Reshuffling Cards: Regulation and Competition in a Capacity Accumulation Game

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Abstract

We study the asymmetric Cournot duopoly game along the capacity accumulation path. The industry evolution is modelled via the differential game approach in which investment choices and capacity utilizations are taken as the control variables and capacities are state variables. The asymmetry of costs may be given by nature but it can also be compensated by the regulator. We discuss on the relative efficiency for the regulator of reallocating capacity or sites at the reform stage. In the open-loop equilibrium, both firms would like to use their full capacity and interaction of both firms appears in their choices of investment rates and the steady state capacities. Moreover, asymmetry of firms' sizes is preferred in the short run, but symmetry of firms' cost structure is preferred in the long run. Specially, when the regulator cannot obtain the symmetry of firms' cost structure, it is optimal to *compensate* the firm with a higher investment cost by allocating a higher initial capacity.

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1 Introduction

Evolution of firms' capacity consists of symmetric and asymmetric outcomes. When available technologies are the same for both firms, symmetric outcomes emerge as an intuitive result. However, historical and geographical factors may cause an asymmetry in the available sites and appropriate technologies. For example, in the case of power generation, regions differ considerably in the comparative merits of wind farms, dams or nuclear plants. This impacts the cost of effective capacity (as opposed to nominal capacity). Such asymmetry may be seen as the responsibility of regulators to reform the structure of the industry (splitting dominant firms) or to authorize investments. In the meantime, asymmetric capacity structures may also arise as the outcome of a game in which firms differ in their economic fundamentals even when regulatory intervention is absent. Many empirical evidences suggest that there are substantial and persistent differences in the capacity size of firms in most industries. For example, industries ranging from photographic film, to mainframe computers, electric generators, color TVs, and steel production, etc., have, or have had large asymmetries in firm sizes. Some researches highlight that substantial asymmetries in firm sizes can arise endogenously for strategic reasons (e.g. Besanko and Doraszelski (2004)). However, there are also other explanations suggesting that cost asymmetries due to size give a special role to size asymmetry, namely more inequality in market size (e.g. Koulovatianos and Mirman (2004)). Here we suggest that *both initial characteristic differences and special regulatory objectives cause capacity asymmetries among firms in the dynamic capital evolution. Regulators may have to balance the trade-off between symmetry and asymmetry in firms' cost and capacity so that to achieve their short-run and long-run objectives.*

The differential game approach is a relatively more practical device to analyze the firms' behavior concerning the accumulation of productive capacity. As capacity accumulation is time consuming, the depreciation of capacity over time and investment cost become important factors to make differences among firms. The time dimension is also important in planning investment efforts toward capacity accumulation. Meanwhile, different strategic interaction among firms can also change firms investment and production choices, consequently, capacity sizes. Here differential game approach can not only combine the results which are obtained by the more traditional approach based on static two-stage games, but also, endow us certain results which are absent from the static model and unique to the dynamic equilibria.

The existing literature on differential games mainly focusses on two kinds of strategies adopted by players, which are also what we study in the present paper: the open-loop and the closed-loop strategies (see Dockner *et al.* (2000)). When players choose to play open-loop game, they will stick to their initial design of the time path concerning the control variable(s) along all the game periods. In other words, a player's strategy does not depend on the other's reaction at any moment. Therefore, time is the only determinant of the action to be done at any instant. The relevant equilibrium concept is the open-loop Nash equilibrium, which is only weakly time consistent and in general, it is not subgame perfect according to Dockner *et al* (2000). When players choose to play a closed-loop game, there is no precommitment for control variable(s) on any path, and their actions at any instant may depend on the history of the game up to that instant, and, in particular, on the values of the state variables. This means one player's decision will definitely take into account the rival's reaction, instead of initially taking it for granted as what it does in the open-loop game. For the sake of simplicity, the information set used by players in setting their actions at any given time is often assumed to be only the current value of the state variables at that time, along with the initial conditions. This specific situation is defined as feedback closed-loop. The relevant equilibrium concept, in this case, is the closed-loop feedback Nash equilibrium, which is strongly time consistent (or subgame perfect) according to Dockner *et al* (2000).

The literature on the evolution of firms sizes focuses on quantity competition and price competition. Besanko and Doraszelski (2004) use numerical simulation to present the key determinants of firms' size distribution: the competition mode, quantity competition or price competition, and the extent of investment reversibility, which is measured by the rate of depreciation. Under price competition, firms have trend to evolve towards asymmetric structures and industry dynamics resemble a rather brutal preemption race. In particular, they found that if firms compete in prices and the rate of depreciation is large, then the industry moves towards an outcome with one dominant firm and one small firm. However, under quantity competition, firms evolve towards equal size independent of whether investment is reversible or not. In their particular setup, where is no adjustment cost of capacity, nor the difference between firms' depreciation rate of capital. Firms' competition modes are highlighted rather than economic characters. Hanig (1986) also studies firms' investment behavior and capacity accumulation over time and he extensively analyzes quantity competition but under the differential game framework. No asymmetries emerge in equilibrium provided that firms are *ex ante* identical. However, asymmetry of firms' sizes will emerge when firms are fundamentally different in their economic characters and when the investment is reversible. Chen (2006) investigates the price and welfare effects of mergers through simulations using a dynamic model of capacity accumulation. He finds that firms are *ex ante* identical but the industry evolves towards an asymmetric size distribution, and asymmetric costs explain mainly the persistent asymmetries in market shares.

The paper which is most related to ours is Reynolds (1987). In Reynolds' paper, the symmetric duopoly is studied in a capacity accumulation game. The symmetry assumption might give some simple results, but it might also lose the insights of the evolution of the asymmetry in firms' investment and capacity accumulation. Our paper avoids losing generality and produces more results which could not be obtained in the symmetric study. In addition, by using the cost indicator, we separate the effects affecting the evolution of asymmetry in firms' investments along the capacity accumulation, which is not studied by Reynolds (1987), neither is studied in other literature to the best of our knowledge.

In this paper we will focus on the open-loop equilibrium. We take choices of investment and capacity utilization as control variables, instead of capacity choices in the traditional models of two-stage static game (e.g. Kreps and Scheinkman (1983)). Indeed, capacity (or capital) is regarded as a state variable and is not directly affected by the capacity of opponents. We start with a thorough development of the open-loop equilibria; this relatively easier case enables us to insist on the impact of the cost structure and capacity allocation. Our findings are the following: first of all, both firms would like to use their full capacity in the open-loop equilibrium. Second, interaction of both firms appears in their choices of investment rates and the steady state capacities, which means, the optimal solutions of each firm depend not only on its own parameters but also on the other's parameters. Third, when the regulator has to allocate the production facilities, it has to balance the trade-off between the asymmetry in cost and the asymmetry in capacity. Fourth, asymmetry of firms' sizes (as measured by installed capacities in steady state) is preferred in the short run, but symmetry of firms' cost structure is preferred in the long run. Specially, when the regulator cannot obtain the symmetry of firms' cost structure, it is optimal to *compensate* the firm with a higher investment cost by allocating a higher initial capacity.

The remainder of this paper is organized as follows: In section 2, we set up the model. Section 3 and 4 present our results under Cournot competition and we study different scenarios with different investment costs and allocation of the initial capacity. Section 5 summarizes and concludes.

2 The model

We look at a game which is played in continuous time. At any instant $t \in [0, \infty)$, there are two firms serving the market, firm 1 and firm 2. The setup is similar to Reynolds' (1987). The main differences are that the two firms may have different technologies (namely, investment costs and depreciation rates). The good which they produce is homogeneous and firms' marginal production costs are constant and equal to 0 for the sake of simplicity (the model can easily be extended to the case of positive constant marginal costs); we denote $q_1(t)$ and $q_2(t)$ the quantities sold by firm 1 and firm 2 at time t respectively. The inverse demand function at time t is

$$P(t) = A - q_1(t) - q_2(t). \tag{1}$$

In the following, i represents a generic firm (1 or 2) and j represents the other one. The accumulation of capacity (or capital) of firm i is

$$\dot{k}_i = \frac{dk_i(t)}{dt} = I_i(t) - \delta_i k_i(t), \tag{2}$$

where $I_i(t)$ is firm i's investment, and δ_i is a constant depreciation rate. The instantaneous costs of investment is quadratic $C_i(I_i) = \frac{c_i}{2} I_i^2$, with c_i positive.[1] We assume that the pair (c_1, c_2) must belong to certain convex set $\Omega \subset \mathbb{R}_+^2$. See subsection 4.1.

We also assume that at any time, firm i's production $q_i(t)$ is a proportion $\alpha_i(t)$ of its capacity $k_i(t)$, with $\alpha_i(t) \in [0, 1]$, $\alpha_i(t)$ could be regarded as the capacity utilization of firm i. We can rewrite the inverse demand functions as

$$P(t) = A - \alpha_1(t)k_1(t) - \alpha_2(t)k_2(t). \tag{3}$$

As firm i's instantaneous profits is $\pi_i(t) = P(t)q_i(t) - \frac{c_i}{2} I_i^2(t)$, firm i's objective is to maximize the present value of the profit flows

$$\begin{gathered} \int_0^\infty \pi_i(t) e^{-\rho t} dt \\ \text{s.t. (2) and its counterpart for } j. \end{gathered} \tag{4}$$

The constant discount rate ρ measures firms' preference to the present gains. The assumption that firms are equally impatient can easily be relaxed.

In the optimization program of firm i, the control variables are the instantaneous investment I_i and the capacity utilization level α_i. We allow firms to

[1] Adding a linear part to the cost does not complicate the analysis, but expressions are heavier.

hold excess capacity, e.g. α_i can be smaller than 1 for some time. A similar approach is taken in the symmetric case by Cellini and Lambertini (2003). This differs from usual assumption of full utilization of capacity (see e.g. the Cournot game in a two-period setting in Kreps and Scheinkman, 1983, or Dockner, 1992, for a differential game).

3 The open-loop Cournot-Nash equilibrium

In this subsection, we examine the open-loop solution of the game. For firm i, the objective and constraints depend on time only through firm j's investment strategy variables and state variables (k_i, k_j).

Dynamics

The analysis of the Hamiltonian of program (4) in Appendix A.1 enables us to calculate the laws of motion followed by the controls:

$$(\rho + \delta_i)c_i I_i - c_i \dot{I}_i = \alpha_i \left[A - 2\alpha_i k_i - \alpha_j k_j\right]. \tag{5}$$

Remark 1 *In this subsection we first consider only trajectories along which firms fully utilize their capacity. This case is not general but is sufficiently rich to illustrate our point on the tradeoff faced by the regulator. More importantly, we will check ex post the validity of this approach in the next subsection on long-run effect.*

In an open-loop equilibrium, α_i has a purely temporary effect since firm i takes the plan of investment and production of firm j as given. In other terms, α_1 and α_2 are directly determined by the static Cournot logic where production cannot overpass currently installed capacity. Consequently, the region in the plane (k_1, k_2) where $\alpha_1 = 1$ and $\alpha_2 = 1$ is the quadrilateral, denoted by Q_α, comprised between the axis and the lower envelope of the best response curves ($I_1 = 0$ and $I_2 = 0$) of the (locally) static game. See Figure 1. Let's denote by Q_k the domain in which $\dot{k}_1 \geq 0$ and $\dot{k}_2 \geq 0$; this quadrilateral is comprised between the axis and the isoclines $\dot{k}_1 = 0$ and $\dot{k}_2 = 0$. We can show that Q_k (the barred area) is a proper subset of Q_α. To do so, it suffices to proves that the stationary equilibrium (crossing point between isoclines mentioned above) is comprised in Q_α and that each isocline cuts $\dot{k}_i = 0$ cuts axis k_i for a value smaller than the monopoly production (which is the limit of Q_α on the axis). This suffices to show that initial conditions such that capacity grows for both firms to reach the steady state is necessarily such that capacity is fully used ($\alpha_1 = \alpha_2 = 1$) along the equilibrium trajectory. The

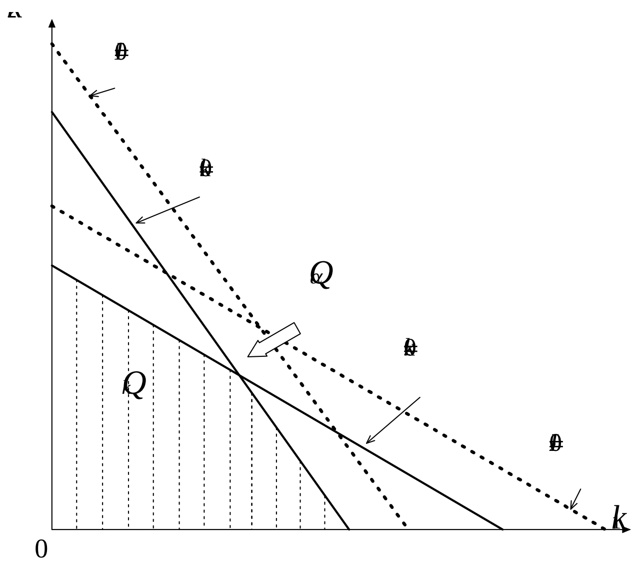

Figure 1: Full capacity utilization

cases where capacity is not always fully used (for example if one of the two firms is very large at the beginning of the game) would necessitate a specific analysis that is not crucial for our main message on the role of regulation.

Using equation (2), $i = 1, 2$, we can rewrite (5) as

$$\ddot{k}_i + \delta_i \dot{k}_i - \left[\frac{2}{c_i} + (\rho + \delta_i)\delta_i \right] k_i + \frac{A - k_j}{c_i} = 0. \tag{6}$$

We define two functions of time $h_1 = \dot{k}_1$ and $h_2 = \dot{k}_2$. The linear second-order system of equations is solved with the classical method by transposing it as a four-dimensional first order system:

$$\dot{H} = MH - N, \tag{7}$$

where $H = (h_1, k_1, h_2, k_2)^T$, $N = (\frac{A}{c_1}, 0, \frac{A}{c_2}, 0)^T$ and

$$M = \begin{pmatrix} -\delta_1 & \frac{2}{c_1} + (\rho + \delta_1)\delta_1 & 0 & \frac{1}{c_1} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{c_2} & -\delta_2 & \frac{2}{c_2} + (\rho + \delta_2)\delta_2 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{8}$$

The eigenvalues of M are denoted by $\lambda_s, s = 1, 2, 3, 4$. At least one of them is negative since $\mathrm{Tr}[M] = -\delta_1 - \delta_2 < 0$. In fact,

$$\mathrm{Det}[M] = (\frac{2}{c_1} + \delta_1(\delta_1 + \rho))(\frac{2}{c_2} + \delta_2(\delta_2 + \rho)) - \frac{1}{c_1 c_2} > 0, \tag{9}$$

meaning that there is an even number of negative eigenvalues, namely, 2 or 4. Moreover, the coefficient of the 2nd order in the characteristic polynomial is

$$\frac{(-1)^2}{2!} \cdot \sum_{\substack{a,b=\{1,2,3,4\} \\ a \neq b}} \prod \lambda_a \lambda_b = -\frac{2(c_1+c_2)+c_1c_2(\delta_1^2+\delta_2^2+\rho(\delta_1+\delta_2)-\delta_1\delta_2)}{c_1c_2} < 0, \tag{10}$$

meaning that eigenvalues can't be all negative. We conclude that in general there are two positive eigenvalues (λ_2 and λ_4) and two negative ones (λ_1 and λ_3).

The weights given to diverging exponentials must be null (otherwise capacity diverges to $\pm\infty$). So capacities, as a function of time, have the form

$$k_i(t) = c_i^0 + c_i^1 e^{\lambda_1 t} + c_i^3 e^{\lambda_3 t}. \tag{11}$$

If we denote initial total capacity as K_0. The evolution of firms' capacities has the following characteristics

$$k_1(0) + k_2(0) = c_1^0 + c_2^0 + c_1^1 + c_2^1 + c_1^3 + c_2^3 = K_0, \tag{12}$$

$$\lim_{t \to +\infty} (k_1(t) + k_2(t)) = c_1^0 + c_2^0 \text{ is finite.} \tag{13}$$

4 Regulating structures

4.1 Sites and investment costs

When available technologies are the same for both firms, investment costs are normally the same. However, historical and geographical factors may cause an asymmetry in the available sites and appropriate technologies. For example, in the case of power generation, regions differ considerably in the comparative merits of wind farms, dams or nuclear plants. This impacts the cost of effective capacity (as opposed to nominal).[2] Though these arguments are clear reasons why firms typically differ, it may be seen as the responsibility of regulators to reform the structure of the industry (splitting dominant firms) or to authorize investments. In this sense, c_1 and c_2 are controls for the regulator, or, though this is not the focus of this study, they may result (in an unregulated market) from a race to sites by firms.

[2] The difference between nominal power and effective power is particularly clear for wind farms. The intensity and variability of wind regimes directly determine the relationship between these two measures of capacity. Effective capacity is of course the economically relevant data, as far as Cournot competition is concerned.

For the analysis to be complete, we propose a theory of the constraints faced by the regulator in the choice of c_1 and c_2. Typically, the choice set Ω (see section 2) can be seen as an upper contour of a quasi-concave differentiable symmetric function $f : \mathbb{R}_+^2 \to \mathbb{R}$:

$$\exists f_\Omega \text{ s.t.: } (c_1, c_2) \in \Omega \Leftrightarrow f(c_1, c_2) \geq f_\Omega. \tag{14}$$

We build a useful example. Assume that we have a continuum of sites parameterized by $\theta \in [\underline{\theta}, \overline{\theta}]$; θ will also denote the site specific investment cost. Namely, the whole nvestment cost corresponding to site θ is

$$C_\theta(z) = \frac{\theta}{2} z^2, \tag{15}$$

where z is site specific rate of investment.

Firm i can be described by the sites it owns. Ownership is summarized by $\omega_i(\theta)$, the size of site θ that firm i owns. When it invests, a firm spreads its capacity augmentation optimally among sites; this choice is represented by function $z(\theta)$.

Let the bilinear operator $\langle \omega, f \rangle$ simply denote

$$\int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta) f(\theta) d\theta \qquad\qquad \forall \omega, f \tag{16}$$

Therefore, $C_i(I)$ as defined in section 2 solves the following program

$$C_i(I) = \min_{z(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \omega_i(\theta) C_\theta(z(\theta)) d\theta \tag{17}$$

$$\text{s.t. } I = \langle \omega_i, z(\theta) \rangle . \tag{18}$$

The first order condition gives

$$z(\theta) = \frac{\lambda}{\theta}. \tag{19}$$

We can now calculate the relationship between λ and I :

$$I = \lambda \left\langle \omega_i, \theta^{-1} \right\rangle \tag{20}$$

We can also express firm i's investment cost

$$C_i(I) = \frac{I^2}{2 \left\langle \omega_i, \theta^{-1} \right\rangle}. \tag{21}$$

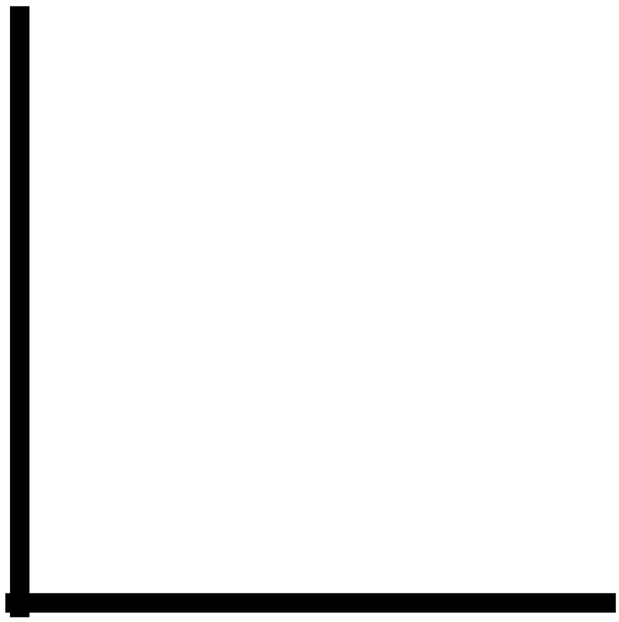

Figure 2: The efficient frontier

We can further identify c_i to $\frac{1}{\langle \omega_i, \theta^{-1} \rangle}$.

Define $h(\theta)$ the initial distribution of sites (whoever owns them). We have $\omega_1(\theta) + \omega_2(\theta) = h(\theta)$ when all sites are allocated, which is true and efficient along the frontier of Ω. Given that $\langle \omega_1, . \rangle + \langle \omega_2, . \rangle = \langle h, . \rangle$, we have

$$\frac{1}{c_1} + \frac{1}{c_2} = \frac{1}{C} = \text{Constant}, \tag{22}$$

where $1/C = \left\langle h, \theta^{-1} \right\rangle$. This constant is independent of the distribution of sites across firms.

This relationship is shown by the efficient frontier in Figure 2. Above the curve, combinations of parameters are inefficient, and below, they are impossible to be reached. Remark that, if we take C equal to 1, c_1 close to 1 (and thus c_2 is very large) means that firm 1 has an overwhelming advantage in investment and it becomes a monopoly in the long run; in the short run, firm 2 may have a non-negligible capacity but it will let it depreciate (see section 4.3).

Given this efficient frontier, we can consider several scenarios in which the total capacity at date 0 is K_0. This is supposed to be exogenous. If K_0

is small compared to the long term value of capital, both firms are expected to engage in accumulation, presumably at different rates. If K_0 is relatively large, one of the two firms, if not both, will let its capital depreciate and invest later to converge to higher values. In between, firms may engage in differentiated strategies (accumulation vs. depreciation), depending on their initial allocation of capital and on their investment costs.

When the regulator has to assign sites to firms, ideally or theoretically, full symmetry gives the most propitious condition for competition. This is true in the long run (the total capacity is maximized) and in the short run (withholding is less likely with similar firms). This can be achieved by assigning similar plants to the two firms so that they are symmetric in their initial capacities and in their investment cost. For example, equal number of plants of the various types of technologies can be given to firms, or, more generally and perhaps more interestingly, equivalent combinations (in the sense of the aggregate cost we examined in the previous subsection).

In practice, a regulator will follow a geographical or technological logic when it comes to defining the boundaries of the two firms. This logic relies on potential economies of scale in the operation of a portfolio of plants (we take this as given). Moreover, various sites may exhibit very diverse installed capacity. These two arguments suggest that fully symmetric allocation of plants at the starting point might not be doable, even though it is optimal to be reached. *In the long run*, symmetric costs are ideal to reach a symmetric equilibrium; *in the short run*, symmetric capacities (for a fixed total initial capital) give the best conditions for effective Cournot competition. The regulator has to balance the trade-off between these two perspectives if both of them could not be satisfied at the same time. Specially when the regulator cannot split the plants in order to obtain both objectives, it has to choose the second best solution and it becomes crucial which objective has more priority. If we consider the interest of consumers, optimal constrained regulation consists in "compensating" the firm with the highest investment costs by allocating higher initial capacity.

In the following analysis, we specify when we vary costs and whether we stay on the efficient frontier or leave it.

4.2 The long run

The steady state condition $\overset{\bullet}{k}_i = 0 \Longleftrightarrow k_i^* = I_i^*/\delta_i$ is used to simplify dynamic equations:

$$\alpha_i \left[A - 2\alpha_i \frac{I_i^*}{\delta_i} - \alpha_j \frac{I_j^*}{\delta_j} \right] = (\rho + \delta_i) c_i I_i^*. \tag{23}$$

We can thus rewrite (44) as

$$\alpha_i \mu_i = \frac{(\rho + \delta_i) c_i I_i^{*2}}{\delta_i}. \tag{24}$$

We conclude that $\mu_i > 0$: at the steady state, $\alpha_i = 1$. Similar calculations can be derived from firm j's program.

Using (23) and the fact that $\alpha_i = \alpha_j = 1$, we can conclude that

$$I_i^* = \frac{(1 + C_j) A \delta_i}{(2 + C_i)(2 + C_j) - 1}. \tag{25}$$

where each firm's parameters enter into the steady state capital only through the cost indicators $C_1 = c_1(\rho + \delta_1)\delta_1$ and $C_2 = c_2(\rho + \delta_2)\delta_2$. Consequently,

$$k_i^* = \frac{(1 + C_j) A}{(2 + C_i)(2 + C_j) - 1}. \tag{26}$$

Remark that a higher c_i or a higher δ_i increases the cost of sustaining any level of capital. Consistently, we find that the equilibrium capacity of each firm is decreasing with respect to its cost indicator. Also, the ratio k_i^*/k_j^* increases (respectively decreases) with respect to C_j (resp. C_i). However, the equilibrium investment of each firm is always decreasing in its own marginal cost, but the effect of the depreciation rate is ambiguous in general.

Given these steady state results, we can retrieve the "non-depreciation" Cournot-Nash equilibrium, which corresponds to $\delta_1 \to 0, \delta_2 \to 0$:

$$I_1^{ND} = I_2^{ND} = 0, \tag{27}$$

$$k_1^{ND} = k_2^{ND} = k^C = \frac{A}{3}. \tag{28}$$

These "non-depreciation" steady-state capacities don't depend on the investment costs and are identical to the static Cournot-Nash equilibrium quantity k^C. This result is analogical to the open-loop equilibrium in Hanig(1986). However, if we assume that $c_1 < c_2$ and $\delta_1 < \delta_2$, it is not possible for the inefficient firm (firm 2) that the steady state capacity, as well as the production, coincides with the static Cournot quantity k^C. The capacity asymmetries in the open-loop equilibrium are

$$k_2^* < k^C \leq k_1^* \tag{29}$$

$$\text{or } k_2^* < k_1^* \leq k^C. \tag{30}$$

It is not difficult to verify that if $C_2 \geq \frac{2C_1}{1 - C_1}$, $k_1^* \geq k^C = k_1^{ND}$, but it is always true that $k_2^* < k^C = k_2^{ND}$.

Recall that the profits of firm i on the equilibrium path can be calculated

$$\pi_i^* = (A - k_i^* - k_j^*)k_i^* - \frac{c_i}{2} I_i^{*2}, i \neq j. \tag{31}$$

Consider first the symmetric case where $c_1 = c_2 = c$ and $\delta_1 = \delta_2 = \delta$ (therefore, $C_1 = C_2 = C = c(\rho + \delta)\delta$), the steady state profit is

$$\pi^* = \frac{A^2(2 + 2C - c\delta)}{2(3 + C)^2}. \tag{32}$$

The comparative statics of this symmetric profit give the following results

$$\frac{\partial \pi^*}{\partial c} = -\frac{A^2\delta(\delta - 2\rho + C(\delta + 2\rho))}{2(3 + C)^3} \lesseqgtr 0, \tag{33}$$

$$\frac{\partial \pi^*}{\partial \delta} = -\frac{A^2 c(\delta - \rho + C(\delta + \rho) + c\rho\delta^2)}{(3 + C)^3} \lesseqgtr 0. \tag{34}$$

The effects of c and δ on the steady state profit are ambiguous in general and strongly depend on the relationship between the depreciation rate δ and the discount factor ρ. In fact, the overall ambiguity (and the difficulty to interpret it) is simply due to the fact that changing cost parameters in the symmetric case changes the characteristics of the two players at the same time. The interest of the asymmetric case is that we separate the effect which comes from firm i's side and the effect which comes from firm j's side.

In the asymmetric case, we have a definite sign for each derivative.

Proposition 2 *The steady state profit*

$$\pi_i^* = \frac{A^2(1 + C_j)^2(2 + 2C_i - c_i\delta_i^2)}{2(3 + 2C_j + C_i(2 + C_j))^2} \tag{35}$$

is such that

$$\frac{\partial \pi_i^*}{\partial c_i} < 0, \frac{\partial \pi_i^*}{\partial c_j} > 0, \frac{\partial \pi_i^*}{\partial \delta_i} < 0, \frac{\partial \pi_i^*}{\partial \delta_j} > 0. \tag{36}$$

Proof. See Appendix A.2. ■

That is, each firm's steady state profit decreases in its own instantaneous investment cost and depreciation rate, but increases in its rival's instantaneous investment cost and depreciation rate. This is invisible in the symmetric case. Asymmetric firms' case endows us the opportunity to explore which effect dominates, the own negative effect, or the rival's positive effect.

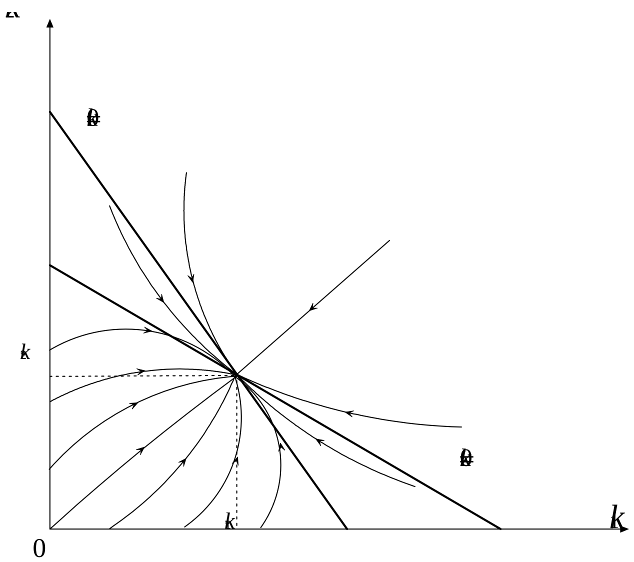

Figure 3: Phase diagram for the open-loop Nash equilibrium

These calculations can completed with mention of the constraint on costs (the allocation of sites):

$$\left.\frac{d\pi_i^*}{dc_i}\right|_{\text{Eff frontier}} = \frac{\partial \pi_i^*}{\partial c_i} + \left.\frac{dc_j}{dc_i}\right|_{\text{Eff frontier}} \cdot \frac{\partial \pi_i^*}{\partial c_j} < 0, \tag{37}$$

$$\left.\frac{d\pi_i^*}{dc_j}\right|_{\text{Eff frontier}} = \frac{\partial \pi_i^*}{\partial c_j} + \left.\frac{dc_i}{dc_j}\right|_{\text{Eff frontier}} \cdot \frac{\partial \pi_i^*}{\partial c_i} > 0. \tag{38}$$

Proposition 3 *Assume that Ω is symmetric. An allocation of sites on the efficient frontier that equalizes costs maximizes long run total capacity and minimizes long run total profits. Symmetry is therefore optimal for consumers.*

Proof. See Appendix A.3. ■

Recall that the steady states of the open-loop Nash equilibrium is derived from (26). Some qualitative properties of the optimal investment path of both firms are illustrated in the phase diagram, i.e. figure 3

If both firms start from zero capacity, they will grow simultaneously until they have reached the steady-state level of capacity. However, if one firm, say firm 1, starts from positive capacity but the other firm, say firm 2, starts from zero capacity, the phase diagram shows that, along the equilibrium path, there will be a period during which firm 1 reduces its capacity level. Still in this case, if firm 1's positive capacity is sufficiently large when firm 2 starts investing, firm 1 will disinvest along the entire equilibrium path. But if firm 1's positive capacity is relatively small when firm 2 starts investing, firm 1 will grow initially, and disinvest only after both firms have grown sufficiently large. Thus the optimal investment of firm 1 involves "overshooting". This result is analogical to Hanig(1986). The intuition behind this overshooting property of the investment path is simple. If firm 1 is not disturbed by any constrains, it would like to grow as fast as possible to its monopoly capacity level, which is larger than the duopoly steady-state capacity for each firm. Now firm 2 starts to invest before firm 1 has grown to the monopoly capacity level. At the moment that firm 2 starts investing, firm 1 still has the desire to expend its capacity to the monopoly level. But as firm 2 grows, firm 1 will have to adjust its capacity and converge to the duopoly steady-state capacity level. Therefore, firm 1's capacity just increases at the beginning but decreases later.

This transition process will be discussed in details in the next subsection and we will analyze more representative numerical examples.

4.3 Transition

Now let's discuss about different scenarios of duopolist firms' capacity evolution and try to understand the regulator's optimal choices in those scenarios. In order to study the effect of asymmetry of costs and initial capacity on the evolution of the market, it will be convenient to focus on asymmetry in c_1 and c_2 while keeping symmetric depreciation rates ($\delta_1 = \delta_2 = \delta$). Indeed, we can then calculate the negative eigenvalues of M :

$$\lambda_1 = -\frac{\delta}{2} - \frac{\sqrt{c_1 c_2(4c_2 - 4\sqrt{c_1^2 - c_1 c_2 + c_2^2} + c_1(4 + c_2\delta(5\delta + 4\rho)))}}{2c_1 c_2}, \tag{39}$$

$$\lambda_3 = -\frac{\delta}{2} - \frac{\sqrt{c_1 c_2(4c_2 + 4\sqrt{c_1^2 - c_1 c_2 + c_2^2} + c_1(4 + c_2\delta(5\delta + 4\rho)))}}{2c_1 c_2}. \tag{40}$$

To see the impact of asymmetry on the consumers' welfare in the different cases, a natural (and simpler) angle is to look at the total capacity over time. This represents, by definition, the total consumption. The trade-off between the long and the short run will appear clearly.

Proposition 4 *Fix total initial capacity to K_0 and costs c_1 and c_2. Denote β the share of K_0 that is allocated to firm 1 (the rest $(1-\beta)$ of K_0 goes to firm 2). Assume without loss of generality that $c_1 < c_2$.*

1. *Total capacity at date 0 and in the long run are independent of β.*

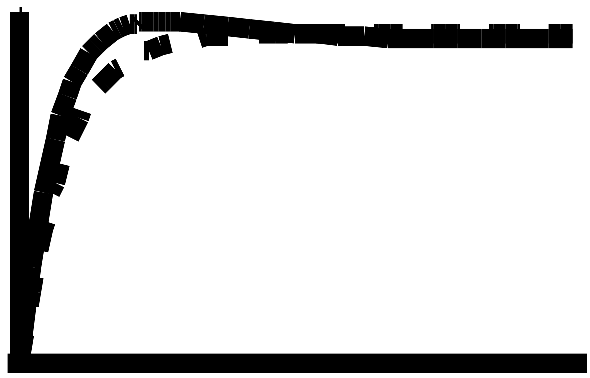

Figure 4: Total capacities in symmetric (dashed line) and asymmetric (solid line) scenarios

2. *Total capacity increases more slowly (or decreases faster) at date 0 as β increases.*

3. *Total capacity at any date $t > 0$ is smaller for larger β.*

Proof. See Appendix A.4 ■

If other things are equal, allocating more capacity to the inefficient firm produces permanent (but vanishing in the long run) benefits. Thus, the inefficient firm leaves the opportunity to invest to the firm which has a comparative advantage in investment. Moreover, the longer survival of the inefficient firm constrains for a longer time the efficient firm, which has to accommodate a non trivial rival in production.

Corollary 1 *If $c_i = c_j$, total capacity as a function of time is independent of the initial allocation of the existing capacity.*

Examples We fix a number of parameters $\delta = 0.1$, $\rho = 0.1$, $A = 1$, $C = 1$ and we set initial total capacity K_0 to 1/2. The initial sharing rate β can vary between 0 and 1 whereas c_1 (or c_2) varies between 1 and $+\infty$.

Figure 4 shows the evolution over 10 years of total capacity under the two scenarios: $c_1 = c_2 = 2$ and $\beta = 1/2$ (fully symmetric initial conditions, dashed line) and $c_1 = 1.3$, $c_2 = 4.33$ and $\beta = 0$ (very asymmetric case, solid line). Remark that differences in the long run are small.

Figure **??** presents total (solid line) and disaggregated (dashed line) capacities.

Figure 4 illustrates the contrast between short term and long term effects. There is a date $T^* > 0$ such that before this date, the total capacity in the asymmetric case is higher, after T^*, the total capacity is smaller than in the symmetric case. We observe also that asymmetry can also cause an overshooting effect in total capacity evolution process.

How to understand these effects? In the case of symmetric costs and capacities, firms simply disinvest and converge monotonically to the long run state. In the asymmetric scenario set above, firm 1 cannot wait too long to take advantage of its lower investment costs (impatience combined with convex investment costs leads to a smooth investment strategy). This is combined with the fact that firm 2 lets its capacity depreciate. Total capacity increases for a while and then decreases to cross the trajectory of the symmetric scenario (at date T^*) and to reach its lower long-run capacity level.

The analysis above directly uncovers the important issue of regulatory consistency. Once the short run benefits of asymmetry are cashed in, it may be tempting to reform the structure of the industry (costs and capacities) to approach the symmetric optimum. The (rational) anticipation of this second (unannounced) stage of regulation would perturb, and invalidate, the notion of equilibrium we have studied.

5 Conclusion

We have studied the open-loop equilibrium in duopoly firms' capacity accumulation. Starting with a thorough development of the open-loop equilibria, we insist on the impact of the cost structure and capacity allocation. We find that both firms would like to use their full capacity in the open-loop equilibrium and interaction of both firms appears in their choices of investment rates and the steady state capacities. Furthermore, when the regulator has to allocate the production facilities, it has to balance the trade-off between the asymmetry in cost and the asymmetry in capacity if symmetry is not practically to be achieved. Moreover, asymmetry of firms' sizes (as measured by installed capacities in steady state) is preferred in the short run, but symmetry of firms' cost structure is preferred in the long run. Specially, when the regulator cannot obtain the symmetry of firms' cost structure, it is optimal to *compensate* the firm with a higher investment cost by allocating a higher initial capacity.

For the future research, we would like to suggest that other regulations may be analyzed, for example, a possible capacity transfer from one firm to the other in a capacity variation may induce a smooth convergence toward stready states. The associated monetary transfer is to be determined. In addition, asymmetry may also emerge from the discount factor of each firm, which means, firms will be differently impatient toward the future profit flow. Regulators have to take into account this asymmetry and to balance more trade-off.

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A Appendix

A.1 Proof

Using the inverse demand function, we can write down the Hamiltonian function of firm i as

$$H_i(I, \alpha, k) = [A - \alpha_i k_i - \alpha_j k_j]\alpha_i k_i - \frac{c_i}{2} I_i^2 + \lambda_i [I_i - \delta_i k_i] - \mu_i \alpha_i. \tag{41}$$

where λ_i is the co-state variable associated to k_i and μ_i is the Lagrangian multiplier of the constraint forcing capacity utilization not to exceed 1. Let $k_i(0)$ define the initial condition for firm i.

The first order conditions are

$$\frac{\partial H_i}{\partial \alpha_i} = 0 \text{ and } \frac{\partial H_i}{\partial I_i} = 0, \tag{42}$$

and the adjoint equation and the associated derivative are

$$-\frac{\partial H_i}{\partial k_i} = \dot{\lambda}_i - \rho \lambda_i. \tag{43}$$

Consequently,

$$\begin{aligned} k_i \left[A - 2\alpha_i k_i - \alpha_j k_j\right] &= \mu_i, & (44) \\ -c_i I_i + \lambda_i &= 0, & (45) \\ \dot{\lambda}_i &= (\rho + \delta_i)\lambda_i - \alpha_i \left[A - 2\alpha_i k_i - \alpha_j k_j\right]. & (46) \end{aligned}$$

From (45), we can derive

$$\dot{\lambda}_i = c_i \dot{I}_i. \tag{47}$$

Plugging these results into (46), we get equation (5) in the text.

A.2 Proof of Proposition 1

Easy calculations give

$$\frac{\partial \pi_i^*}{\partial c_i} = \tag{48}$$
$$-\frac{A^2\delta_i(1+C_j)^2(2\rho + c_i\delta_i^2(2+C_j)(\delta_i+3\rho) + \delta_i(1+2(2+C_j)(1+c_i\rho^2))}{2(3+2C_j+C_i(2+C_j))^3} < 0,$$

$$\frac{\partial \pi_i^*}{\partial c_j} = \frac{A^2\delta_j(\delta_j+\rho)(1+C_j)(2+(3+C_i)C_i + c_i\delta_i\rho(1+C_i))}{(3+2C_j+C_i(2+C_j))^3} > 0, \tag{49}$$

$$\frac{\partial \pi_i^*}{\partial \delta_i} = \tag{50}$$
$$-\frac{A^2c_i(1+C_j)^2(\rho + c_i\delta_i^2(2+C_j)(\delta_i+3\rho) + \delta_i(1+(2+C_j)(2+c_i\rho^2))}{(3+2C_j+C_i(2+C_j))^3} < 0,$$

$$\frac{\partial \pi_i^*}{\partial \delta_j} = \frac{A^2c_j(2\delta_j+\rho)(1+C_j)(2+(3+C_i)C_i + c_i\delta_i\rho(1+C_i))}{(3+2C_j+C_i(2+C_j))^3} > 0. \tag{51}$$

A.3 Proof of Proposition 2

Maximize total long run capacity

$$k_{1+2}^* = \frac{(2+C_1+C_2)A}{(2+C_1)(2+C_2)-1}. \tag{52}$$

Minimize total profits

$$\pi_{1+2}^* = \frac{A^2}{2}\left(\frac{(1+C_2)^2(2+2C_1-c_1\delta_1^2)}{(3+2C_2+C_1(2+C_2))^2} + \frac{(1+C_1)^2(2+2C_2-c_2\delta_2^2)}{(3+2C_1+C_2(2+C_1))^2}\right). \tag{53}$$

If in both case we prove the superiority of the symmetric situation, we prove that symmetry is optimal in the long run.

A.4 Proof of Proposition 3

We can rewrite each firm's trajectory as

$$k_i(t) = c_i^0 + c_i^1 e^{\lambda_1 t} + c_i^3 e^{\lambda_3 t}.$$

where $i = 1, 2$ and λ_1, λ_3 are negative eigenvalues of matrix M.

At date 0, the total capacity is $K(0) = k_1(0) + k_2(0) = K_0$. In the long run, the total capacity $K(t)$ is determined by c_1^0 and c_2^0:

$$\lim_{t\to+\infty} K(t) = \lim_{t\to+\infty} K(k_1(t)+k_2(t)) = c_1^0 + c_2^0$$

c_1^0 and c_2^0 can be solved by using (12), (13) and eigen-system of matrix M. Injecting the values of c_1^0 and c_2^0, we will get the steady state of the total capacity

$$\lim_{t\to+\infty} K(t) = c_1^0 + c_2^0 = \frac{A(2+(c_i+c_j)\delta(\delta+\rho))}{3+2c_j\delta(\delta+\rho)+c_i\delta(\delta+\rho)(2+c_j\delta(\delta+\rho))}$$

It is obvious that the total capacities at date 0 and in the long run are independent of the initial allocation rule β.

At date 0 the slope of the total capacity with the sharing rate β is denoted as ξ

$$\xi = \left.\frac{\partial K(t)}{\partial t}\right|_{t=0} = \lambda_1(c_i^1+c_j^1)+\lambda_3(c_i^3+c_j^3)$$

by injecting the values of c_1^0 and c_2^0, we will find the effect of β on the slope ξ

$$\left.\frac{\partial \xi(t)}{\partial \beta}\right|_{t=0} = -\frac{1}{4c_ic_j\sqrt{c_i^2-c_ic_j+c_j^2}}\left\{(c_i-c_j)\,K_0\,[R_1-R_2]\right\}$$

where

$$\begin{aligned} R_1 &= \sqrt{c_ic_j\left(4c_j-4\sqrt{c_i^2-c_ic_j+c_j^2}+c_i\left(4+c_j\delta\left(5\delta+4\rho\right)\right)\right)} \\ R_2 &= \sqrt{c_ic_j\left(4c_j+4\sqrt{c_i^2-c_ic_j+c_j^2}+c_i\left(4+c_j\delta\left(5\delta+4\rho\right)\right)\right)} \end{aligned}$$

Therefore, the slope variation is positively proportional to difference (c_i-c_j).

If firm i is a firm with a higher cost and with a higher initial capacity, then the total capacity increases faster at date 0, or decreases more slowly. It seems that giving more capacity to inefficient firm has short term benefits.

If we look at the total capacity $K(t)$ at any date $t>0$, the derivative of slope $\xi(t)$ with respect to β will be

$$\begin{aligned} \frac{\partial \xi(t)}{\partial \beta} &= \frac{1}{2\sqrt{c_i^2-c_ic_j+c_j^2}}\left\{(c_i-c_j)\,K_0\exp t\left[-\frac{2c_ic_j\delta+R_1+R_2}{2c_ic_j}\right]\right. \\ &\quad \left.\left[\exp t\left[\frac{c_ic_j\delta+R_2}{2c_ic_j}\right]-\exp t\left[\frac{c_ic_j\delta+R_1}{2c_ic_j}\right]\right]\right\} \end{aligned}$$

Therefore, the impact of β on slope $\xi(t)$ is positively proportional to (c_i-c_j) at any date t. The more asymmetry, i.e. the more initial capacity allocated to the inefficient firm, the better for consumers since the total capacity at any date will be permanently superior.