Emotions and Political Unrest
Supplementary Material

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Second Order Conditions

Periods 1 and 2 - Entitled policies, $\hat{\tau}_t^i$, $\hat{b}^i$.

If $\delta \leq -\tau_2^* L_\tau(\tau_2^*) T_b(0)$ (cf. Lemma 4 in the main text), $\hat{b}^p$, $\hat{b}^r$, and $\hat{\tau}_t^r$ are corner solutions. Thus we only have to verify the SOC for $\hat{\tau}_t^p$, $(t = 1, 2)$. Consider period 2. As pointed out by Lemma 2, the fair policy is an interior solution (i.e. $b^* < \bar{b}$) if $-\pi^{ir} l_2^* + \pi^{ip}(\tau_2 L_\tau(\tau_2) + l_2^*) = 0$. Let us rewrite it as:

$$\delta - \frac{1}{2}(1+\delta)\,\eta(\tau_2) = 0$$

Differentiating wrt τ_2 yields the following SOC, which is negative at $\hat{\tau}_2^i$:

$$SOC_{\hat{\tau}_2^p} \equiv -\frac{1}{2}(1+\delta)\,\eta_\tau(\hat{\tau}_2^i) < 0$$

As for $\hat{\tau}_1^p$, the calculation of $SOC_{\hat{\tau}_1^p}$ parallels $SOC_{\hat{\tau}_2^p}$.

Period 2 - Equilibrium policy, τ_2^*

Let us rewrite the optimality condition $\tau_2 L_\tau(\tau_2) \leq \varsigma[P_{2\tau}^p(\tau_2, b) + P_{2\tau}^r(\tau_2, b)]$ as:

$$W_{2\tau}(\tau_2^*, b) - \varsigma P_{2\tau}^p(\tau_2^*) - \varsigma P_{2\tau}^r(\tau_2^*, b) \leq 0$$

that holds with equality if $S(b) > 0$, or $b < \tilde{b}$. In this case, the second order condition, call it SOC_{τ_2}, is satisfied at the equilibrium point:

$$SOC_{\tau_2} \equiv W_{2\tau\tau}(\tau_2^*, b) - \varsigma P_{2\tau\tau}^p(\tau_2^*) - \varsigma P_{2\tau\tau}^r(\tau_2^*, b) < 0$$

where

$$W_{2\tau\tau}(\tau_2^*, b) = -l_2^* \eta_\tau(\tau_2^*) < 0$$

$$P_{2\tau\tau}^p(\tau_2^*) = \begin{cases} \frac{\omega}{2(\sigma-\mu)} P_2^p[-2\left(\hat{V}_2^p - V_2^p\right) l_2^*(1-\eta(\tau_2^*))P_{2\tau}^p + P_2^p l_2^{*2}(1-\eta(\tau_2^*))^2 + \\ P_2^p\left(\hat{V}_2^p - V_2^p\right) l_2^* \eta_\tau(\tau_1^*)] > 0 \end{cases}$$

$$P_{2\tau\tau}^r(\tau_2^*, b) = \frac{\omega}{2(\sigma-\mu)} P_2^r l_2^* \left[2\left(\hat{V}_2^r - V_2^r\right) P_{2\tau}^r + P_2^r l_2^*\right] > 0$$

If $b \geq \tilde{b}$, inequality (23) is strict: the non-negativity constraint on s_2 binds. Then τ_2^* is determined by $\tau_2^* L(\tau_2^*) = b^*$.

Period 1 - Equilibrium tax rate, τ_1^*

Differentiating (35) yields the following second order condition, $SOC_{\tau 1}$, which is negative at the point (τ_1^*, b^*):

$$SOC_{\tau 1} \equiv W_{1\tau\tau}(\tau_1^*, b^*) - \varsigma P_{1\tau\tau}^p(\tau_1^*, b^*) - \varsigma P_{1\tau\tau}^r(\tau_1^*, b^*) < 0$$

where

$$W_{1\tau\tau}(\tau_1^*) = -l_1^* \eta_\tau(\tau_1^*) < 0$$

$$P_{1\tau\tau}^p(\tau_1^*, b^*) = \begin{cases} \frac{1}{2(\sigma-\mu)} P_1^p[-2\left(\hat{V}_1^p - V_1^p\right) l_1^*(1-\eta(\tau_1^*))P_{1\tau}^p + P_1^p l_1^{*2}(1-\eta(\tau_1^*))^2 + \\ P_1^p\left(\hat{V}_1^p - V_1^p\right) l_1^* \eta_\tau] > 0 \end{cases}$$

$$P_{1\tau\tau}^r(\tau_1^*, b^*) = \frac{\omega}{2(\sigma-\mu)} P_1^r l_1^* \left[2\left(\hat{V}_1^r - V_1^r\right) l_1^* P_{1\tau}^r + P_1^r l_1^*\right] > 0$$

Period 1 - Equilibrium debt, b^*

By Proposition 4, $b^* \geq \tilde{b}$, thus the optimality condition is (38), which holds with equality if the equilibrium is interior (i.e. $b^* > \tilde{b}$). In this case the second order condition can be obtained by differentiating the following equation:

$$-\frac{\eta(T(b))}{(1-\eta(T(b)))} - \varsigma\left[P_{1b}^p(\tau_1^*, b) + P_{2\tau}^p(\tau_2^*, b)\frac{1}{(1-\eta(T(b)))l_2^*}\right] = 0$$

Evaluated at b^*,

$$SOC_b \equiv -\frac{\eta_\tau(T(b^*))T_b(b^*)}{(1-\eta(T(b^*)))^2} -$$
$$-\varsigma\left[P^p_{1bb}(\tau^*_1, b^*) + \frac{P^p_{2\tau\tau}(\tau^*_2,b^*)T_b(b^*)}{(1-\eta(T(b^*)))l^*_2} + \frac{P^p_{2\tau}(\tau^*_2,b^*)\eta_\tau(T(b^*)))T_b(b^*)l^*_2}{(1-\eta(T(b)))l^*_2}\right] < 0$$

where the second and the third terms in the square brackets are positive (notice that, as shown earlier, $P^p_{2\tau\tau} > 0$) and, as for the first term, it is positive too:

$$P^p_{1bb}(\tau^*_1, b^*) = -\frac{\omega}{(\sigma-\mu)}P^p_1 P^p_{1b} + \frac{\omega}{2(\sigma-\mu)}\left(P^p_1\right)^2 > 0$$

Comparative Statics

Period 1 - Tax rate

Riot disruptions, ς^i

Differentiating the optimality condition, $\tau_1 L_\tau(\tau_1) - \varsigma\left[P^p_{1\tau}(\tau_1, b) + \varsigma P^r_{1\tau}(\tau_1, b)\right] = 0$, yields,

$$\begin{aligned}\frac{\partial \tau_1^*}{\partial \varsigma^p} &= \frac{P^p_{1\tau}}{SOC_{\tau 1}} > 0 \\ \frac{\partial \tau_1^*}{\partial \varsigma^r} &= \frac{P^r_{1\tau}}{SOC_{\tau 1}} < 0\end{aligned}$$

Higher ability to inflict social costs results in more favorable tax rate in the first period.

Frustration, ω^i

$$\begin{aligned}\frac{\partial \tau_1^*}{\partial \omega^p} &= \frac{\varsigma^p P^p_{1\tau\omega}}{SOC_{\tau 1}} > 0 \\ \frac{\partial \tau_1^*}{\partial \omega^r} &= \frac{\varsigma^r P^r_{1\tau\omega}}{SOC_{\tau 1}} < 0\end{aligned}$$

where $P^i_{1\tau\omega} = \frac{1}{2(\sigma-\mu)}\left(P^i_1\right)^2 A^i_{1\tau\omega} + \frac{1}{\sigma-\mu}\left(P^i_1\right) A^i_{1\tau} P^i_{1\omega}$ is negative for $i = p$ and it is positive for $i = r$. In both groups, higher frustration yields more sensitivity of riots to tax rate. Thus higher frustration of a group leads to more favorable τ_1^*.

Self-serving bias - ideology, δ^i

$$\begin{aligned}\frac{\partial \tau_1^*}{\partial \delta^p} &= \frac{\varsigma^p P^p_{1\tau\delta}}{SOC_{\tau 1}} > 0 \\ \frac{\partial \tau_1^*}{\partial \delta^r} &= 0\end{aligned}$$

where $P^p_{1\tau\delta} = \frac{1}{2(\sigma-\mu)}\left(P^p_1\right)^2 A^p_{1\tau\hat{\tau}} T^p_\delta + \frac{1}{\sigma-\mu}\left(P^p_1\right) A^p_{1\tau} P^p_{1\hat{\tau}} T^p_\delta < 0$. Recall that the entitlements of the rich at time 1 are zero, for any $\delta \geq 0$. By contrast, the entitlement tax rate of the poor is increasing in δ. Higher δ yields higher $\hat{\tau}^p_1$. Thus the threat of riots by the poor increases. This yields higher τ_1^*.

Ideological heterogeneity, σ^i

Consider the poor sector:

$$\frac{\partial\tau_1^*}{\partial\sigma^p} = \varsigma^p \frac{-\frac{4}{\left(4\sigma^p - A_1^p\right)^2} P_1^p A_{1\tau}^p}{SOC_{\tau 1}} < 0$$

More heterogeneous groups are less coordinated in riots. Higher σ^p yields smaller threat of riots by the poor. Thus τ_1^* is lower.

As for the rich sector, higher σ^r implies lower P_1^r and higher τ_1^*.

Debt, and period 2 Tax rate

Recall that at the second period taxes only service the debt in equilibrium, while subsidies are zero. Debt fully determines the equilibrium tax rate at period 2: $\tau_2^* = T(b^*)$. Thus the comparative statics for b^* parallels the one for τ_2^*. Moreover, since in equilibrium the rich sector is not aggrieved, changes in the parameters of this sector do not affect the equilibrium. We can consider only the poor sector. Recall that, if the solution is interior (i.e. $b^* > \tilde{b}$), then the optimality condition (38) holds with equality. Differentiating it yields what follows.

Riot disruption, ς^p

$$\frac{\partial b^*}{\partial\varsigma^p} = \frac{P_{1b}^p + P_{2\tau}^p \left[(1-\eta(T(b^*)))l_2^*\right]^{-1}}{SOC_b} > 0 \quad \rightarrow \quad \frac{\partial\tau_2^*}{\partial\varsigma^p} > 0$$

Higher sensitivity to policy by the poor sector means bigger dampening impact of b on first period riots. Thus, the government issues more debt. Higher debt implies higher taxes in period 2.

Frustration parameter, ω^p

$$\frac{\partial b^*}{\partial\omega^p} = \frac{\varsigma^p \left\{P_{1b\omega}^p + P_{2\tau\omega}^p \left[(1-\eta(T(b^*)))l_2^*\right]^{-1}\right\}}{SOC_b} > 0 \quad \rightarrow \frac{\partial\tau_2^*}{\partial\omega} > 0$$

where $P_{1b\omega}^p = \frac{1}{2(\sigma-\mu)}\left(P_1^p\right)^2 A_{1b\omega}^p + \frac{1}{\sigma-\mu}\left(P_1^p\right) A_{1b}^p P_{1\omega}^p < 0$; analogously, $P_{2\tau\omega}^p = \frac{1}{2(\sigma-\mu)}\left(P_2^p\right)^2 A_{2\tau\omega}^p + \frac{1}{\sigma-\mu}\left(P_2^p\right) A_{2\tau}^p P_{2\omega}^p < 0$. When ω is higher, aggrievement of the poor sector is more sensitive to debt. Then the government has a stronger incentive to issue debt.

Self-serving bias - ideology, δ^p

$$\frac{\partial b^*}{\partial\delta^p} = 0 \quad \rightarrow \frac{\partial\tau_2^*}{\partial\delta^p} = 0$$

Under the condition on δ in Lemma 4, both sectors want zero debt. Higher self-serving bias does not affect the government policy. See the main text for a discussion of what happens if the condition on δ does not hold.

Ideological heterogeneity, σ^p

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$$\frac{\partial b^*}{\partial \sigma^p} = \varsigma^p \frac{P^p_{1b\sigma^p} + P^p_{2\tau\sigma^p}\left[(1-\eta(T(b^*)))l_2^*\right]^{-1}}{SOC_b} < 0 \quad \rightarrow \frac{\partial \tau_2^*}{\partial \sigma^p} < 0$$

This result comes from the fact that $P^p_{tx\sigma^p} = -\frac{1}{(\sigma^p-\mu)}\left(P^p_t\right)^2 A^p_{tx} + \frac{1}{\sigma^p-\mu}\left(P^p_t\right) P^p_{t\sigma^p} A^p_{t\tau} < 0$ $(x = b, \tau;\ t = 1, 2)$. Since $P^p_{t\sigma^p} = -\frac{A^p_t}{\left(4\sigma^p - A^p_t\right)^2} < 0$, and $A^p_{t\tau} > 0$, then $P^p_{tx\sigma^p} < 0$. Higher σ^p makes the poor sector less cohesive in riots. The debt has a lower impact on aggrievement reduction at the first period. Thus the government has a smaller incentive to issue debt.

Period 2 - Subsidies

By definition, $S(b) = \tau_2^* L(\tau_2^*) - b$, where $\tau_2^* = T(b)$ is given by the optimality condition $\tau_2 L_\tau(\tau_2) \leq \varsigma[P^p_{2\tau}(\tau_2, b) + P^r_{2\tau}(\tau_2, b)]$. By Proposition 2, $S(b) > 0$, for any $b < \tilde{b}$. For these values of b, $\tau_2 L_\tau(\tau_2) \leq \varsigma[P^p_{2\tau}(\tau_2, b) + P^r_{2\tau}(\tau_2, b)]$ holds with equality. Thus,

$$\frac{\partial s_2^*}{\partial x} = l_2^*(1-\eta(\tau_2^*))\frac{\partial \tau_2^*}{\partial x}$$

where x is any parameter in the model. Observe that $\frac{\partial s_2^*}{\partial x}$ and $\frac{\partial \tau_2^*}{\partial x}$ have the same sign. Moreover, following the same steps for the comparative statics on τ_1^*, it is easy to show that also $\frac{\partial \tau_2^*}{\partial x}$ and $\frac{\partial \tau_1^*}{\partial x}$ have the same sign. Thus, exploiting the above comparative statics on τ_1^*, the following results are straightforward:

$$\begin{array}{ccc} \frac{\partial s_2^*}{\partial \varsigma^p} > 0 & \frac{\partial s_2^*}{\partial \varsigma^r} < 0 \\ \frac{\partial s_2^*}{\partial \omega^p} > 0 & \frac{\partial s_2^*}{\partial \omega^r} < 0 \\ \frac{\partial s_2^*}{\partial \delta^p} > 0 & \frac{\partial s_2^*}{\partial \delta^r} = 0 \\ \frac{\partial s_2^*}{\partial \sigma^p} < 0 & \frac{\partial s_2^*}{\partial \sigma^r} > 0 \end{array}$$