect to  $\pi$  as given by

$$\frac{dP_H}{d\pi} = -\frac{r(r-1)}{(r-\pi)^2} + \frac{\pi(1-\pi)}{(r-\pi)^2} \frac{dv^*}{d\pi}.$$

The first term represents the (negative) direct effect of a change in  $\pi$ , while the second term is the indirect one through a change in  $v^*$ . Since  $\frac{dv^*}{d\pi} > 0$ , the second term is positive and, depending on the value of  $\pi$ , may dominate so that  $\frac{dP_H}{d\pi}$  is not monotonic in  $\pi$ . However, even when this is the case, as long as the indirect effect is small enough, the monotonicity of  $P_H - \lambda_H c_1^{nd}$  is guaranteed.

## 4 Extensions

In this section we consider various extensions of our basic model. First, we consider alternative consumers' utility functions to analyze the effects of risk aversion on our results. Second, we study the case when deposits are fully insured. Lastly, we extend the analysis to the case where there is competition also on the deposit side. For sake of brevity, we only focus on analyzing whether the main results of the basic model still hold without characterizing the two equilibria of the model in detail again. This means also that we will refer to a defaulting bank interchangeably as a bank deviating from the no default equilibrium and as a risky bank in the mixed equilibrium for  $v = v^*$ .

### 4.1 Risk aversion

So far we have assumed that depositors have a logarithmic utility function. This implies a constant relative risk aversion equal to one. We now assume that depositors have a constant relative risk aversion utility function (CARA) given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where  $\sigma \geq 1$  represents the degree of risk aversion. All the rest of the model remains the same. We have the following result.

**Proposition 7** *There exists a value  $\bar{\sigma}$  of risk aversion such that the defaulting banks hold positive reserves for  $\sigma \geq \bar{\sigma}$  and zero otherwise.*

The intuition behind the proposition is simple. A higher relative risk aversion implies that depositors require more liquidity insurance and thus a larger promised repayment from their banks at the intermediate date. This holds for both defaulting and non-defaulting banks. The former still satisfy depositors' increased early withdrawal demands by holding enough reserves. The latter sell part of their loans in state  $L$  and default in state  $H$ . However, as  $\sigma$  increases beyond  $\bar{\sigma}$ , depositors' early demands become so large –and thus  $P_H$  so low– that also the defaulting banks find it convenient to start holding a positive amount of reserves. This allows them to pay more to withdrawing depositors in state  $H$  and thus to reduce the promised repayment in state  $L$ .

The main insight of Proposition 3 still hold even when defaulting banks hold positive reserves. In particular, there still exists a critical value  $v_\sigma^* \in (0, \bar{v})$  such that the mixed equilibrium exists for any  $v \leq v_\sigma^*$  and the no default equilibrium exists for any  $v \geq v_\sigma^*$ . For brevity, we show this with a numerical example. We first compute the threshold  $v_\sigma^*$  and we then look at banks' expected profits at  $v = v_\sigma^* - \varepsilon$  and  $v = v_\sigma^* + \varepsilon$ . For the set of parameters  $\pi = 0.8$ ,  $\lambda_L = 0.8$ ,  $\lambda_H = 0.81$ ,  $V = 6$ ,  $\sigma = 3.9$  and  $\varepsilon = 0.1$ , we have:

	$v_\sigma^*$	$\Pi_{(v=v_\sigma^*)}$	$R_{(v=v_\sigma^*)}$	$c_{1(v=v_\sigma^*)}$	$c_{2(v=v_\sigma^*)}$	$\Pi_{(v=v_\sigma^*-\varepsilon)}$	$\Pi_{(v=v_\sigma^*+\varepsilon)}$
No default	0.784	0.602	0.827	1.02	1.56	0.62	0.585
Mixed	0.784	0.602	0.457	2.537	2.537	0.632	0.572

Table 1: The equilibrium for  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$  and  $\sigma > 1$ .

As the table shows, at  $v = v_\sigma^* = 0.784$ , non-defaulting banks hold more reserves and offer a more volatile deposit contract than defaulting banks but all of them have the same expected profits equal to 0.602. At  $v = v_\sigma^* - \varepsilon$  defaulting banks have higher expected profits, suggesting that the mixed equilibrium exists. The opposite happens at  $v = v_\sigma^* + \varepsilon$

where then only the no default equilibrium exists. This confirms that the main result of Proposition 3 remains valid when defaulting banks hold a positive amount of reserves.

## 4.2 Deposit Insurance

So far we have assumed that deposits are not insured. This implies that when their bank defaults in the bad state, depositors receive the liquidation value  $P_H$ . We now consider the case where deposits are insured. This means that the government now guarantees depositors of a risky bank to receive  $c^I > P_H$  in state  $H$ . The insurance scheme is assumed to be fairly priced. The government transfers an amount  $t = c^I - P_H$  to the depositors of risky banks in state  $H$  in exchange for the payment of a premium  $C$  equal to

$$C = (1 - \pi)t = (1 - \pi)(c^I - P_H), \quad (41)$$

which reflects the fact that risky banks default only with probability  $(1 - \pi)$ . For simplicity, we consider that the premium is paid ex post, but the same result holds if it is instead paid ex ante.

The effect of deposit insurance on the threshold  $v^*$  is not clear-cut. Safe banks are not affected by the presence of deposit insurance as they offer safe deposit contracts. By contrast, risky banks pay now a higher repayment  $c^I > P_H$  to depositors in state  $H$  and can thus reduce the promised repayments  $c_{1di}^d = c_{2di}^d = c_{di}^d$  to early and late depositors in state  $L$ . Whether this increases banks' incentives to behave risky depends on whether the advantage in terms of lower promised repayments to depositors in the good state dominates the premium  $C$  required for the insurance. We have then the following result.

**Proposition 8** *In the case of fairly-priced deposit insurance, the threshold  $v_{di}^*$  is smaller than that without deposit insurance (i.e.,  $v_{di}^* < v^*$ ) if*

$$(1 - \pi)(c^I - P_H) > c^d - c_{di}^d, \quad (42)$$

and it is greater otherwise.

The proposition simply states that the introduction of fairly-priced deposit insurance reduces (enlarges) the range of parameter for which the mixed equilibrium exists if the premium  $(1 - \pi)(c^I - P_H)$  required for the provision of the insurance is larger (smaller) than the reduction  $c^d - c_{di}^d$  in the promised repayments to consumers relative to the case without insurance. This is the case for example when  $\pi$  is sufficiently small since (42) is more likely to hold as  $\pi$  decreases. The result holds for any guaranteed repayment  $c^I > P_H$ .

### 4.3 Competition in the deposit market

So far we have considered that consumers can either deposit in the bank or store for a return of 1. In this section, we extend the analysis to the case where their opportunity cost is  $\alpha \in (1, \bar{\alpha})$  so that consumers' participation constraint is now given by

$$E[u(c_1, c_2, \lambda_\theta)] \geq u(\alpha) > 0. \quad (43)$$

The higher  $\alpha$  the more banks will have to promise to depositors in terms of higher  $c_1$  and  $c_2$ . In this sense, the parameter  $\alpha$  can be interpreted as representing the degree of competition in the deposit market, similarly to the parameter  $v$  in the credit market.

The introduction of  $\alpha > 1$  affects the split of surplus between banks and depositors and thus the incentives for banks to behave risky. The main insight of Proposition 3 remains valid, although, as illustrated in the following result, the size of the threshold  $v_\alpha^*$  changes relative to the basic model.

**Proposition 9** *There exists a value  $\alpha^* \in (1, \bar{\alpha})$  such that the threshold  $v_\alpha^*$  increases in  $\alpha$  (i.e.,  $\frac{\partial v_\alpha^*}{\partial \alpha} > 0$ ) for  $\alpha \leq \alpha^*$ , and it decreases in  $\alpha$  (i.e.,  $\frac{\partial v_\alpha^*}{\partial \alpha} < 0$ ) otherwise.*

The proposition shows that there is a non-monotonic relationship between the degree of competition in the deposit market  $\alpha$  and financial stability. The range of parameters

$v \leq v_\alpha^*$  where the mixed equilibrium exists is larger when competition on deposits is not intense ( $\alpha \leq \alpha^*$ ) and it is smaller otherwise. The result relies on the different effects that a greater  $\alpha$  has on the promised repayments to depositors offered by the different types of banks. As  $\alpha$  increases, all banks need to increase the promised repayment to consumers to satisfy their participation constraints. However,  $\alpha$  also affects the compensation that a risky bank needs to pay to depositors in state  $L$  for the fact that it defaults in state  $H$  where all depositors receive  $P_H$ . Such a compensation is small when  $\alpha$  is low, and it becomes larger as  $\alpha$  increases. As a consequence, when competition in the deposit market is intense enough (i.e., for  $\alpha \geq \alpha^*$ ), loans have to be more profitable to induce banks to behave risky. The "compensation" effect introduced with the competition on the deposit side is reminiscent of the margin effect in Martinez-Miera and Repullo (2010), since, as shown in Figure 2, it introduces a non-monotonicity in the relationship between  $\alpha$  and  $v_\alpha^*$ .

Insert Figure 2

## 5 Empirical predictions

The model has several important implications in terms of the relationship between competition and stability. The first insight is that credit market competition makes the banking system more stable. When credit markets are competitive, banks are less profitable and have more incentives to behave prudently. Each bank holds enough reserves to insure itself against the risk of large liquidity shocks. This result is consistent with the findings in Berger and Bouwman (2009) that banks enjoying greater market power as a result of a process of mergers and acquisitions are more likely to hold fewer reserves and grant more loans; and with the finding in Petersen and Rajan (1995) that banks grant more loans as competition becomes less intense.

The second insight of the model is that the cutoff value of credit market competition at which default emerges in equilibrium crucially depends on the structure of the liquidity

shocks. A low probability of experiencing large liquidity shocks increases the opportunity cost of holding reserves and thus induces banks to behave imprudently. A first implication is that crises start to emerge in more competitive credit markets and, although they are less likely to occur, they are more severe as they involve a larger number of defaulting banks. Another implication is that economies with similar levels of credit market competition may differ in terms of stability depending on the structure of liquidity shocks. The result may provide an important explanation for the mixed empirical evidence on the relationship between competition and stability.

The model also delivers some implications concerning the relationship between competition, financial stability and credit availability. When large liquidity shocks are unlikely, the instability of the banking system is more likely to be associated with greater credit availability. In contrast, when large liquidity shocks are more likely to occur, stable banking systems are able to provide more credit than unstable ones. These results are consistent with the finding in Acharya, Shin and Yorulmazer (2011) that bank liquidity is countercyclical as banks economize on reserves when the fundamentals of the economy are more positive. Extending our reasoning to a more dynamic framework, our results are also consistent with the observation, as reported in Castiglionesi, Feriozzi and Lorenzoni (2010), that market liquidity has decreased over time during the boom phase preceding the recent crisis.

The main prediction of the basic model that more competition in the credit market enhances financial stability remains valid irrespective of the degree of consumers' risk aversion, the presence of fairly-priced deposit insurance and of more competition in the deposit market. However, the critical level of credit market competition above which the banking system is stable decreases with the size of the deposit insurance premia and with intense deposit market competition. Thus, the model predicts that crises emerge only in more monopolistic credit markets when banks operate in risky environment and thus have to pay large fairly-priced deposit insurance premia or are subject to intense competition in the deposit market.

## 6 Concluding remarks

In this paper we have developed a model where banks face liquidity shocks and can invest in liquid reserves and safe loans. The latter can be sold on an interbank market at a price that depends on the demand and supply of liquidity. We have shown that two types of equilibria exist, depending on the degree of credit market competition. In the no default equilibrium, all banks are self sufficient as they hold enough reserves to withstand any liquidity shock they may face. In the mixed equilibrium, banks behave asymmetrically. Some banks, defined as risky, do not hold reserves and raise liquidity when needed by selling loans on the interbank market. In the good state with low liquidity shocks, they are able to satisfy their liquidity demands and remain solvent. In the bad state where liquidity shocks are large, they sell all their loans and default. Thus, in the mixed equilibrium crises are observed with positive probability. The no default equilibrium exists when competition in the credit market is intense, while the mixed equilibrium exists in more monopolistic credit markets. This implies that competition is beneficial to financial stability. This basic result is robust to a number of extensions of the basic model although the critical value of credit market competition defining the existence of either type of equilibrium varies with the structure of liquidity shocks, consumers' risk aversion, the presence of deposit insurance and the intensity of deposit market competition.

The model can be extended further. For example, the analysis assumes that in the mixed equilibrium consumers can observe the type of bank they deposit at so that safe and risky banks offer different deposit contracts. If this assumption was removed, there would be a pooling in deposit contracts and consumers would be promised the same deposit terms irrespective of their bank's type. This would lead to ex post differences among consumers as those at the safe banks would enjoy a positive rent while those at the risky banks would suffer a loss. This would in turn lower the desirability of default, thus reducing the range of parameters in which the mixed equilibrium exists.

The assumption that banks' type is observable to consumers guarantees also that the

safe banks remain solvent when the risky banks default. Removing this assumption would lead to the possibility of contagion across types of banks, in particular when the number of risky banks in the economy is large. This may induce safe banks to hold greater reserves initially and develop strategies to signal their types to depositors. Modelling an economy with unobservable types of banks would constitute an interesting future research topic.

We have also assumed that bank default is costless. Introducing bankruptcy costs that reduce what depositors obtain in the case their bank defaults would increase the cost of default for the risky banks and would therefore again reduce the range of parameters where the mixed equilibrium exists.

A final remark regards the way we have modeled competition. Both in the credit market and in the deposit market, we consider that the degree of competition determines only the split of the surplus generated by investing in loans between banks and entrepreneurs on the one hand, and between banks and depositors on the other. This reduced form is convenient as it isolates the effect of competition on banks' profitability, similarly to the charter value literature, while maintaining the demand for loans and deposits inelastic and thus banks' size constant. An interesting alternative specification would be to consider a more general framework where competition affects also the demand for loans and deposits. This would generate additional trade-offs between bank size, liquidity and financial stability. We leave this for future research.

## Appendix

**Proof of Proposition 1:** The bank's maximization problem is convex and, given the concavity of the profit function, it has a unique solution. In order to avoid default, each bank keeps enough reserves to cover its demand for liquidity at date 1 in either state. Given  $\lambda_H > \lambda_L$ , in equilibrium (8) must hold. This implies that (4) is satisfied with equality in state  $H$  and with strict inequality in state  $L$ . The only other binding constraint in equilibrium is the consumers' participation constraint as given by (7). Solving it with respect to  $c_2$ , we obtain

$$c_2^{ND} = (c_1^{ND})^{-\left[\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)}\right]}. \quad (44)$$

Substituting (44) and (8) into (2) gives

$$\Pi_i = r(1-\lambda_H c_1^{ND}) + \pi(\lambda_H - \lambda_L) c_1^{ND} - [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)] (c_1^{ND})^{-\left[\frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)}\right]}.$$

Differentiating this with respect to  $c_1$ , we obtain

$$-r\lambda_H + \pi(\lambda_H - \lambda_L) + [\pi\lambda_L + (1-\pi)\lambda_H] (c_1^{ND})^{-\frac{1}{\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)}} = 0,$$

from which we obtain  $c_1^{ND}$  as (9) in the proposition. Substituting this into (44) gives  $c_2^{ND}$  as in (10). The proposition follows.  $\square$

**Proof of Proposition 2:** We derive the vector  $\{R^S, L^S, c_1^S, c_2^S, R^R, L^R, c_1^R, c_2^R, \rho, P_L, P_H\}$  characterizing the mixed equilibrium as the solution to the maximization problem of the safe and risky banks, the market clearing conditions and the equality between the expected profit of risky and safe banks. The only binding constraints in the banks' maximization problems are the consumers' participation constraints given by (20) and (26). It can be shown that all other constraints representing the resources constraints at dates 1 and 2 are satisfied with strict inequality.

Given this, the problem of the safe banks using the Lagrangian can be written as

$$S = \Pi^S - \mu^S \left[ [\pi\lambda_L + (1-\pi)\lambda_H] \ln(c_1^S) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)] \ln(c_2^S) \right].$$

The first order conditions with respect to  $R^S, c_1^S, c_2^S$  and  $\mu^S$  are as follow:

$$\frac{\pi}{P_L} + \frac{1-\pi}{P_H} = 1, \quad (45)$$

$$\left[ \frac{\pi\lambda_L}{P_L} + \frac{(1-\pi)\lambda_H}{P_H} \right] r = -\frac{\mu^S}{c_1^S} [\pi\lambda_L + (1-\pi)\lambda_H] \quad (46)$$

$$c_2^S = -\mu^S \quad (47)$$

$$[\pi\lambda_L + (1-\pi)\lambda_H] \ln(c_1^S) + [\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)] \ln(c_2^S) = 0. \quad (48)$$

For the risky banks, the problem becomes

$$R = \Pi^R - \mu^R \left[ \pi[\lambda_L \ln(c_1^R) + (1-\lambda_L) \ln(c_2^R)] + (1-\pi)[\ln(R^R + P_H(1-R^R))] \right].$$

The first order conditions with respect to  $R^R$ ,  $c_1^R$ ,  $c_2^R$  and  $\mu^R$  are:

$$-\pi r + \frac{\pi r}{P_L} = \frac{\mu^R(1-\pi)(1-P_H)}{R^R + P_H(1-R^R)} \quad (49)$$

$$\frac{r}{P_L} = -\frac{\mu^R}{c_1^R} \quad (50)$$

$$c_2^R = -\mu^R \quad (51)$$

$$\pi[\lambda_L \ln(c_1^R) + (1-\lambda_L) \ln(c_2^R)] + (1-\pi)[\ln(R^R + P_H(1-R^R))] = 0. \quad (52)$$

The equilibrium is the solution to the system of all the first order conditions (45)-(48) and (49)-(52) together with (27), (28) and (29).

We solve the system by first using (45) to derive  $P_H$  as in (14). Substituting (14) into (46), (47) and (48) gives  $c_1^S$  and  $c_2^S$  as in (31) and (32).

Using (50) and (51), we obtain  $c_1^R$  as in (33) and  $R^R$  from (49) as

$$R^R = \frac{(1-\pi)c_1^R}{\pi(P_L-1)} - \frac{P_H}{1-P_H}. \quad (53)$$

Substituting  $P_H$  from (14) into (53) and rearranging it gives

$$R^R = \frac{(1-\pi)}{\pi(P_L-1)}(c_1^R - P_L) \leq 0$$

for any  $P_L - 1 > 1$  and  $c_1^R - P_L \leq 0$ . The former follows from (14), as otherwise  $P_H > 1 > P_L$ , which cannot hold given the excess of liquidity in state  $L$ . The latter follows from the fact that the profits of the risky banks must be non-negative in equilibrium. To see this, we rewrite (22) as follows:

$$\Pi^R = \pi \left[ r + \left( \frac{1}{P_L} - 1 \right) R^R r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L} r \right].$$

As  $\left( \frac{1}{P_L} - 1 \right) R^R r < 0$  for  $P_L > 1$ ,  $\Pi^R \geq 0$  requires

$$r - (1-\lambda_L)c_2^R - \frac{\lambda_L c_1^R}{P_L} r > 0.$$

Rewriting  $r$  as  $\lambda_L r + (1 - \lambda_L)r$  and rearranging the terms gives

$$(1 - \lambda_L)(r - c_2^R) + \lambda_L r \left( \frac{P_L - c_1^R}{P_L} \right). \quad (54)$$

This is positive if  $P_L - c_1^R > 0$  as this implies also that  $r - c_2^R > 0$ . Consider  $P_L - c_1^R < 0$ . Then, from  $c_2^R = \frac{r}{P_L} c_1^R$ , it is  $c_2^R > r$  and (54) is negative. Then, in equilibrium  $P_L - c_1^R > 0$  must hold. It follows that  $R^R = 0$  as in the proposition. Then, we use (29) to derive  $R^S$  as in (30).

To find  $c_2^R$  as in the proposition we first rearrange  $\Pi^S = \Pi^R$  from (27) as

$$R^S \left[ -1 + \frac{\pi}{P_L} + \frac{1 - \pi}{P_H} \right] r - \left[ \frac{\pi \lambda_L}{P_L} + \frac{(1 - \pi) \lambda_H}{P_H} \right] r c_1^S + r(1 - \pi) + \pi c_2^R - [\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)] c_2^S = 0.$$

From (45) it is  $[-1 + \frac{\pi}{P_L} + \frac{1 - \pi}{P_H}] = 0$ . From (46) it holds

$$\left[ \frac{\pi \lambda_L}{P_L} + \frac{(1 - \pi) \lambda_H}{P_H} \right] r c_1^S = [\pi \lambda_L + (1 - \pi) \lambda_H] c_2^S.$$

Substituting these into the expression above for  $\Pi^S - \Pi^R$ , we have  $c_2^R$  as in (34).

Finally, from (26), (28) and (30), we have  $P_L$  and  $\rho$  as in (35). The proposition follows.

□

**Proof of Proposition 3:** We divide the proof in two parts. First we prove that the no default equilibrium exists only in the range  $v \geq v^*$ . Second, we prove that the mixed equilibrium exists only when  $v \leq v^*$ .

The no default equilibrium exists as long as it is not profitable for a bank to deviate and choose a portfolio allocation resulting in default in state  $H$ . The maximization problem of the deviating bank, denoted with the superscript  $d$ , is the same as the one of a risky bank in (22)-(25) with  $P_L$  and  $P_H$  as in (12) and (14). Using the first order conditions with respect to  $R^d$ ,  $c_1^d$ ,  $c_2^d$  and  $\mu^d$ , which are as in (49), (50), (51) and (52), we obtain:

$$R^d = 0 \quad (55)$$

$$c_1^d = c_2^d = c^d = (P_H)^{-\frac{(1-\pi)}{\pi}} = \left( \frac{r(1-\pi)}{r-\pi} \right)^{-\frac{(1-\pi)}{\pi}}, \quad (56)$$

where we have used  $P_L = r$ . Substituting (55) and (56) into (22) gives

$$\Pi^d = \pi(r - c^d). \quad (57)$$

Deviating is profitable if and only if  $\Pi^d \geq \Pi^{ND}$ , where  $\Pi^{ND}$  is as in (11). Define  $f(\pi, v) = \Pi^{ND} - \Pi^d$ . When  $v \rightarrow \bar{v}$  and  $r \rightarrow 1$ ,  $f(\pi, v) \rightarrow 0$  since from (9), (10) and (56),  $c_1^{ND} = c_2^{ND} = c^d \rightarrow 1$ . Differentiating  $f(\pi, v)$  with respect to  $v$  gives

$$\frac{\partial f(\pi, v)}{\partial v} = \frac{\partial \Pi^{ND}}{\partial v} - \frac{\partial \Pi^d}{\partial v}$$

where

$$\frac{\partial \Pi^{ND}}{\partial v} = -(1 - \lambda_H c_1^{ND}) < 0$$

$$\frac{\partial \Pi^d}{\partial v} = -\pi \left( 1 - \frac{(1 - \pi)c^d}{(r - \pi)r} \right) < 0.$$

The profits  $\Pi^{ND}$  and  $\Pi^d$  are monotonically decreasing in  $v$ . For  $v \rightarrow \bar{v}$ ,  $\frac{\partial f(\pi, v)}{\partial v} \rightarrow -(1 - \lambda_H) < 0$ . Thus, there exists a unique threshold  $v^* \in (0, \bar{v})$  such that  $f(\pi, v) = \Pi^{ND} - \Pi^d = 0$  if and only if  $f(\pi, v) \leq 0$  for  $v \rightarrow 0$ . A sufficient condition for this is that  $\pi$  is sufficiently high. To see this, we first show that  $f(\pi, v)$  is monotonically decreasing in  $\pi$ . Differentiating  $f(\pi, v)$  with respect to  $\pi$  gives

$$\frac{\partial f(\pi, v)}{\partial \pi} = -(r - c^d) - \frac{\partial c_2^{ND}}{\partial \pi} + \pi \frac{\partial c^d}{\partial \pi}, \quad (58)$$

where

$$\begin{aligned} \frac{\partial c_2^{ND}}{\partial \pi} &= (\lambda_H - \lambda_L) c_2^{ND} \left[ \ln \left( \frac{\pi \lambda_L + (1 - \pi) \lambda_H}{\pi \lambda_L + (r - \pi) \lambda_H} \right) + \frac{\lambda_H (r - 1)}{\pi \lambda_L + (r - \pi) \lambda_H} \right], \\ \frac{\partial c^d}{\partial \pi} &= -\frac{c^d}{\pi} \left[ \frac{1}{\pi} \ln \left( \frac{r - \pi}{r(1 - \pi)} \right) - \frac{r - 1}{(r - \pi)} \right]. \end{aligned} \quad (59)$$

For  $\lambda_H \rightarrow \lambda_L$ ,  $\frac{\partial c_2^{ND}}{\partial \pi} \rightarrow 0$ . The sign of  $\frac{\partial c^d}{\partial \pi}$  is negative if the difference in the square bracket in (59) is positive. It is easy to see that such a difference is increasing in  $r$  and it equals zero for  $r \rightarrow 1$ . This implies  $\frac{\partial c^d}{\partial \pi} < 0$  for any  $r > 1$ . Also, it then holds that  $\frac{\partial f(\pi, v)}{\partial \pi} < 0$  for any  $v$ . Given the monotonicity of  $f(\pi, \gamma)$  in  $\pi$ , there exists a value  $\underline{\pi}$  such that  $f(\pi, v) = \Pi^{ND} - \Pi^d = 0$  for  $v \rightarrow 0$  and  $f(\pi, v) < 0$  for  $v \rightarrow 0$  for any  $\pi > \underline{\pi}$ . Then it follows that it exists a value  $v^* \in (0, \bar{v})$  above which  $\Pi^{ND} > \Pi^d$  and below which

$\Pi^{ND} < \Pi^d$ . It follows that the no default equilibrium only exists when  $v \geq v^*$ .

We turn now to prove that the mixed equilibrium only exists for  $v \leq v^*$ . For the mixed equilibrium to exist, safe banks must be willing to buy loans on the interbank market. Recall that in the mixed equilibrium  $\Pi^S = \Pi^R$  and note from (22) that  $\Pi^R$  increases with  $P_L$  for any given  $v$ . Moreover,  $\Pi^R = \Pi^d$  and  $\Pi^S = \Pi^{ND}$  when  $P_L = r$ . Thus, as  $\Pi^{ND} = \Pi^d$  at  $v = v^*$  and  $\Pi^R = \Pi^S$  in the mixed equilibrium, it must also hold  $\Pi^{ND} = \Pi^d = \Pi^R = \Pi^S$  at  $v = v^*$ . Consider now a value of  $v > v^*$ . In this range we have shown, in the first part of the proof, that  $\Pi^d < \Pi^{ND}$ . It follows that  $\Pi^R|_{P_L=r} < \Pi^S|_{P_L=r}$ . For  $\Pi^R = \Pi^S$  to hold as required in the mixed equilibrium, it must be  $P_L > r$ . Consider now a value of  $v < v^*$ . Above we have shown that  $\Pi^d > \Pi^{ND}$  holds in this range. It follows that  $\Pi^R|_{P_L=r} > \Pi^S|_{P_L=r}$ . Thus, it must be  $P_L < r$  for  $\Pi^R = \Pi^S$ . Given that  $1 < P_L < r$  must hold for the safe banks to be willing to provide their excess liquidity on the market, the mixed equilibrium exists only for  $v \geq v^*$ . The proposition follows.  $\square$

**Proof of Proposition 4:** Recall that  $v^*$  is the solution to  $f(\pi, v) = \Pi^{ND} - \Pi^d = 0$ , where  $\Pi^{ND}$  and  $\Pi^d$  are given respectively by (11) and (57). The solution depends on  $\pi$  and, from the implicit function theorem,  $\frac{dv^*}{d\pi} = -\frac{\partial f(\pi, v)/\partial \pi}{\partial f(\pi, v)/\partial v}$ . The numerator is the same as (58), which is negative for  $\lambda_H \rightarrow \lambda_L$ . So the sign of  $\frac{dv^*}{d\pi}$  is given by the sign of the denominator  $\partial f(\pi, v)/\partial v$ . As shown in the proof of Proposition 3,  $\partial f(\pi, v)/\partial v < 0$  for  $v \rightarrow \bar{v}$ , and  $\partial f(\pi, v)/\partial v > 0$  in the range of  $v$  where  $v^*$  exists. Thus,  $\frac{dv^*}{d\pi} > 0$ , as in the proposition.  $\square$

**Proof of Proposition 5:** Differentiating the expression for  $\rho$  in (35) with respect to  $\pi$  and rearranging it gives

$$\frac{d\rho}{d\pi} = (\lambda_H - \lambda_L) \left[ \frac{d(\lambda_L c_1^R - P_H)}{d\pi} c_1^S - \frac{dc_1^S}{d\pi} (\lambda_L c_1^R - P_H) \right].$$

Differentiating  $c_1^S$  as in (31) with respect to  $\pi$  after substituting  $P_L = r$  at  $v = v^*$  gives

$$\frac{dc_1^S}{d\pi} = (\lambda_H - \lambda_L) c_1^S \left[ \ln \left( \frac{\pi \lambda_L + (1 - \pi) \lambda_H}{\pi \lambda_L + (r - \pi) \lambda_H} \right) - \frac{[\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)] \lambda_H (r - 1)}{[\pi \lambda_L + (r - \pi) \lambda_H] [\pi \lambda_L + (1 - \pi) \lambda_H]} \right],$$

which is zero for  $\lambda_H \rightarrow \lambda_L$ . Thus, a sufficient condition to have  $\frac{d\rho}{d\pi} < 0$  is that  $\frac{d(\lambda_L c_1^R - P_H)}{d\pi} < 0$ , which holds because of the concavity of consumers' utility functions.  $\square$

**Proof of Lemma 1:** The lemma follows immediately from the difference between (37) and (38) at  $v = v^*$ .  $\square$

**Proof of Proposition 6:** We first define the cutoff value  $\bar{\pi}$ . From (14) and (9),

differentiating the difference  $P_H - \lambda_H c_1^{nd}$  with respect to  $\pi$  at  $v = v^* = 0$  gives:

$$\left. \frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} \right|_{v=v^*=0} = -\frac{V(V-1)}{(V-\pi)^2} - \lambda_H(\lambda_H - \lambda_L)c_1^{nd} \left[ \ln \left( \frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (V-\pi)\lambda_H} \right) + \right. \\ \left. - \lambda_H \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)](V-1)}{[\pi\lambda_L + (1-\pi)\lambda_H][\pi\lambda_L + (V-\pi)\lambda_H]} \right].$$

For  $\lambda_H \rightarrow \lambda_L$ , it is  $\frac{\partial(P_H - \lambda_H c_1^{nd})}{\partial\pi} < 0$ . This implies that it exists a unique solution for the equation  $P_H - \lambda_H c_1^{nd} = 0$  at  $v = v^* = 0$  defined as  $\bar{\pi}$  such that  $P_H - \lambda_H c_1^{nd} \geq 0$  for  $\pi \leq \bar{\pi}$  and  $P_H - \lambda_H c_1^{nd} < 0$  for  $\pi > \bar{\pi}$ . Also,  $P_H - \lambda_H c_1^{nd} < 0$  for  $\pi \rightarrow 1$ . Differentiating then  $P_H - \lambda_H c_1^{nd}$  with respect to  $\pi$  at any  $v^* > 0$  after substituting (14) and (9) gives:

$$\left. \frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} \right|_{v=v^*>0} = -\frac{r(r-1) - \pi(1-\pi)\frac{dv^*}{d\pi}}{(r-\pi)^2} - \lambda_H(\lambda_H - \lambda_L)c_1^{nd} \left[ \ln \left( \frac{\pi\lambda_L + (1-\pi)\lambda_H}{\pi\lambda_L + (r-\pi)\lambda_H} \right) + \right. \\ \left. - \lambda_H \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)](r-1)}{[\pi\lambda_L + (1-\pi)\lambda_H][\pi\lambda_L + (r-\pi)\lambda_H]} \right] + \\ - \lambda_H^2 c_1^{nd} \frac{[\pi(1-\lambda_L) + (1-\pi)(1-\lambda_H)]}{[\pi\lambda_L + (r-\pi)\lambda_H]} \frac{dv^*}{d\pi},$$

where  $\frac{dv^*}{d\pi} > 0$  from Proposition 4. For  $\lambda_H \rightarrow \lambda_L$ , the expression above simplifies to

$$\left. \frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} \right|_{v=v^*>0} = -\frac{r(r-1)}{(r-\pi)^2} - \left( -\frac{\pi(1-\pi)}{(r-\pi)^2} + \lambda_L(1-\lambda_L)\frac{c_1^{nd}}{r} \right) \frac{dv^*}{d\pi}.$$

Assuming  $-\frac{\pi(1-\pi)}{(r-\pi)^2}\frac{dv^*}{d\pi}$  is sufficiently small,  $\frac{d(P_H - \lambda_H c_1^{nd})}{d\pi} < 0$ . Then, there exists a value  $\hat{\pi} \in (\underline{\pi}, 1)$  such that  $P_H - \lambda_H c_1^{nd} \geq 0$  for any  $\pi \leq \hat{\pi}$  and  $P_H - \lambda_H c_1^{nd} < 0$  otherwise. The proposition follows.  $\square$

**Proof of Proposition 7:** The problem of a bank deviating from the no default equilibrium and defaulting in state  $H$  is again the same as the one of a risky bank in (22)-(25) with  $P_L$  and  $P_H$  as in (12) and (14) while depositors' participation constraint is now given by

$$\pi \left[ \lambda_L \frac{(c_1^d)^{1-\sigma}}{1-\sigma} + (1-\lambda_L) \frac{(c_2^d)^{1-\sigma}}{1-\sigma} \right] + (1-\pi) \left[ \frac{R^d + P_H L^d}{1-\sigma} \right]^{1-\sigma} = 1.$$

Using the Lagrangian, the first order conditions with respect to  $R^d$ ,  $c_1^d$ ,  $c_2^d$  and  $\mu^d$  are as follows:

$$-\pi r + \frac{\pi r}{P_L} = \mu^d (1 - P_H) [R^d + P_H (1 - R^d)]^{-\sigma} \quad (60)$$

$$(c_1^d)^\sigma \frac{r}{P_L} = -\mu^d \quad (61)$$

$$(c_2^d)^{-\sigma} = -\mu^d \quad (62)$$

$$\pi[\lambda_L(c_1^d)^{1-\sigma} + (1-\lambda_L)(c_2^d)^{1-\sigma}] + (1-\pi)(R^d + P_H L^d)^{1-\sigma} = 1. \quad (63)$$

Since  $r = P_L$ , from (61) and (62) it follows that

$$c_2^d = c_1^d = c^d.$$

Substituting it into (60) and using the expression for  $P_H$  from (14), we obtain

$$R^d = \frac{(r-\pi)}{\pi(r-1)} [(c^d)^\sigma - r]. \quad (64)$$

The sign of (64) depends on the sign of the square bracket. Given that  $[(c^d)^\sigma - r]$  is monotonic in  $\sigma$ , it follows that there exists a threshold value of  $\sigma$ , defined as  $\bar{\sigma}$  and given by

$$\bar{\sigma} = \frac{\text{Log}(r)}{\text{Log}(c^d)} > 1,$$

such that  $R^d > 0$  for  $\sigma > \bar{\sigma}$  and  $R^d = 0$  for  $\sigma \leq \bar{\sigma}$ . The proposition follows.  $\square$

**Proof of Proposition 8:** We first show that there exists an equivalent threshold  $v_{di}^*$  to the one in Proposition 3 when deposit insurance is in place. Then, we analyze how this compares to the threshold  $v^*$  when there is no insurance.

The no default equilibrium remains as in Proposition 1. The expected profits  $\Pi_{di}^d$  of a bank deviating from the no default equilibrium and defaulting in state  $H$  with deposit insurance are given by

$$\Pi_{di}^d = \pi \left[ r L_{di}^d - r \left( \frac{\lambda_L c_{1di}^d - R_{di}^d}{P_L} \right) - (1-\lambda_L) c_{2di}^d - C \right] \quad (65)$$

where  $C$  is as in (41) while the participation constraint is equal to

$$\pi \left[ \lambda_L u(c_{1di}^d) + (1-\lambda_L) u(c_{2di}^d) \right] + (1-\pi) u(c^I) = u(1).$$

The first order conditions and the solution to the problem are the same as in the case without deposit insurance with the only difference that consumers promised repayments

in state  $L$  are now

$$c_{1di}^d = c_{2di}^d = c_{di}^d = (c^I)^{-\frac{(1-\pi)}{\pi}}$$

instead of (56) so that (65) simplifies to

$$\Pi_{di}^d = \pi(r - c_{di}^d - C) = \pi \left[ r - c_{di}^d - (1 - \pi)(c^I - P_H) \right]. \quad (66)$$

To prove that  $v_{di}^*$  exists, we need to show that  $\Pi^{ND} \leq \Pi_{di}^d$  for  $v \leq v_{di}^*$  and  $\Pi^{ND} \geq \Pi_{di}^d$  otherwise, where  $\Pi^{ND}$  is as in (11). We follow the same steps as in the proof of Proposition 3. Define  $f_{di}(\pi, v) = \Pi^{ND} - \Pi_{di}^d$ . When  $v \rightarrow \bar{v}$  and  $r \rightarrow 1$ ,  $f_{di}(\pi, v) \rightarrow \pi(c_{di}^d + c^I) \geq 0$  since from (9), (10) and (13),  $c_1^{ND} = c_2^{ND} = P_H \rightarrow 1$ . Differentiating  $f(\pi, v)$  with respect to  $v$  gives

$$\frac{\partial f_{di}(\pi, v)}{\partial v} = \frac{\partial \Pi^{ND}}{\partial v} - \frac{\partial \Pi_{di}^d}{\partial v}$$

where

$$\frac{\partial \Pi^{ND}}{\partial v} = -(1 - \lambda_H c_1^{ND}) < 0$$

and

$$\frac{\partial \Pi_{di}^d}{\partial v} = -\pi \left[ \frac{(r - \pi)^2 - (1 - \pi)^2 \pi}{(r - \pi)^2} \right] < 0.$$

The profits  $\Pi^{ND}$  and  $\Pi_{di}^d$  are monotonically decreasing in  $v$ . For  $v \rightarrow \bar{v}$ ,  $\frac{\partial f_{di}(\pi, v)}{\partial v} \rightarrow -(1 - \lambda_H) < 0$ . Thus, there exists a unique threshold  $v_{di}^* \in (0, \bar{v})$  such that  $f_{di}(\pi, v) = \Pi^{ND} - \Pi_{di}^d = 0$  if and only if  $f_{di}(\pi, v) \leq 0$  for  $v \rightarrow 0$  assuming that  $\pi$  is sufficiently high so that  $\Pi^{ND} < \Pi_{di}^d$  at  $v = 0$ . Then it follows that the threshold  $v_{di}^* \in (0, \bar{v})$  exists.

To see how the introduction of deposit insurance affects the threshold  $v_{di}^*$ , we then simply compare the deviating bank's expected profit without deposit insurance as given by (57) with that with deposit insurance as in (65). It follows that  $v^* > v_{di}^*$  when  $\Pi^d > \Pi_{di}^d$  that is when

$$(1 - \pi)(c^I - P_H) > c^d - c_{di}^d.$$

The proposition follows.  $\square$

**Proof of Proposition 9:** We divide the proof in two parts again. We first show that there exists an equivalent threshold  $v_\alpha^*$  as in Proposition 3 when consumers' opportunity cost is  $\alpha > 1$ . Then, we study how this varies with  $\alpha$ .

The bank's problem in the no default equilibrium is as in (2)-(6) with the only difference that consumers' participation is now given by (43). Following the same steps as in the

proof of Proposition 1, we have  $R^{ND} = \lambda_H c_1^{ND}$  again and

$$c_1^{ND} = \alpha \left( \frac{\pi \lambda_L + (1 - \pi) \lambda_H}{\pi \lambda_L + (r - \pi) \lambda_H} \right)^{\pi(1 - \lambda_L) + (1 - \pi)(1 - \lambda_H)} \quad (67)$$

$$c_2^{ND} = \alpha \left( \frac{\pi \lambda_L + (r - \pi) \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H} \right)^{\pi \lambda_L + (1 - \pi) \lambda_H}. \quad (68)$$

The problem of the deviating bank is also as before with the only difference that consumers' participation constraint is now given by

$$\pi \left[ \lambda_L u(c_1^d) + (1 - \lambda_L) u(c_2^d) \right] + (1 - \pi) u(R + P_H L^d) \geq u(\alpha).$$

The deviating bank then still chooses  $R^d = 0$  and promises depositors in state  $L$

$$c_1^d = c_2^d = c^d = \alpha^{\frac{1}{\pi}} (P_H)^{-\frac{1 - \pi}{\pi}} = \alpha^{\frac{1}{\pi}} \left( \frac{r(1 - \pi)}{r - \pi} \right)^{-\frac{1 - \pi}{\pi}}. \quad (69)$$

Banks expected profits  $\Pi^{ND}$  and  $\Pi^d$  are then still as in (11) and (57), respectively, but with  $c_2^{ND}$  and  $c^d$  as in (68) and (69). We can then compute  $v_\alpha^*$  as the solution to

$$f(\pi, v, \alpha) = \Pi^{ND} - \Pi^d = 0,$$

from which we have

$$f(\pi, v, \alpha) = r(1 - \pi) - \alpha \left( \frac{\pi \lambda_L + (r - \pi) \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H} \right)^{\pi \lambda_L + (1 - \pi) \lambda_H} + \pi \alpha^{\frac{1}{\pi}} \left( \frac{P_L(1 - \pi)}{P_L - \pi} \right)^{-\frac{1 - \pi}{\pi}} = 0.$$

When  $v \rightarrow \bar{v}$  and  $r \rightarrow 1$ ,  $f(\pi, v, \alpha) > 0$  since from (67) and (69),  $c_1^{ND} = c_2^{ND}$  and  $c^d \rightarrow \alpha^{\frac{1}{\pi}}$ . Both  $\Pi^{ND}$  and  $\Pi^d$  are decreasing in  $v$ . The expressions for the derivatives are similar to those in the proof of Proposition 3 and equal to

$$\frac{\partial \Pi^{ND}}{\partial v} = -(1 - \lambda_H c_1^{ND}) < 0,$$

and to

$$\frac{\partial \Pi^d}{\partial v} = -\pi \left[ 1 - c^d \frac{(1 - \pi)}{r(r - \pi)} \right] < 0.$$

For  $v \rightarrow \bar{v}$ ,  $\frac{\partial f(\pi, v, \alpha)}{\partial v} \rightarrow -(1 - \lambda_H) + \pi(1 - \alpha^{\frac{1}{\pi}}) < 0$  as  $\alpha^{\frac{1}{\pi}} > 1$ . Thus, there exists

a unique threshold  $v_\alpha^* \in (0, \bar{v})$  such that  $f(\pi, v, \alpha) = \Pi^{ND} - \Pi^d = 0$  if and only if  $f(\pi, v, \alpha) \leq 0$  for  $v \rightarrow 0$  assuming that  $\pi$  is sufficiently high so that  $\Pi^{ND} < \Pi^d$  at  $v = 0$ .

To analyze the effect of  $\alpha$  on the  $v_\alpha^*$ , we use the implicit function theorem and have

$$\frac{dv_\alpha^*}{d\alpha} = -\frac{\frac{\partial f(\pi, v, \alpha)}{\partial \alpha}}{\frac{\partial f(\pi, v, \alpha)}{\partial v}}.$$

The existence of the threshold  $v_\alpha^*$  requires that  $\frac{\partial f(\pi, v, \alpha)}{\partial v}$  is positive, i.e., the profits function of a deviating bank is steeper than that of a non-defaulting bank. This implies that the sign of  $\frac{dv_\alpha^*}{d\alpha}$  is equal to the inverse sign of  $\frac{\partial f(\pi, v, \alpha)}{\partial \alpha}$ . The derivative  $\frac{\partial f(\pi, v, \alpha)}{\partial \alpha}$  is given by

$$\frac{\partial f(\pi, v, \alpha)}{\partial \alpha} = -\frac{1}{\alpha} \left[ \alpha \left( \frac{\pi \lambda_L + (r - \pi) \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H} \right)^{\pi \lambda_L + (1 - \pi) \lambda_H} - \alpha^{\frac{1}{\pi}} \left( \frac{P_L(1 - \pi)}{P_L - \pi} \right)^{-\frac{1 - \pi}{\pi}} \right] = -\frac{1}{\alpha} (c_2^{ND} - c^d).$$

The sign of  $\frac{\partial f(\pi, v, \alpha)}{\partial \alpha}$  depends on the difference  $c_2^{ND} - c^d$ , which in turn depends on  $\alpha$ . In the benchmark case  $\alpha = 1$ , we know that  $c_2^{ND} - c^d > 0$ . As  $\alpha$  increases both  $c_2^{ND}$  and  $c^d$  increases. However, while  $c_2^{ND}$  increases linearly as

$$\frac{\partial c_2^{ND}}{\partial \alpha} = \left( \frac{\pi \lambda_L + (r - \pi) \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H} \right)^{\pi \lambda_L + (1 - \pi) \lambda_H},$$

$c^d$  increases more than proportionally as

$$\frac{\partial c^d}{\partial \alpha} = \frac{1}{\pi \alpha} c^d.$$

Thus, there exists a level of  $\alpha^* > 1$  at which  $c_2^{ND} - c^d = 0$ . This is given by

$$\alpha^* = \frac{P_L(1 - \pi)}{P_L - \pi} \left[ \frac{\pi \lambda_L + (r - \pi) \lambda_H}{\pi \lambda_L + (1 - \pi) \lambda_H} \right]^{\frac{(\pi \lambda_L + (1 - \pi) \lambda_H) \pi}{1 - \pi}} = P_H \left( \frac{c_2^{ND}}{c_1^{ND}} \right)^{\frac{(\pi \lambda_L + (1 - \pi) \lambda_H) \pi}{1 - \pi}}.$$

Thus, for  $\alpha < \alpha^*$ ,  $\frac{\partial f(\pi, v, \alpha)}{\partial \alpha} < 0$ , while for  $\alpha > \alpha^*$ ,  $\frac{\partial f(\pi, v, \alpha)}{\partial \alpha} > 0$  and the proposition follows.  $\square$

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