HOW TO SIMULATE THE COORDINATED EFFECT OF A MERGER

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Abstract

In order to evaluate the collusive impact of a merger only descriptive tools are generally employed, as the application of econometric techniques is sporadic and simulation models virtually absent. The antitrust authorities therefore seem to make inefficient use of the information they usually gather on the demand and cost structure. In this paper we show how it is possible, by using a specific collusive equilibrium, to simulate the coordinated effect of a concentration. We apply Friedman's notion of "balanced temptations", which refers to a model characterized by a unique collusive equilibrium and by only one parameter - the common critical discount factor - which can measure the stability of a cartel. It appears that this equilibrium - a great deal similar to the Nash bargaining solution – accommodate demand and cost asymmetries. A merger that modifies such asymmetries would therefore not exercise any relevant effect on cartel stability. Finally, we expect from our analysis to detect a coordinated effect especially in those cases where a unilateral effect arises.

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1-Introduction

In the last decade we witnessed a decisive improvement in the empiric analysis applied to antitrust merger control. Such development has mainly affected the unilateral effect evaluation, which is now much more precise than a few years ago thanks to the adoption of simulation models. On the contrary, the empirical analysis of the coordination effect is less advanced, as methods currently in use turn out to be quite unsophisticated and controversial. There is no doubt that the incentives to collusion, and their modification by a merger, are much more complex to analyse than the unilateral effect. However casual observation of leading law cases suggests that, considering the impressive information already gathered for the unilateral effect evaluation, quantitative tools may merit a wider application also for detecting the coordinated effect.

Starting from the seminal article by Stigler (1964) it has become common to identify - among the factors which make reaching and maintaining collusive agreements easier - the number of competitors, the market transparency, cost symmetry and product homogeneity. Unfortunately, with the concentration index exception, in applied economic analyses as well as in antitrust practices, the reference to these variables is generally vague, as it is not supported by specific indicators. In particular, in order to evaluate the collusive risk associated with concentration, in most of the relevant cases only descriptive analyses have been employed. The collusive propensity of a market has been appraised without using any of the analytical tools which are common in the estimation of unilateral effects. However, these techniques could prove to be very useful when dealing with two of the above-mentioned variables: costs asymmetries and product differentiation.

In the event of cost symmetry and homogenous products, achieving a cartel agreement is relatively simple, as the division of the collusive extra-profits (following the familiar joint profits maximization) becomes uniform. In this framework, the number of competitors and the concentration index probably play a major role in evaluating the collusive impact of a merger. But in the other cases, reaching and maintaining a collusive agreement prove to be strongly dependent on the specific demand and cost structure (from which incentives to cooperate or to deviate are derived). It is by modifying this structure that a merger can be either pro or anti-collusive. Therefore, the demand and cost estimation must provide useful information on the probability of collusion among firms before and after a merger. In this paper we will show how to develop such an analysis.

First we review the main methods currently used by antitrust authorities (or proposed by experts) to evaluate the coordination effect of a merger. Secondly, we identify the main variables that should influence the probability of collusion and the manner in which they can be modified by concentration. We complement this analysis by devoting particular attention to the collusive scheme adopted by a cartel. What emerges is that the Friedman equilibrium, based on the balanced temptation assumption, is not at odds with the expected behaviour of cartels and has some suitable features for being used in the analysis. In particular it is unique and allows us to monitor one single parameter – the common critical discount factor – in order to ascertain the stability of a cartel. So we have all the ingredients for delineating the procedure to be followed, in order to predict a coordinated effect. In short, this procedure is based on a double simulation (*ex ante* and *ex post*) of the Friedman equilibrium. We then show, through numeric examples, that this equilibrium accommodates, as the Nash solution does, the demand and cost asymmetries, so as to immunize the critical discount factor (an indicator of cartel stability) from their changes due to a merger. However the degree of differentiation, but only in the Chamberlin sense, can affect stability, a result which is coherent with former contributions, although obtained in a different framework. It follows that a merger which shows a substantial unilateral effect can also have, by increasing the “average distance” among products, a strong coordinated effect. In concluding the paper we discuss the main assumptions on which this procedure rests.

2 - Empirical methods to identify the coordinated effect

The traditional way of assessing the coordinated effect of a merger has long been to make use of a check list of elements – as originally listed by Stigler – considered relevant to the probability of collusion. Thanks to recent developments in cartel theory this approach has been further refined, giving rise to what has been defined as the *Consensus-Detection-Punishment* paradigm. This means verifying whether the market conditions make it possible to reach an agreement, and whether they also allow for monitoring of the cartel members’ behaviour and subsequent punishment should some deviation occur[3]. In practice, empirical methods applied to identify coordinated effects have been focussed on a narrow number of parameters: the number of competitors, price parallelism, deviant behaviour, some sort of asymmetry (capacity, market shares, costs, etc.) and product differentiation.

The ***number of competitors*** (and its reduction due to a merger) is generally considered the major cause for a coordinated effect to arise. It is difficult to disagree with Stigler that this number must play a crucial role in the probability of reaching a collusive agreement, as the more firms in the market, the more difficult it is to find consensus on a specific price or quantity structure. A second line of causation is based on the stability of cartels, which also seems negatively associated with the number of colluders: the larger this number is, the higher must be the profitability, and therefore the probability of cheating (Tirole, 1990,pp.247-248: Ivaldi-Jullien-Rey-Seabright-Tirole, 2003, p.p.12-14). Nevertheless when looking for an exact relationship between number of competitors and probability of collusion so as to forecast the increase in this probability with one less firm in the market, we experience difficulties. By the way this is the fundamental question in an antitrust merger case.

[3] Airtour/First Choice case is the most important European case law which refers rather extensively to this paradigm.

Beyond the provocative article of Selten (1973), there are no contributions to this specific issue. Also the Stigler intuition, although 40 years old, still awaits formal proof, not surprisingly considering the propensity of current analysis to investigate the stability of cartels rather than their formation. Also the argument that connects the number of firms to the cartel stability, although it has some merit, is not conclusive and must be integrated with other analyses. Otherwise the theory risks some peculiar results in that it attributes the maximum coordinated effect to those mergers which involves the smallest firms. The change in the number of the competitors must jointly be valued with the other modifications produced by the concentration and therefore does not deserve separate analysis.

Parallelism - In the practical applications of the method (Colemann-Sheffmann, 2003 and Colemann-Meyer-Sheffmann, 2003) the analysis adopted is essentially descriptive of the market environment, particular attention being paid to the propensity for collusion as manifested in past behaviour. Essentially, this is a matter of assessing the degree of parallelism in terms of prices, quantities and production capacity characterising the market in the period immediately preceding the concentration. Concrete applications of the method also find room for econometric techniques which, unlike those based on assessing unilateral effects, make broader use of the temporal dimension of the data, with regressions in time series rather than in panel or cross section. This is of course fairly obvious, given that the object of investigation is parallelism in behaviours over time.

Generally speaking, such analyses should prove particularly effective in cases where a collusion-prone environment existed before the concentration occurred. Here further reduction in the number of competitors can only increase the risk of coordination. In the other cases, where analysis reveals a lack, both of structural conditions favouring collusion and parallelism in behaviours, investigation can hardly be considered exhaustive. The reason for this is simple enough, and recognised also by those who advocate the approach[4]: the analysis fails to pinpoint changes which will be caused by the concentration. Thus, the chances are very high that analysis of this type will not be able to identify those cases in which concentration transforms a market previously characterised by pro competitive conditions into a market typified by pro collusive characteristics.

The maverick firm – Clearly, a more promising approach, less subject to the limitations of the previous method, involves identification of a maverick firm, which generally disrupts the collusive equilibria. Should such a firm become involved in a concentration, the consequence would evidently be greater harmony among the remaining firms on the market. This approach has been acknowledged in the US Merger Guidelines since 1984, but it has only recently been theorized and, to some

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4 "*The analysis of current competition will show what other impediments to coordination exist (...) The question then arises whether the merger would change this outcome. We have not done much analysis in these areas and encourage more work to be done*" (Scheffmann-Colemann, 2003, p.21)

extent, also exemplified with reference to concrete contexts (Baker 2002). This approach is based on the observation that cartels are generally not perfect (i.e. not set on the objective of maximising joint profits) but they are imperfect, incomplete, and converging on focal points according to the interests of the various participants. Given such a picture, an important role is taken on by the firm that has the following two characteristics: a) different product positioning or cost structure from that of competitors; b) enough economic weight to inhibit a collusive agreement in which it has no part. This firm, usually smaller than the market leader, typically displays different behaviour from that of its direct competitors. Thus, it will prevent the formation of a cartel or at any rate condition the way it is formed and managed[5].

On the practical level, the first thing to do is to verify whether a maverick firm is present before the concentration, and secondly to ascertain the impact of the concentration on the behaviour of this firm. The impact will be maximum if the firm is directly involved in the merger. In such a case we could expect a sort of reorientation of the maverick firm behaviour as it, after the merger, must take in consideration the interests of the group which it belongs to. A pro-collusive effect could also emerge as a consequence of the modifications generated in the incentive structure when two non-maverick firms merge. Baker's analysis seems to represent a move forward from the above-mentioned one in that it takes into account the way in which the merger modifies the structure of incentives of firms.

Symmetry of capacities – Baker's method is based on ascertaining asymmetry between a firm and its competitors, which is a sufficient circumstance to make collusive agreement hard to achieve. However, this analysis fails to raise any questions about asymmetries which can be associated with capacities, costs and demand.

As to capacity, in an article prompted by a well-known European Community case (Nestlè-Perrier), Comte-Jenny-Rey (2002) examine the distribution of productive capacity between competitors and how it can affect the probability of collusion in the market. The conclusions they reached are somewhat surprising in that in certain circumstances it is actually the asymmetry between productive capacities that proves the catalysing force for collusive agreements. Let us see the reasoning for

5 Baker gives the example of the American airlines tariffs. During the year 2000 Continental Airlines took the initiative to apply a fuel surcharge of $ 40 and $ 20, respectively for flights of more and less than 1000 miles. Five other companies immediately imitated Continental but not Northwest, American West or Southwest. This fact proved to be decisive in forcing Continental to abandon the price increase. Subsequently, Continental repeated its attempt with the same consequences. After the second attempt, Northwest launched an increase in the fuel surcharge by the same amount but modulated it in a different way (not according to distance but to the distinction between tourist and business class fights). This increase was immediately applied by the other companies (with the exception of Southwest) and remained in effect in the successive period. This example - according to Baker - shows quite clearly the role a maverick firm can play in conditioning the choices of competitors.

their findings. The probability of collusion depends on two mechanisms: a) the incentives to cheat, or in other words the profit to be made by deviating from the collusive agreement; b) retaliation capacity. When the productive capacity is relatively modest (as compared with aggregate demand) deterrence is reduced, and the major factor thus becomes the incentive to deviate. In such a situation, therefore, greater asymmetry between productive capacities reduces the risk of collusion (accentuating incentives to deviate). When, on the other hand, the productive capacity proves far greater than the aggregate demand (in particular when any subset of firms is able to cover the entire demand) deterrent capacity is at the maximum. In this case, if the major firm is able to cover the entire market, then the transference of productive capacity to it from minor firms can only reduce the overall incentives to deviate. In fact, the incentives for the minor firms decrease while showing no increase for the major firm, which was already able to cover the market. Thus, in these circumstances an increase in the degree of asymmetry proves to be pro-collusive. Basically, the argument rests on the assumption that incentives (for the biggest firm) to cheat are not affected by increased availability of productive capacity.

Symmetry of product number - Khun-Motta (2000), Motta (2004) and Khun (2004) have presented models which, while being different for the analytical treatment (different demand functions and punishment mechanisms), are mainly convergent as regards the economic environment (differentiated products), the type of asymmetry which has been considered (number of products of each firm) and the results (pro-collusive nature of the symmetry of product ownership).

First of all, notice that, unlike the previous approach, collusion is evaluated in a differentiated products environment. However, the use of symmetric demand functions (so the "distance" among products is always the same) mitigates the consequences of this choice. Under this assumption the only difference among firms is in the number of products. Secondly, in these models the problem of reaching a collusive agreement (and of finding the right price for everybody) is not properly discussed. The rather standard, but in this context implausible, assumption of joint profit maximization is adopted. The results obtained by Motta and Khun can be summarized as follows.

The firm which holds the greater number of products has little incentive to deviate from the collusive agreement as it is the greatest beneficiary. This conclusion is based on demand externalities and in particular on the fact that the product price increase produces a positive impact on the margin of the others. So the one who holds the greatest number of products is also the one who obtains the largest share of profits from collusion. Instead, for the same reason, the largest firm has less incentive to repress deviant behaviours (his repression cost is higher than that of competitors). Therefore, the stability of a collusive agreement is limited on the one hand by the incentive to cheat, by the smallest firm, and on the other hand by the incentive to punish defection, by the largest firm. The more symmetric the firms (with respect to the number of products) the more such limits are slackened, the higher the probability of collusion and the higher the associated profits.

The prescriptions of the model in terms of merger control are evident. If the concentration involves a more uniform product distribution a pro-collusive effect is produced. This conclusion brings the authors to point out the strong contrast between single (that is, unilateral effect) and collective (that is, coordinated effect) dominant position. Each concentration which causes the reinforcement of the one automatically weakens the other.

Cost symmetry - A merger can modify the cost structure of a market so as to make it more or less asymmetric. This could be the consequence of vertical as well as horizontal mergers when efficiency is improved (e.g. by allowing a firm to have access to technology which is used by the other competitors, a case of symmetry increase). However, we don't have to assume a change in the cost structure for a merger to have an effect through costs. Simply by combining firms with different costs, a merger could modify the incentive structure which controls the probability of collusion. It is a widely shared opinion that this asymmetry makes collusive agreements more difficult to achieve and maintain[6]. One paper (Rothschild, 1999), for instance, shows that cost asymmetry has a clear anti-collusive impact. This result is obtained by employing a *grim trigger* model and assuming that cartels maximize joint profits. But it is sufficient to remove this last assumption and adopt different models of collusion, like the Nash bargaining solution or Friedman equilibrated temptations (Bae,1987; Harrington, 1991), to obtain very different results (Collie, 2003): cost asymmetry is no longer a factor that hinders collusion but rather favours it.

Product differentiation - The opinions of economists on collusion and differentiated products are still relatively inconclusive. The initial Stigler observation, concerning the difficulties in reaching an agreement, reigned uncontested for decades until it was opposed by another kind of consideration tending more towards evaluation of cartel stability. In particular, several models (Deneckere, 1983; Ross, 1992; Chang, 1991) have reached the opposite conclusion, that the stability of the collusive agreement is generally strengthened by product differentiation because of smaller incentives to deviation (see also Martin,2002, and the literature quoted there). The intuition is the following: the "*distance*" among differentiated products is higher in comparison to the case of perfect homogeneity. It follows that profits from cheating are lower, making collusion more stable. It is true that punishment of deviant behaviour is less severe with differentiation as Bertrand competitive equilibrium allows, in this case, firms to retain positive profits. But this second effects is dominated by the first.

6 Here the reference is to the *Premdor/Masonite* case investigated by the US Department of Justice. As a consequence of this vertical merger the cost structure of the two firms remaining in the market would have become similar, so as to weaken an important obstacle to co-ordinate behaviour.

However, all these models make the crucial assumption of symmetric demand function so that – both in the case of the competitive Nash equilibrium and in the case of optimal collusion obtained by maximizing joint profits – prices, profits and market shares of all firms are the same. Furthermore, this feature does not permit one to deal with the main problem (the one stressed by Stigler) which firms have to solve in order to reach an agreement, namely, how to share collusive gains in an equitable way. The choice of symmetrical demand functions (also common to Motta and Khun models, as already stressed) makes questioning the realism of the joint profit maximization assumption employed in all these contributions, redundant. With symmetrical demand, and with symmetrical costs as well, the collusive profit shares are in fact uniform and the problem, dear to Stigler, of the inherent difficulties in reaching an agreement doesn't emerge. It is only when demand is not symmetric that an agreement to increase prices can generate very unequal profit changes so that firms, unable to resort to side payments, must solve the not negligible problem of identifying that price (or price structure) on which a broad consent would converge. The results of these models are therefore inconclusive. It is certainly not by chance that the only contribution (Häckner, 1994) which deals with asymmetry (within a model of vertical differentiation) gets opposite results, establishing a clear negative relationship between the degree of differentiation and cartel stability. It must be noted, however, that the result was reached thanks to the assumption of joint profit maximization, not very plausible in a context of strong asymmetries[7].

Looking at antitrust intervention we obtain contrasting evidence. On the one hand it is certainly true that cartels with differentiated products are very rare. The US DOJ, for instance, has not prosecuted this sort of cartel for many decades. On the other hand, however, turning to cases where the coordinating effect of a merger has been carefully scrutinized [8], we discover that they have almost always concerned markets with differentiated products. Suffice to consider the cases dealt with by American jurisprudence (cruises, breakfast cereals and food for children[9]) not to mention the most famous case of collective dominant position (that is, coordination effect) dealt with by the European Commission, which concerned a widely advertised market (mineral water).

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7 The author seems however to be perfectly aware of the problem. He discusses the possibility of adopting a different rule of collusive profit sharing (e.g. Nash bargaining solution) but he rejects it because it seems to be analytically intractable (Häckner, 1994, p.161).

8 It must be remembered, however, that most of them were primarily unilateral effect cases.

9 The *New York/Kraft General Food* case is quoted by some commentators (Dick, 2002 and 2003) in relation to the judgement of the Appeal Court which based its verdict in favour of the concentration on the degree of product differentiation. The *FTC/H.J.Heinz* case is still more revealing because the US Federal Trade Commission (FTC) was able to show, on the basis of the business plan, how the concentration would have reduced the product variety in the market.

3. How the probability of collusion is modified by a merger

In the light of economic literature and antitrust practices it is not clear what role is played by the most important variables (on cartel formation and stability) because the results seem strongly conditioned by the collusive scheme employed by colluders. This problem will be dealt in the following section. Let's divert our attention to the specific effects which mergers could induce on collusion. We detect three different classes of effects. The first one deals with asymmetry, the second with differentiation (*à la* Chamberlin) and the third with the delimitation of the set of cheating profits.

Asymmetry can be modified by a merger in two different ways. It gives rise to a sort of "*average effect*". Assume a wide distribution of firms along cost or demand characteristics. A merger between two firms located at the opposite boundaries of this distribution could increase the homogeneity of the market.

There is another consequence of a merger on the degree of symmetry and on the probability of collusion - we could define it the "*absorbing effect*" - that emerges thanks to the unilateral impact of the operation. Let us imagine, for instance, that before the merger two firms had benefited from collusion more than others, because of the high reciprocal cross elasticities: therefore, by colluding, they would have increased prices more than other competitors thanks to the mutual compensation of the demand losses. We assume that the unequal distribution of the advantages had constituted an obstacle for reaching a collusive agreement. A merger between these two firms would clearly have, because of the high cross elasticities, a strong unilateral effect. At the same time, because the *ex ante* closest firms are now only one unit, the degree of asymmetry among the firms in the market must decrease thereby increasing the incentive to collude.

The second class of effects also materializes in connection with the unilateral effect. When two merging firms are very close in the product space, we can expect not only a strong unilateral effect but also an increase in the average distance among all firms belonging to a market. So, if it is the case that the degree of differentiation (*à la* Chamberlin) plays an important role on cartel stability (because it lowers the probability of cheating), we must expect concomitant unilateral and coordinated effects.

The conclusions of the former two paragraphs is not familiar to antitrust practitioners[10]. Generally they don't recognize any positive relationship between unilateral and coordinated effect. In Europe there are no cases of mergers vetoed because of single dominance which were also forbidden for collective dominant position (the legal European "translation" of the notion of coordinated effect). In the United States, where the standard of evaluation (substantial lessening of competition) leaves room for an overlap between the two effects, no special attention has been devoted to this relationship[11]. The preceding arguments show the fallacy of this

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10 We have already mentioned the point of view of Comte-Jenny-Rey and Kuhn-Motta.

11 See, for example, the recent paper by Coleman,M-Scheffman,D (2003).

opinion. In some cases a greater collusive risk arises as a consequence of the unilateral effect of a merger.

We come now to the final class of merger effects on collusion. When two firms merge they join their collusive profit but do not join the incentives to deviate to the same extent. Profits from deviation depend on the additional demand which a firm can capture by lowering its collusive price. After a concentration there is one less firm (the merged one) from which the merging firm can attract this additional demand. We can expect therefore that a merger must always have a positive influence on cartel stability, because, for the new entity, cheating profits do not sum as much as collusive profits do. The smaller the number of competitors, the higher this effect is. It is therefore evident why the number of firms influences the stability of cartels.

In order to measure all these effects we must employ an analytical framework able to deal with the not cooperative stage (before the merger) and with the collusive phase (after the merger). To do this we resort to three main behavioural assumptions about: 1) how firms compete before the merger and after it (if they do not collude); 2) what cartels maximize; 3) the incentive compatibility constraint which preserves the cartel stability.

Very standard answers to the first and third questions can be provided. We are going to assume that before the merger firms compete according to the Bertrand model. This assumption seems appropriate for the case of differentiated products we are considering here[12]. Then we must handle the problem of finding the conditions that preserve cartel stability, that is, the incentive compatibility constraint which excludes the possibility of cheating. Two classes of models generally deal with this problem. The first type, the so-called "trigger strategies" model, assumes that the punishment for cheating consists of all firms restoring the competitive equilibrium forever (Friedman, 1971). The second type of model (optimal penal codes) is a combination of stick (punishment) and carrot (collusive profits) strategies (Abreu,1986; Abreu-Pearce-Stachetti,1986). In this article we will examine only the first type of models.

Each collusive agreements is characterized by a set of parameters $(\delta_1, \delta_2, \ldots, \delta_i, \ldots, \delta_n)$, one for every cartel member, which represents the critical discount ratio of each firm. Such a variable specifies the minimum discount ratio (used to evaluate future collusive earnings) which allows collusive behaviour. For each firm this ratio is calculated on the basis of the following relationship:

$$\delta_i = ({}_i\pi^s - {}_i\pi^c) / ({}_i\pi^s - {}_i\pi^b)$$

where:

${}_i\pi^c$ = collusive profit of firm *i*

${}_i\pi^s$ = profit which firm *i* obtains by deviating from the collusive agreement

${}_i\pi^b$ = competitive (Bertrand) profit of firm *i*

12 In the case of homogeneous products (a case we don't deal with in this paper) it would be better to resort to the Cournot model.

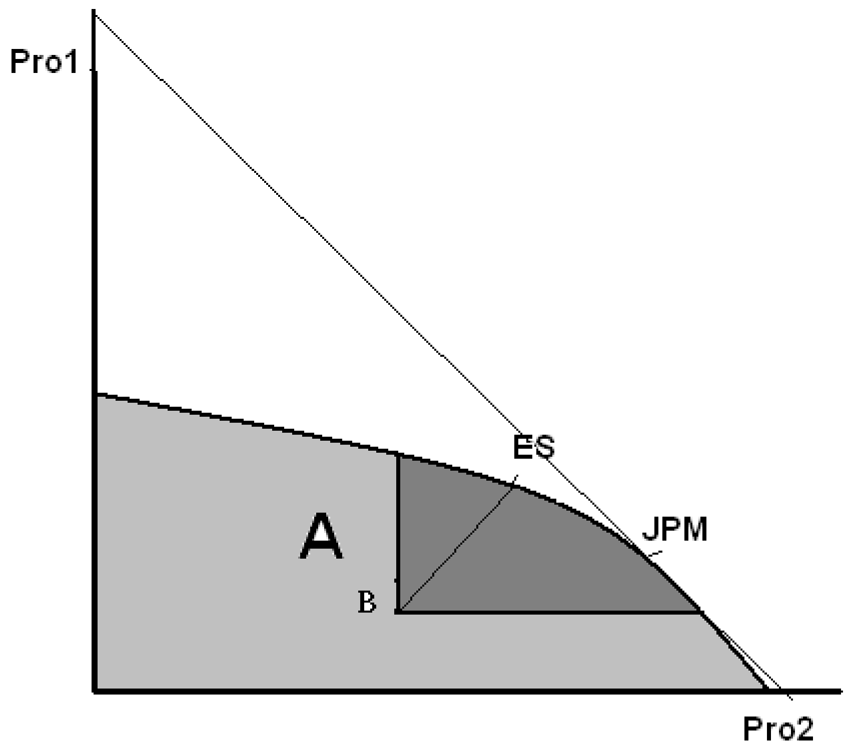
Such parameters therefore constitute an indicator of cartel stability: the lower they are, the higher the cartel stability. The plurality of the parameters - as with the plurality of possible collusive equilibria – represents a problem for the evaluation of the merger coordinated effect, unless a merger modifies all these parameters in the same direction, a very implausible hypothesis. We show how to deal with this problem in the following section.

An answer to the second question – what does a cartel maximize? – must be less standard. It is a widely shared opinion that cartels maximize joint profits of colluders. There are however good reasons to reject this view and resort to different, more plausible rules for cartels. The next section is devoted to the discussion of this point.

4 – What does a cartel maximize?

There will generally be many possible collusive agreements, each allowing cartel members to increase their profits in comparison with the competitive profits. In the case of two firms, we define the set of all possible collusive profits as *A* (Chart. 1).

Chart 1 – Set of possible collusive profits



The upper right boundary of this set represents the so-called "Pareto efficient frontier", that is, the points that constitute the maximum achievable profit of a firm for every level of profit of the other. The point "*B*" corresponds to the profits obtained in absence of collusion. The dark grey area corresponds to a subset of *A* where collusive profits of both firms are higher than competitive profits.

To evaluate the coordinated effect we need a theoretical model which clearly prescribes a unique collusive equilibrium and the stability of which can be inferred by looking at only one parameter which measures cheating probability. If we don't find a theory which satisfies the property of having only one collusive equilibrium configuration out of set *A,* we would have some difficulty in appraising the coordinated effect of a merger: as a consequence of a merger some possible collusive configuration could be easier to reach and maintain but others more difficult. In short, a merger would have conflicting effects on the probability of colluding. Without the reference to a specific and unique collusive equilibrium every possibility to foresee a merger coordinated effect would irremediably be jeopardized. We can imagine three different rules alternatively been adopted by a cartel: 1) joint profit maximization; 2) equalitarian solution; 3) stability preserving rule.

Let's start by considering the rather innocent assumption that colluders fix prices in order to <u>maximize their joint profits</u>. According to the *Chart 1* this solution corresponds to the JPM point located on the Pareto line. Such assumption, still popular in recent literature[13], is generally attributed to an article by Patinkin (1947). Although apparently intuitive, this hypothesis is generally not correct. As Bain observed, in a prompt comment on Patinkin's article (Bain, 1948), "perfect collusion" (that is, maximization of joint profits) is not an option whereas firms have different costs and side payments are not allowed. The reason is intuitive. Joint profit maximization generates, with different costs, an unequal distribution of collusive earnings (in the case of homogeneous products, the strict adherence to joint profit maximization principle implies that only the most efficient firm remains in activity). In our example the joint profit maximization makes firm 2 better off compared to firm 1. It could also be possible to give an example where joint profit maximization shrinks the profits of some firms with respect to competitive equilibrium. This rule is therefore unfeasible. Bain reaches the conclusion that sub-optimal collusive agreements, which he doesn't specify, are more plausible. With reference to oil cartels Bain observed:

> *"Conservationists [that is those people who want to preserve oil reserves] wish to determine individual quotas according to engineering considerations, so as to maximize the yield from an entire pool. But considerations of "equity" – i.e. the assumed right of each firm to produce and sell an "equitable" share of the output drawn from the common pool - dominate the allocation. So long as each firm is left dependent for earnings on its own output [i.e. there are no side payments], as*

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13 See, for example, the already mentioned articles by Chang (1991), Ross (1992) and Häckner (1994).

it is in absence of "unitization", output allocation must be dominated by "equity" and pay only secondary heed to considerations of conservation and of aggregate costs" (Bain, 1947, p.621).

The case taken into examination by Bain refers only to homogeneous products, but the logic of the argument is more general and it also extends to collusion with differentiated products. The only way to maintain the hypothesis of joint profit maximization is therefore to assume a complete symmetry (market shares, costs, demand) among colluders. If so, joint profit maximization is a plausible assumption. By the way all other rules (belonging to the remaining two categories) generate the same results. The case of perfect symmetry cannot be excluded but we should be rather sceptical about this possibility.

With asymmetries the only possibility for colluders to adopt a joint profit maximization rule is to resort to side payments. But this possibility is highly hypothetical. First, antitrust legislation, which is now widespread all over the world, makes it very difficult for colluders to sustain a system of side payments. But there is a much deeper reason for not considering this possibility plausible. Collusion, by its nature, is a temporary phenomenon and there is no reason for firms not being aware of it. Each firm will only choose to collude if its relative position with respect to the competitors does not worsen. If, for instance, a firm is able directly to capture by colluding a greater share of the extra profits (which are then redistributed to the other colluders), it will turn out to be in a better position to compete with the other firms when the cartel breaks down. That is why the other firms will be reluctant to accede to this agreement. In the words of Friedman:

In an abstract single-period model there is no objection to making side payments which redistribute profits: however in a fuller model with a long time horizon a perhaps some kind of consumer loyalty or explicit capital stock, an agreement which has some firms producing little or nothing may put them in a atrophied position over time. Then it later becomes possible for the others to reduce the subsidy to this group. (Friedman, 1977, p.29)

The <u>egalitarian solution</u> seems a natural way to overcome the limits of the former rule. Following this sharing method firms obtain an equal portion of the collusive extra-profits. This rule is compatible with many solutions, each belonging (in our example of two firms) to the bisector line which departs from the competitive equilibrium. The most profitable solution is obtained at the intersection between this line and the Pareto frontier (ES, in *Chart 1*).

Despite the apparent simplicity, this rule is very difficult to implement as it requires a common knowledge of cost and demand characteristics of all products. Furthermore it seems a very extreme, and implausible solution to the former problem: why should weaker (in terms of costs or quality of products) firms obtain the same share of collusive extra-profits than the most successful and efficient firms? In the very end this solution reveals the same flaw as the other namely, instability. The first scheme is opposed by the weakest firms, the second by the strongest: every cartel following one of these two rules will soon be defeated by cheating.

The third class of possible collusive schemes – based on stability preserving rules – is in between the two former rules. As suggested by empirical literature we know that cartels generally adopt some rather simple rules which allow their members to obtain a share of cartel profits which is in proportion to their competitive profits: common price increases or uniform reductions of quantities usually belong to this category. In order to find a plausible unique collusive equilibrium, suitable for detecting coordinated effects, we will concentrate our attention on a couple of formal models which belong to this category: the Nash bargaining solution and the balanced temptations model by Friedman.

The solution proposed by Nash refers to a cooperative game which deals with the problem of distribution of the gains from negotiation. The problem to which Nash answers is therefore more general than how to reach a collusive agreement and the applications of the Nash model to cartels are relatively and surprisingly limited: to my knowledge there have been only the articles by Schmalensee (1987) and Harrington (1971), dealing with homogeneous products, and the article by Fershtman-Pakes (2000), which refers to the case of differentiated products. These authors, well aware that the maximization of joint profits is implausible, decided to employ instead a Nash bargaining solution.

The Nash approach is based on several assumptions, the most important being: a) equilibrium must be on the Pareto frontier; b) symmetry; c) independence of the irrelevant alternatives. Thanks to these assumptions Nash shows (in the case of two players) that the cooperative equilibrium is reached maximizing the following function ("Nash product"):

$$(U_1 - U\hat{}_1)\,(U_2 - U\hat{}_2)$$

where: U_i = utility of agent i in case of agreement
$U\hat{}_i$ = utility of agent i without agreement

It appears (Mas Colell-Whinston-Green,1995, pp.842-843) that such solution is in an intermediary point between a solution that maximizes total utility and an egalitarian solution that evenly shares gains from negotiation. It has been observed that Nash bargaining equilibrium can be interpreted as the end of a process of mutual concessions. The firm with the greatest gain from negotiation is the one which gives concessions; each concession makes the Nash product higher. That is why the end of the process corresponds to the Nash equilibrium (Harsanyi, 1977 and 1987).

The transposition of the Nash solution to a cartel is immediate and is drawn by substituting utilities with profits. The threat point, which corresponds to the level of profits achieved in absence of collusion, is the Bertrand equilibrium (with differentiated products) or Cournot equilibrium (with homogeneous products).

The Nash solution allows us to solve the problem of the multiplicity of collusive equilibria but not that of multiplicity of parameters which control cartel stability. We can easily verify that in the Nash bargaining equilibrium firms have different critical discount thresholds (δ). These thresholds are, however, contained within a narrow interval. Such an interval shrinks as costs or demands become more similar. With symmetrical demands and costs, the Nash solution generates the same critical discount threshold for all firms. It must also be noticed that with demands and costs becoming more symmetrical the γ parameter (defined here after), which locates the equilibrium point on the Pareto frontier, converges to 0.5.

Friedman (1971) supplied another solution to the multiplicity problem. Differently from Nash, he employed a non cooperative game and focused on the cartel stability instead of the cartel formation; moreover he didn't impose, as Nash did, the independence of irrelevant alternatives assumption. But similarly to Nash, the collusive Friedman equilibrium is on the Pareto frontier and is based on the symmetry assumption. The solution emerges by equalling the critical discount thresholds of all firms:

$$\delta_1, = \delta_2 = \delta_i = \delta_n = \delta^*$$

that is

$$({}_i\pi^s - {}_i\pi^c) / ({}_i\pi^s - {}_i\pi^b) = ({}_j\pi^s - {}_j\pi^c) / ({}_j\pi^s - {}_j\pi^b) \quad i \neq j$$

Where π^s = cheating profit
π^c = collusive profit
π^b = Bertrand profit

By adding this constraint to the Pareto frontier condition we are able to get the collusive prices for all the products. The Pareto frontier is obtained by maximizing a weighted function of joint profits. In the case of two firms, each producing only one product, the average profit function is:

$$\pi_t = \gamma * \pi_1 + (1-\gamma) * \pi_2$$

The prices on the Pareto frontier are a function of γ and are obtained thanks to the two first order conditions:

$$d\,\pi_t / d\,p_1 = 0 \quad \text{and} \quad d\,\pi_t / d\,p_2 = 0$$

The Pareto frontier is shaped by varying γ in the interval (0,1).

The satisfactory property of the Friedman solution not only rests on the uniqueness of equilibrium but also on the fact that it generates only one parameter (δ^*) which controls the cartel stability. It therefore can be used as an appropriate indicator for measuring merger coordinated effects.

The Friedman solution could also be interpreted as a cooperative game in which firms make mutual concessions in order to arrive at a satisfactory collusive equilibrium. We could describe this game in the following way. The game starts with a firm proposing a collusive price structure which is associated with specific collusive and cheating profits for each firm. We expect that the firm with the lowest critical discount (δ) is the most inclined to make concessions (that is, to change the initial collusive prices structure in order to increase collusive profits of other firms). That is because this firm attaches the greatest importance to the agreement. On the contrary, a firm with a relatively high critical discount ratio will benefit, compared to others, more from cheating than from colluding. It therefore will show a relatively poor commitment to the agreement and thus will be more reluctant to make concessions. So the firm with the lowest discount ratio changes its price in order to decrease the critical ratio of the others. The process of mutual concessions ends when all incentives to collude are equal. There is no reason why this point is on the efficient frontier of Pareto. We suspect that in order to obtain the Friedman equilibrium the process of mutual concessions must start from a point already located on this frontier.

As Friedman himself noticed, the equilibrium of balanced temptations turns out to be very similar to that of Nash, because of their common assumptions. This appears to be confirmed by the results obtained by Harrington e Bae[14]. In practical terms, using linear demand functions and constant marginal costs, the difference between Nash bargaining profits and BTE are of second order. Moreover the discount critical factor obtained with the equilibrium of balanced temptations is always inside the interval of variation (particularly narrow) of critical factors obtained with the Nash solution.

These considerations markedly improve the appeal of employing the balanced temptations equilibrium in that the justifications advanced by Friedman can be complemented by other motivations which make the Nash solution reasonable.

4 - Procedure to follow for controlling the coordinated effect of a merger

We now have all the ingredients for a procedure that allows us to simulate the coordinated effect of a concentration. We accomplish this task on the basis of three fundamental assumptions: 1) Bertrand competition, when firms follow a not cooperative strategy; 2) Freedman balancing temptation equilibrium, when firms shift

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14 Harrington,1991,p.770.

to collusion; 3) grim trigger strategies for preserving the cartel stability. Our approach allows us to detect a coordinated effect generated by one of the several causes mentioned in paragraph 2 by looking at evolution of only one parameter: the common critical discount ratio. Our procedure is based on three distinguished stages. The implementation of these needs a preliminary inquiry on demand and cost functions, which we will assume, therefore, are known to the researcher.[15]

1° stage – It is necessary to estimate the balanced temptations equilibrium for the *ex ante* market structure. Notice that critical discount ratios must be calculated for each firm and not for each product. By equalizing the ratios of all firms we will then be in a position to calculate the collusive prices and profits. Such estimation will provide useful indications on the market's degree of symmetry (parameter γ) and on cartel stability (parameter δ *) before the concentration.

In particular, we consider the case of *M* firms, each producing n_i products, the total number of products sold in the market being N (= $\Sigma\, n_i$). The profit function of the generic "j" firm is:

$$\Pi_j = (p_{j1} - c_{j1})^* q_{j1} (p_1 \ldots p_N) + (p_{j2} - c_{j2})^* q_{j2} (p1 \ldots p_N) + \ldots + (p_{j\,nj} - c_{j\,nj})^* q_{j\,nj} (p_1 \ldots p_N)$$

The weighted total profit function is:

$$\Pi_t = \Pi_1 * \gamma_1 + \ldots + \Pi_J * \gamma_J + \ldots + \Pi_m * (1 - \gamma_1 - \gamma_2 - \ .. - \gamma_{m-1})$$

$$\text{subject to: } \gamma_J > 0 \text{ and } 0 < \Sigma\, \gamma_J < 1$$

We calculate the N first order conditions to obtain the Pareto frontier constraint:

$d\Pi_t / dp_1 = 0$

…….

$d\Pi_t / dp_i = 0$

…….

$d\Pi_t / dp_N = 0$

Then we add the equations imposing the constraint of equal critical discount ratio:

$\delta_1 = \delta^*$

………..

$\delta_j = \delta^*$

……..

$\delta_m = \delta^*$

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[15] It is common practice to extract information on costs directly from the competitive equilibrium conditions.

where the generic δ_j parameter is the ratio:

$$\delta_j = ({}_j\pi^s - {}_j\pi^c) / ({}_j\pi^s - {}_j\pi^b)$$

Friedman equilibrium is found by solving the system of *m* + *n* unknown *(the* m-1 γ_j parameters, δ^* and the n product prices) and *n* + *m* equations (from Pareto and Friedman constraints).

2° stage – The unilateral effect of the merger must be simulated. As it is now common practice such examination is implemented by an evaluation of the competition equilibrium based on the market structure which will emerge from the concentration. We are therefore in a position to use the new structure of prices and profits coherent with Bertrand competition for the following analysis.

The unilateral effect is simulated in the standard way by assuming Bertrand competition (with differentiated products). Let's shall, for example, consider a merger between the first two firms. The profit function of the new firm will be

$$\Pi_c = \Pi_1 + \Pi_2$$

Prices, due to the unilateral effect of the merger, are obtained by solving the following system of first order conditions,

$$d\Pi_c / dp_1 = 0$$
$$d\Pi_c / dp_2 = 0$$
$$d\Pi_3 / dp_3 = 0$$
…….
$$d\Pi_i / dp_i = 0$$
…….
$$d\Pi_N / dp_N = 0$$

Limits of merger simulations are well known. Not only because the results crucially depend on assumptions not easily verifiable. But also because simulations generally focus on short term effects (on prices and quantities) neglecting to pay the due attention to long term effects of mergers such entry or repositioning. In a way this paper is an attempt to provide an answer to this criticism by exploring some features of oligopolies in a protracted time dimension. Other recent contributions have thrown some lights on repositioning (Ghandhi-Froeb-Tschantz-Werden,2004) and entry process (Pakes,2006).

3° stage – We re-estimate the balanced temptations equilibrium with the new structure of expected prices (as changed by the merger unilateral effect). From this examination we get the new critical discount ratio of the market. If it is lower than the one previously calculated, the concentration has a pro-collusive effect. We can also control the parameter γ to assess whether, because of the merger, the market structure has become more or less symmetrical (regarding costs and demand).

It could also be advisable to implement an additional check. We might ask whether the common discount ratio is in a range which is coherent with the specific characteristics of the market under analysis. It might be the case that, as a consequence of the merger, the critical common discount ratio decreases but it is still too high to justify a collusive attitude in that specific market. In this case the incentive compatibility constraint is not fulfilled. Therefore, with this control we can avoid a false positive error. Davis (2005), which adopts a methodology for detecting coordinated effects which is very close to the one used here[16], attributes to this check a pivotal role. But we have some doubt on its self-sufficiency. We should not forget that the discount ratio is not so easy to calculate, as it depends not only on accounting data or stock exchange prices which can be easily collected but also on more vague information about subjective perceptions of firms. I also wonder how we could deal with the case, not implausible, of a pre-merger critical ratio already compatible with collusion.

An example of the procedure can be found in the Appendix. Notice how close the Nash equilibrium is to the Friedman equilibrium. Also notice the positive relationship between coordinated and unilateral effect, a result which we are going to discuss in the following section. And finally notice the positive impact of the merger on the cartel stability due to the shrinking of the cheating profit set. As a consequence of a merger, the critical ratio of the new entity is always lower than the *ex ante* ones. If firms collude by following Friedman equilibrium, this effect also depresses the common critical ratio, making collusion more stable.

5 - Balanced temptations equilibrium and asymmetry

For the purpose of evaluating whether the selected indicator, i.e. the common critical discount ratio, can usefully be employed for appraisal of the coordinated effect of a concentration, we now examine how it works in different market environments. Because this ratio is calculated on the basis of several components, it is not easy to handle the problem analytically. Rather we will appraise the relationship between this parameter and the degree of cost or demand asymmetry by looking at some numerical examples. However, if the analysis is restricted to linear demand functions and constant marginal costs, our results appear rather general. In particular we will show that: 1) the balanced temptations equilibrium is strongly influenced by the degree of demand symmetry; 2) this first conclusion also sheds some light on the absence of any relationship between demand symmetry and cartel

16 Differently from our analysis, Davis assumes joint profit maximization as cartel rule. Davis and me are presently writing a joint paper on this topic.

stability; 3) cartel stability is clearly influenced by product differentiation (*à la* Chamberlin); 4) cost symmetry is substantially insignificant for stability.

Shall we examine the case of two firms (x,y) with linear demand:

$Q_x = a_x + b_{xx} P_x + b_{xy} P_y$
$Q_y = a_y + b_{yx} P_x + b_{yy} P_y$

And constant marginal costs

$C_x = C_{mx} Q_x$
$C_y = C_{my} Q_y$

We start by evaluating the relationship between demand asymmetry and the balanced temptations equilibrium. To do this we attribute the same marginal cost to the two firms ($C_{mx} = C_{my} = 5$) and we employ two demand functions that are equal in all ($a_x= a_y = 140$ e $b_{xx}= b_{yy} = -3.5$) except in the coefficients b_{xy} and b_{yx}. By varying these coefficients we can modify the degree of demand asymmetry and control its effects on the equilibrium. Some results are shown in the Tab. A1 in the Appendix.

First, it can easily be verified that the collusive equilibrium is considerably influenced by demand asymmetry. The closer the values of the two coefficients, the more γ, which measures the point of the frontier of Pareto where the equilibrium is obtained, converges towards the median value (0.5). Notice, in particular, that the collusive equilibrium always favours the firm which has the smaller coefficient with respect to the price of the competitor (in our examples, firm *x*). Such firm is the one that would benefit less - in relative terms - from joint profit maximization because an increase in prices induces a smaller increase in internal demand (coming from consumers of the other firm). Therefore the balanced temptations equilibrium achieves the result of equalizing the cartel stability by increasing the collusive incentives for those firms that otherwise (in the case of, for instance, joint profit maximization) would get smaller collusive benefits.

This first point explains the second observation that the degree of demand asymmetry is irrelevant to the stability of the cartel. We note that the parameter which measures cartel stability (δ^*) moves in diverging directions according to whether the reduction of the degree of asymmetry is produced by increasing the smaller price coefficient or by decreasing the greatest price coefficient. Cartel stability emerges only in this second case while in the first one δ^* coefficient increases, showing a lower probability of collusion. In particular, when the symmetry is obtained by increasing the smaller price coefficient, and the "*average distance*" between products shrinks, the probability to collude decreases. On the contrary, when the symmetry is obtained by lowering the coefficient (the "*average distance*" between the two products increases) the cartel becomes more stable. This shows that the "*average distance*" effect (*à la* Chamberlin) clearly prevails over the symmetry effect.

A more direct way to show the same property is by applying a particular demand function that contains two specific parameters, one related to the degree of symmetry the other related to the degree of differentiation ("*average distance*" among products) so as to check the effects of these two parameters on cartel stability. This function was used for the first time by Shubik-Levitan[17] and here it will be employed in a slightly different version which is, in the case of two firms competing in a market,

$$Q_x = 0.5(v * a - p_x (1+m/2) + m/2\ p_y)$$
$$Q_y = 0.5((1-v) * a - p_y (1+m/2) + m/2\ p_x)$$

As we want to control the relationship between demand symmetry and the critical discount ratio, we will assume that the marginal costs of firms are equal. Notice, in this case, that the value of *a* (a scale variable for the markets of the two products) does not affect in any way the critical ratio, because in all components of this ratio (π^c, π^s, π^b) *a* only appears as a multiplicative (and squared) coefficient: we therefore can simplify the relationship by equalling *a* to *1*. The "*m*" parameter measures the differentiation degree: when it is equal to 0 the differentiation is highest and the demand function of a product only depends on its own price. As "*m*" grows, products become more similar and their substitutability increases.

The parameter "*v*" measures the specific degree of preference for the first product: when it assumes the intermediate value (0.5) consumers have the same degree of preference for the two products and the demand functions are perfectly symmetrical. As this parameter approaches the extreme values of the interval of variation (0,1) the preference towards a specific product grows and the demand functions become more asymmetric. Therefore, by modifying these two parameters we can appraise the property of the equilibrium of the balanced temptations.

Also in this case we can easily notice (Tab. A2) that the *m* parameter, which controls differentiation, has a paramount influence on cartel stability; on the contrary the *v* parameter, proxy for symmetry, does not produce any effect at all.

These results are coherent with economic literature, previously quoted, which stress the role of the degree of product differentiation. Notice however that they are obtained in a different way, by controlling for the asymmetry and by assuming that cartels do the same, i.e. they share collusive profits according to the demand characteristics so as to lower the risk of cheating .

Checking for the relationship between cost symmetry and cartel stability we get similar results (Tab.A3 in Appendix). The exercise is conducted by maintaining the demand structure unchanged and symmetrical and by modifying marginal costs in

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[17] For a comment about this function see Motta (2004).

such a way that from an initial situation of elevated asymmetry, we approach the point where marginal costs are the same. Also in this case the balanced temptations equilibrium favours the more disadvantaged firm to the point that its incentive to collude becomes aligned with that of the more efficient firm. Therefore cost asymmetry influences collusive prices (and profits) but, not unrelated, it doesn't affect cartel stability.

It deserves to be noted that these observations related to the balanced temptations equilibrium must also be valid for the Nash bargaining solution, because of the great similarity between these two equilibria. They both are influenced by asymmetries so as to immunize cartel stability from this phenomenon.

From this analysis we must expect that the parameter selected to appraise the effect of coordination (δ^*) will generally provide indications concordant with the unilateral effect simulation. A strong unilateral effect implies that the merging firms are very close in the product space. So we can expect that after the merger the market will be characterized by an higher "average distance" among the remaining competitors. Our indicator (δ^*) will decrease showing a more pro-collusive market structure. Therefore, the appraisal of unilateral effect should in general result concordant with that of the coordinated effect.

6 - Conclusion

The procedure that we have proposed here allows us to evaluate in a more rigorous framework the merger coordinated effect. The simulation of the pro-collusive effect is obtained by imposing some assumption, on cartel *modus operandi*, which proves to be essential for exploiting the available information on demand and costs. The results of the exercise are conditioned therefore by the realism of the assumptions which, at the conclusion of the paper, is worthwhile noting.

First, we have assumed that, in reaching a collusive agreement, firms pay attention to cheating probability so as to fix a price structure compatible with the cartel stability. The second assumption, less important conceptually, but not of negligible importance from a practical point of view, consists of the cartel being in a position to calibrate prices in a differentiated way, in order to maximize its own objective function. The third assumption, not independent from the first two, is that there is a remarkable degree of transparency in the market which allows mutual monitoring of behaviours.

The first assumption is the less obvious. We have seen how it dramatically modifies the analytical framework, implying that cost or demand symmetry does not influence the stability of a collusive agreement any longer. Therefore we wonder – this is the problem emphasised by Stigler - how much firms want to control the probability of cheating. It seems reasonable that stability should be considered a crucial element by members of a cartel. The opposite assumption - that firms show poor interest in cartel stability – would be rather surprising: why should colluders ignore the rather extensive economic literature on cartel stability? But we cannot exclude the possibility that firms do not control the risk of cheating. In this case the analysis would have to be different and would be based on the hypothesis that the cartel maximizes joint profits. Such assumption is not necessarily realistic but the joint profit maximization is the more obvious (and extreme) sharing criteria which will be adopted by a cartel which is indifferent to stability. It must be emphasized that in this case the outcome of the analysis also reflects the changes into the degree of asymmetry produced by the merger. Therefore a concentration that increases such symmetry (of demand or costs) will clearly turn out to be pro-collusive. The dichotomy between Patinkin and Bain emerges once again. But it is the Bain approach which must correspond to optimal collusion, because it also accounts for the cartel stability. The Patinkin approach - that cartels are not interested in stability problems - appears to be sub-optimal[18] and therefore confined to contexts with relatively primitive behaviours.

More complicated instead is evaluating if firms are effectively in a position to control the cartel stability. Here the degree of transparency in the market plays an obviously important role. Under conditions of perfect information, firms are in a position to calculate every sort of collusive equilibrium. That does not imply however, that the lack of transparency jeopardizes this result completely. We have noticed that the Friedman equilibrium turns out to be similar to that of Nash which can be considered an iterative process of a cooperative game (Harsany,1977, p152-153). In this process, firms make mutual concessions and the level of knowledge required is not necessarily elevated. Therefore with poor information dissemination Nash equilibrium could emerge.

We still have to address the problem of procedures to be implemented by cartels. In this paper we have assumed, in agreement with majority of recent literature, that a cartel, when maximizing its own objective function fixes the price of products of colluders. Certainly this is a possible procedure, above all when the typologies of products are limited and their demand relatively stable. In such circumstances the agreement is relatively simple and does not need frequent revisions. But in many other cases the number of products is so high that the task of calibrating price increases for all of them appears prohibitive. Also, consider that demand changes force cartel to frequently modify the structure of the prices. Therefore a collusive agreement or a tacit cooperation often uses simpler rules which probably are not optimal, although essential for reaching an agreement.

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18 Assuming that all effective discount ratios of colluders are the same.

The problem has been tackled by Baker (1993) who, by stressing the intrinsically imperfect nature of a cartel, pointed out the importance of these behavioural rules, which he calls "*focal rules*". They can be very different, the most common being uniform (in per cent or absolute value) price increases or the observance of market (or capacity) shares. The importance of the focal rules stems from the fact that they allow the choice set of a cartel to return to a single (or a manageable limited number of) parameter. Such rules are therefore compatible with the mildest forms of cooperation up to the tacit ones.

Baker does not seem to think that it is possible to find an optimal rule for every economic environment. The reason why one of them prevails is essentially due to the institutional market features and, in some way, consolidates itself with the habit. Each market can therefore adopt a rule which best suits its product characteristics, the portfolio of the firms products, the degree of disclosure of information on relevant competitive choices. He gives the example of a market in which each firm trades a multiplicity of products with prices indicated in a price book (known by all competitors). From the example we can infer that all firms have almost the same basket of products. In such circumstances, the practice based on the communication by a firm of a single percentage price increase for all its products must be considered an appropriate focal rule.

Therefore in practical applications, if the cartel rule which allows price to change in differentiated pattern has to be discarded, great attention must be payed to detecting collusion technology (that is, the appropriate *focal rule*), an investigation which does not seem, however, in most cases, unfeasible. Generally from the specific characteristics and the history of a market, it would be possible to extract enough information on the typical price pattern to forcast the focal rule in case of collusion. In such a case, however, we must modify the proposed test so as to consider this additional constraint. We cannot forget, however, that Friedman equilibrium belongs to the same class of schemes which other simpler rules (common price increases, uniform production contractions) belong to. Therfore we can consider the Friedman solution as a proxy for these simpler methods.

Two final notes of caution seem opportune. First, contrary to the unilateral effect simulation where we assume that firms play the same game (Bertrand) before and after the merger, in the case of coordinated effect we want to infer the probability of a change from one game (Bertrand) to another (collusion). We try to accomplish this task by looking at how mergers modify cartel stability. For a better procedure we should consider some threshold of the critical ratio. The question arises whether this threshold should be fixed by following some general rules or, otherwise, according to the characteristics of the market under investigation. Second, in the equilibrium proposed by Friedman, and suggested here for the simulation of a coordinated effect, it is assumed that firms select the collusive equilibrium on the efficient frontier of Pareto. This is, after all, also the approach followed by Nash. However, once we accept the idea that firms take into consideration the risk of cheating, we must also question whether it is reasonable to assume that they limit their choice to equilibria located on the Pareto frontier or whether they also contemplate equilibria under this frontier. This last strategy could greatly reduce the critical discount ratio and therefore make the cartel much more stable. If we proceed in this direction we must model the trade off between stability and level of collusive profits. This seems to be an important line of development for future research.

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Table A1 – Simulation of a collusive equilibrium (with different values for demand parameters)

${}_{t+1}d^*$	*Bertrand profits*	*collusive profits BTE*	*cheating profits BTE*	*critical discount ratio (δ)*	*symmetry indicator (γ)*
$b_{xy}=1$ $b_{yx}=3$	X→1755 Y→2992	X→1933 Y→3429	X→2133 Y→3919	.529	.720
$b_{xy}=2$ $b_{yx}=3$	X→2737 Y→3501	X→3795 Y→5009	X→5287 Y→7133	.585	.592
$b_{xy}=3$ $b_{yx}=3$	X→4135 Y→4135	X→9453 Y→9453	X→21607 Y→21607	.696	.5
$b_{xy}=0.5$ $b_{yx}=2$	X→1346 Y→2106	X→1374 Y→2188	X→1404 Y→2272	.507	.750
$b_{xy}=1$ $b_{yx}=2$	X→1665 Y→2216	X→1765 Y→2389	X→1872 Y→2574	.516	.640
$b_{xy}=1.5$ $b_{yx}=2$	X→2033 Y→2332	X→2268 Y→2629	X→2532 Y→2962	.529	.561
$b_{xy}=2$ $b_{yx}=2$	X→2457 Y→2457	X→2926 Y→2926	X→3483 Y→3483	.543	.5
$b_{xy}=0.3$ $b_{yx}=1$	X→1212 Y→1525	X→1218 Y→1540	X→1225 Y→1555	.502	.789
$b_{xy}=0.5$ $b_{yx}=1$	X→1311 Y→1540	X→1325 Y→1564	X→1340 Y→1588	.503	.628
$b_{xy}=0.7$ $b_{yx}=1$	X→1415 Y→1556	X→1440 Y→1588	X→1465 Y→1621	.504	.568
$b_{xy}=1$ $b_{yx}=1$	X→1580 Y→1580	X→1626 Y→1626	X→1672 Y→1672	.507	.5
$b_{xy}=0.6$ $b_{yx}=3.4$	X→1132 Y→2875	X→1136 Y→2932	X→1140 Y→2989	.503	.937

Table A2 – Critical discount ratios for different *v* (degree of asymmetry) and *m* (degree of differentiation)

V	*.1*	*.3*	*.5*
M			
1	.513	.511	.510
2	.534	.530	.529
3	.556	.552	.551
4	.577	.573	.571
5	.597	.593	.591
6	.615	.611	.610
7	.632	.628	.627
8	.648	.644	.643
9	.662	.659	.658
10	.676	.672	.671

Table A3 – Simulation of a collusive equilibrium (with different marginal costs)

	Bertrand profits	*joint profit maximization*	*collusive profits BTE*	*cheating profits BTE*	*critical discount ratio (δ)*	*symmetry indicator (γ)*
$c_x=5$ $c_y=1$	X→2343 Y→2807	X→2749 Y→3381	X→2817 Y→3312	X→3381 Y→3913	.544	.512
$c_x=4$ $c_y=2$	X→2456 Y→2688	X→2903 Y→3219	X→2938 Y→3185	X→3511 Y→3777	.544	.506
$c_x=3$ $c_y=3$	X→2570 Y→2570	X→3060 Y→3060	X→3060 Y→3060	X→3643 Y→3643	.544	.5

Simulation of coordinated effect: an example

In this example we simulate the coordinated effect of a merger in a three goods (each initially produced by mono-product firms) market. For a better assessment of the collusive effect we examine three different collusive equilibria: join profit maximization, Nash bargaining solution and balanced temptations equilibrium. We employ linear demand functions and constant marginal costs whose parameters are:

Demand system

$Q_1 = 80 - 3 p_1 + 2 p_2 + p_3$

$Q_2 = 140 + 2 p_1 - 4 p_2 + p_3$

$Q_3 = 100 + p_1 + p_2 - 3.5 p_3$

Costs

$C_1 = 5 Q_1$

$C_2 = 10 Q_2$

$C_3 = 7 Q_3$

Notice that the chosen parameters imply that substitutability between products *1* and *2* is higher than substitutability between *2* and *3* or substitutability between *1* and *3*. In this setting we obtain the following *ex ante* competitive and collusive equilibria:

Tab.A4 - Ex ante competitive and collusive equilibria

	Prices			***Profits***			***Critical discount ratios***			***Total profits weights***		
	P_1	P_2	P_3	π_1	π_2	π_3	δ_1	δ_2	δ_3	γ_1	γ_2	γ_3
Bertrand Equilibrium	31.62	33.8	27.13	2126	2265	1418	_	_	_	_	_	_
Joint Profit Maximization Equilibrium	83.7	77.6	61.73	3619	3957	2477	.67	.62	.59	.33	.33	.33
Nash Bargaining solution	84.2	78.6	60.39	3564	3729	2747	.68	.67	.50	.33	.32	.35
Balanced temptations equilibrium	83.1	77.7	62.3	3786	3905	2359	.63	.63	.63	.34	.33	.33

Total profits weights refer to the Pareto efficient frontier:

$$\Pi_t = \Pi_1 * \gamma_1 + \Pi_2 * \gamma_2 + \Pi_3 * \gamma_3$$

$$\text{subject to: } 0 < \gamma_J < 1 \text{ and } \Sigma\, \gamma_J = 1$$

Let us now simulate the unilateral and coordinated impact of A) a merger between the first and the second firm; B) a merger between the second and the third firm.

Table A5 - Ex post competitive and collusive equilibria: A) merger between firm 1 and 2

	Prices			***Profits***		***Critical discount ratios***		***Total profits weights***	
	P_1	P_2	P_3	π_{12}	π_3	δ_{12}	δ_3	γ_{12}	γ_3
Bertrand Equilibrium	52.2	51.4	32.6	5731	2293	_	_	_	_
Joint Profit Maximization Equilibrium	83.7	77.6	61.7	7576	2477	.36	.89	.50	.50
Nash Bargaining solution	86.5	80.0	58.72	6817	3153	.57	.54	.44	.56
Balanced temptations equilibrium	86.3	79.8	58.8	6860	3119	.56	.56	.44	.56

In commenting tables A4 and A5 I want to stress, first, the strong similarity between Nash bargaining solution (NBS) and Friedman's balanced temptation equilibrium (BTE): it emerges from all variables under control (prices, profits, discount ratios, and weights). Second, the merger between the first two firms shows a strong unilateral effect as the market average price increases by more than 40%. That is not surprising considering the proximity of the two firms products. Evaluated according to the BTE, the merger has also a strong pro-collusive effect: the critical discount ratio decreases, from 0.62 to 0.56, showing that a cartel after the merger is more stable than before. This result does not arise if we assume that colluders maximize their joint profit. After the merger the critical discount ratios calculated for the joint profits maximization equilibrium (JPM) diverge substantially, as one of them grows very fast (up to . 89). In this condition we can assume that one firm has strong incentives to cheating and the cartel is therefore very unstable.

Shall we examine now a merger between two firms (2 and 3) whose products are less substitutable (Tab.A6).

Table A6 - Ex post competitive and collusive equilibria: A) merger between firm 2 and 3

	Prices			*Profits*		*Critical discount ratios*		*Total profits weights*	
	P_1	P_2	P_3	π_1	π_{23}	δ_1	δ_{23}	γ_1	γ_{23}
Bertrand Equilibrium	33.9	38.1	32.1	2504	3955	_	_	_	_
Joint Profit Maximization Equilibrium	83.7	77.6	61.7	3619	6434	.73	.51	.50	.50
Nash Bargaining solution	82.0	78.9	62.7	4204	5822	.60	.62	.52	.48
Balanced Temptations equilibrium	82.0	78.8	62.6	4168	5861	.61	.61	.52	.48

As expected, the unilateral effect is modest compared to the former merger: the average competitive price (Bertrand) increases by 12 %. In this case also the parameter (δ), which controls the cartel stability, decreases very gently when evaluated with BTE. That confirms the strong association between unilateral and coordinated effect.