

# The pro-collusive effect of increased cartel detection probabilities

Jeroen Hinloopen,<sup>\*†</sup>

*University of Amsterdam (and Tinbergen Institute)*

and

*Economics Network for Competition and Regulation  
(ENCORE)*

November 2004

## Abstract

An increase in cartel discovery probability due to irregular price movements that result from cartel defection is shown to increase cartel stability as short-run defection profits are less likely to be earned.

**Key words:** cartel stability, detection probabilities, defection incentives.

**JEL classification:** L12, L41.

---

\*Correspondence: University of Amsterdam, Faculty of Economics and Econometrics, Department of Economics, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands; J.Hinloopen@uva.nl; <http://www.fee.uva.nl/io/jhinloopen>.

†Thanks are due to Massimo Motta, Karl Schlag, and seminar participants at the European University Institute. The usual disclaimer applies.

# 1 Introduction

Unexpected price movements trigger the attention of antitrust authorities as these price movements might signal the existence of some collusive agreement that fails (see Abrantes-Metz, Froeb, and Taylor, 2004). Cheating on a cartel thus increases the probability that the (former) cartel is discovered by antitrust authorities.

In this note we formalize the intuition that this increase in cartel discovery probability reduces the incentive to cheat on the cartel as it reduces the probability of earning short-term defection profits. In the limit, when cartel discovery after defection is certain, cheating on a collusive agreement is shown never to be observed.

We show further that cartel defection is less likely to be observed if the per-period detection probability increases, provided that the increase in cartel detection probability in case of defection is ‘large enough’. This somewhat surprising result is due to the relative importance of two off setting forces. An increase in all per-period detection probabilities increases in particular the detection probability in the defection period, making defection less likely. At the same time it lowers the probability of being able to defect as the cartel is less likely to exist, thus increasing the incentive to defect in order to earn defection profits. The larger is the increase in detection probability due to defection, the stronger is the former effect, making defection less likely in case of increased per-period detection probabilities.

## 2 Cartel compliance

At some point in time, say  $t = 1$ , a group of firms engages in a collusive agreement to sustain higher than non-cooperative Nash prices. This agreement is illegal and therefore there is no formal contract that members of the cartel can use to exact compliance. Rather, all cartel members are assumed to behave as grim reapers: sustain collusion as long as all other cartel members did so up to the last period, revert to independent Nash behavior otherwise. The concomitant trigger strategy profile reads as (see Friedman, 1971):

$$\begin{aligned} s_i^1 &= p^C, \\ s_i^t &= \begin{cases} p^C & \text{if } p_j^k = p^C, \quad k = 1, \dots, t-1, \quad j = 1, \dots, m, \\ p^N & \text{otherwise;} \end{cases} \end{aligned} \tag{1}$$

$t = 2, \dots, i = 1, \dots, m$ , where  $p^C$  is the collusive price, and where  $p^N$  is the non-cooperative Nash price. For each cartel member this collusive agreement yields per period profits  $\pi^C$ , which strictly exceed the alternative under non-cooperative Nash behavior,  $\pi^N$ .

From the moment on that the cartel is created all members realize that there is a per-period detection probability  $p \in (0, 1)$ ; the probability that the cartel is discovered during some period.<sup>1</sup> Accordingly, the probability that the cartel is discovered from periods 1 through period  $k$  equals:

$$P(k) = 1 - (1 - p)^{k+1}. \quad (2)$$

If the cartel is discovered it ceases to exist forever after and firms earn each period  $\pi^N$  only. Hence, profits earned in period  $k$  as expected at the beginning of period 1 are given by:

$$v^C(k) = [1 - P(k)] \pi^C + P(k) \pi^N. \quad (3)$$

The expected present discounted value of cartel compliance, given that future periods are discounted with rate  $\delta \in (0, 1)$  and that all cartel members adhere to the trigger strategy, is thus equal to:

$$V_t^C = \sum_{i=0}^{\infty} \delta^i v^C(i). \quad (4)$$

### 3 Incentive compatibility

Alternatively, a cartel member defects from the collusive agreement to earn short-run defection profits  $\pi^{defect} > \pi^C$  during the defection period. As defection typically yields price movements of greater amplitude which trigger the attention of antitrust authorities (see Abrantes-Metz, Froeb, and Taylor, 2004) the per-period detection probability after defection increases to  $p + \varepsilon > p$ , where  $p + \varepsilon < 1$ . Accordingly, expected defection profits when defecting in period  $d$  equal:

$$v^{defect}(d) = [1 - P(d - 1)] [(1 - (p + \varepsilon)) \pi^{defect} + (p + \varepsilon) \pi^N] + P(d - 1) \pi^N \quad (5)$$

The discounted value of defection in period  $d$  given that all members adhere to the trigger strategy is then:

---

<sup>1</sup>See Hinloopen (2003) for an analysis where per-period detection probabilities can vary each period.

$$V^{defect} = \sum_{i=0}^{d-1} \delta^i v^C(i) + \delta^d v^{defect}(d) + \sum_{i=d+1}^{\infty} \delta^i \pi^N. \quad (6)$$

Solving  $V^C \geq V^{defect}$  yields as incentive compatibility constraint for collusion to be sustained as a subgame perfect Nash equilibrium:

$$\frac{\pi^{defect} - \pi^N}{\pi^C - \pi^N} \leq \frac{1}{(1-p-\varepsilon)} \sum_{i=0}^{\infty} \delta^i (1-p)^{i+1} = S(p, \varepsilon). \quad (7)$$

## 4 Cartel stability and detection probabilities

Note that  $\forall p, \delta \in (0, 1)$  and  $p + \varepsilon < 1$  we have that  $S(p, \varepsilon) \neq \emptyset$ , and  $S(p, \varepsilon) > 0$ . Accordingly, even if there is a positive probability of cartel detection in every period there is a range of parameter values and relative profit levels for which the cartel will exist.

The first result of interest is:

**Proposition 1** *The larger is the increase in the cartel detection probability during the defection period, the more likely the incentive compatibility constraint is met to sustain an equilibrium price above the non-collusive Nash equilibrium price.*

**Proof.**  $\partial S(p, \varepsilon) / \partial \varepsilon = \sum_{i=0}^{\infty} \delta^i (1-p)^{i+1} / (1-p-\varepsilon)^2 > 0$ . ■

Indeed, an increased detection probability due to defection makes it less likely for a cheating firm to earn short-run defection profits thus reducing the incentive to defect. In the limit defection will never be observed, as  $\lim_{\varepsilon \rightarrow 1-p} S(p, \varepsilon) = \infty$ .

At the same time, when the per-period detection probability would not be affected by some cartel member defecting from the collusive agreement, an increase in the per-period detection probability increases the incentive to defect:

**Proposition 2** *In case defection does not affect the per-period cartel detection probability, the larger are per-period cartel detection probabilities the less likely the incentive compatibility constraint is met to sustain an equilibrium price above the non-collusive Nash equilibrium price.*

**Proof.**  $\partial S(p, \varepsilon) / \partial p|_{\varepsilon=0} = - \sum_{i=0}^{\infty} i \delta^i (1-p)^{i-1} = - \delta / [1 - \delta(1-p)]^2 < 0$ . ■

There are two opposite forces that underlay Proposition 2. An increase in the detection probability during the defection period reduces the incentive to defect (Proposition 1). However, an increase in all other per-period detection probabilities makes it less likely that there is still the option to defect as the cartel is less likely to still exist. According to Proposition 2 the latter effect outweighs the former in case defection does not increase in particular the cartel detection probability during the defection period. This, however, is a special case, as shown in the next proposition:

**Proposition 3** *For all  $\varepsilon > \varepsilon^* = \delta(1 - p)^2$ , the larger are per-period cartel detection probabilities the more likely the incentive compatibility constraint is met to sustain an equilibrium price above the non-collusive Nash equilibrium price.*

**Proof.**  $\partial S(p, \varepsilon) / \partial p|_{\varepsilon > 0} = (\sum_{i=0}^{\infty} \delta^i (1 - p)^i [\varepsilon - i(1 - p - \varepsilon)]) / (1 - p - \varepsilon)^2 > 0 \iff \varepsilon > \delta(1 - p)^2$ . Note also that  $\varepsilon^* = \delta(1 - p)^2 < 1 - p$ . ■

Although increased per-period detection probabilities make it less likely that defection is still an option, if defection increases ‘enough’ the detection probability during the defection period an increase in all per-period detection probabilities makes the cartel more stable. In this case the reduced probability of earning defection profits outweighs the reduced probability of still being able to defect.

## 5 Conclusions

Antitrust authorities benefit from a reputation of being an effective cartel discovery agency; increased per-period cartel detection probabilities reduce the stability of cartels. However, stressing the fact that cartel defection yields price movements that facilitate the discovery of (former) cartels makes defection less attractive as defection profits are less likely to be earned. Indeed, if the increase in detection probability due to defection is large enough, increased per-period cartel detection probabilities increase the stability of cartels. In this case the reduction in likelihood of earning defection profits outweighs the reduction in likelihood of still being able to defect, making defection less attractive.

## References

- [1] Abrantes-Metz, Rosa, Luke M. Froeb, and Christopher Taylor (2004), “A variance screen for collusion”, US Federal Trade Commission.

- [2] Friedman, James W. (1971), “A non-cooperative equilibrium for supergames”, *Review of Economic Studies*, **38**, pp. 1 – 12.
- [3] Hinlopen, Jeroen (2003), “Cartel stability with time-dependent detection probabilities”, Tinbergen Institute Discussion Paper, TI 2003-104/2.