Do stand-alone private antitrust damages improve welfare?

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Abstract

A game theoretical model is built, where both public and private antitrust enforcement play a role. The economy consists of a continuum of industries, in each of which a monopolist can engage into anti-competitive activity. The national competition authority (NCA) only unveils a proportion of such abuses (and imposes a monetary penalty on these). Plaintiffs face information acquisition costs (evidence-gathering) if they want to bring a damages' action but do not internalise the opportunity cost of the courts' efforts.

Whilst deterrence cannot decrease after the introduction of stand-alone private damages, the impact on total welfare is actually ambiguous and will be heterogeneous across market sizes (consumer welfare, by contrast, cannot decrease and will in fact improve for most parametric conditions). Second, I show that class actions have an ambiguous effect on total welfare. Third, the results are robust to allowing for type I and II errors by the NCA. Finally, I endogenise the NCA's optimal policy subject to a resource constraint.

1 Introduction and motivation

1.1 Legal framework

Antitrust damages are the monetary payments determined by courts that companies infringing antitrust rules should pay to compensate those entities that have suffered from such illegal behaviour. In the European Union,

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such illegal behaviour is assessed against Article 81 and Article 82 of the Treaty of Rome, or the equivalent legal instruments in the Member States' legislation. Calculating damages clearly entails the estimation of the "but for" or counterfactual market outcome, namely the "competitive" outcome. Whilst this benchmark is relatively clear in theory, in practice it is particularly complex to pin down. Total damages are usually computed as the difference between the actual price and the counterfactual one, multiplied by the quantity previously purchased by the customers (to which interest is added).

As for policy, although (treble) damages have been a feature of US antitrust for a long time (cfr. Sections 4, 5 and 16 of the Clayton Act), private damages actions are a relatively new reality in Europe. In Europe, National Competition Authorities (NCAs) and the European Commission (EC) have almost exclusively made use of the fines' regime to punish wrongdoers and to deter further behaviour incompatible with competition law (some notable cases being the fines to the participants in the vitamins' cartel or those to Microsoft).

However, the EC has very recently published a White Paper precisely on this topic.[1] The EC is mostly promoting actions by key entities and stakeholders; yet, in the White paper, it recalls of the existence of the right of an individual to sue for damages, as the European Court of Justice noted in 2001 and 2006.[2]

The EC is also currently acting as a plaintiff itself and seeking damages from a cartel between elevators' companies that has affected EC premises themselves. One of the reasons of this lawsuit, admittedly, is precisely to show this new avenue to citizens and businesses.[3]

The above action is an example of follow-on private damages (FOPDs), as the complaint is lodged after a 'guilty' finding by the NCA (or the EC). By contrast, stand-alone private damages (SPDs) are those sought in absence of intervention by the NCA (or the EC).

1.2 Objective of this paper

In this paper I investigate under what conditions the introduction of SPDs should (i) increase deterrence and (ii) increase welfare.

In light of the EC White Paper, one can expect that both follow-on and stand-alone damages will play a role in European competition policy and enforcement. Incidentally, it is worth stressing that very little information about actual cases can be found in the public domain. Contrary to NCA decisions of which non-confidential versions are made public, private litigation will typically involve very strict confidentiality clauses on all parties, so very little empirical analysis can be performed in this field.

Whilst, intuitively, damages also grant a further level of deterrence of anti-competitive behaviour (alongside antitrust fines), current EC policy makes clear that the main aim of antitrust damages should be to compensate customers for the damage received (single damages).[4]

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[1] Commission of the European Communities, White Paper on Damages actions for breach of the EC antitrust rules, COM(2008) 165 final, 2 April 2008; and accompanying Commission Staff Working Paper.

[2] Case C-453/99, Courage and Crehan, [2001] ECR I-6297, and Joined Cases C-295-298/04, Manfredi, [2006] ECR I-6619.

[3] "This case represents an important example of a coherent attitude of the Commission in acting both as market regulator and as diligent administrator of EU taxpayers' money. As a result of the anticompetitive behaviour of these companies, the EU institutions, and so the European taxpayer, have suffered financially by paying over the odds for the installation and maintenance of lifts and escalators. It is our duty to seek compensation for these damages. We act firmly where we find contractors act illegally [. . .] the Commission is doing its utmost to encourage and facilitate actions for damages before national courts by victims of anticompetitive behaviour. In this case, we are leading by example". Neelie Kroes, European Commissioner, quoted by EC Press Release IP/08/998 of 24 June 2008.

[4] See the EC White Paper on Damages actions (at 1.2). As a confirmation of this, the UK High Court has recently ruled that in damages claims following an infringement decision by the European Commission, the correct claim is for compensatory and not

The basic model will take the probability of the NCA unveiling anti-competitive activity as exogenous (it will then be endogenised).[5] Hence, the only actions in the basic model concern (1) whether the upstream firm wants to behave competitively, given exogenous parameters and (2) whether the customer (or plaintiff) wants to go to court.

For clarity purposes, I focus on (potentially illegal) unilateral behaviour by the upstream firm (Art. 82 infringements), so as to only shed light on the implications of private antitrust enforcement. The effect that private enforcement has on illegal collective behaviour by the upstream oligopolists (Art. 81 infringements) could become an extension of this paper.

I proceed as follows: Section 2 briefly reviews the key existing literature in the field; Section 3 introduces the basic model; Section 4 derives the equilibrium outcome of the basic model; Section 5 provides a welfare analysis; Section 6 introduces the possibility of class actions and it assesses the welfare impact; Section 7 adds the possibility of type I errors by the NCA (false convictions) and of type II errors (false acquittals) and investigates whether the results previously found still hold; Section 8 endogenises the NCA's behaviour by introducing a resource constraint with a trade-off on the proportion of industries investigated vs. the error sizes once the investigation has begun; Section 9 concludes, draws some policy implications and suggests further modelling extensions.

2   Literature

A review of the literature related to my work cannot but begin with the seminal work by Becker (1968) on optimal punishment. This has generated a wealth of research in different fields, including of course antitrust enforcement. Becker introduced what has become known as the "Beckerian (optimal) fine", namely the penalty that wrongdoers should face. This should be calculated as the product of (i) the total harm done to society through that behaviour and (ii) the inverse of the probability that the wrongdoer is caught and convicted. Of course, it remains debatable how exactly to compute this fine in reality or if this *ex ante* optimal fine remains *ex post* optimal (the wrongdoer might go bankrupt as a consequence, potentially leaving the industry less competitive).

Next, I need to briefly provide an overview of two, separate, strands of literature: first, tort law and law and economics, especially insofar as information acquisition is concerned; second, (optimal) antitrust enforcement. Part of the contribution of this paper is to start bridging aspects of these two literatures. It is in this light that I construct the basic model discussed in the next Section, through which I will address policy questions which I believe to be important.

As for a brief review of the former literature, Milgrom and Roberts (1986) find that – in a litigation context – if the decision-maker is not sufficiently informed, competition between litigants leads to efficient decision-making only if the parties' interests are sufficiently opposed.

Dewatripont and Tirole (1999) discuss the role of advocates and their positive impact on policy-making decisions thanks to the competition that emerges from agents representing opposing special interests (as opposed to a single non-partisan decision-making body). This effect works via two channels: the alignments of advocate's and party's interests; and the right incentives to appeal a wrong decision by the arbitrating body.

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exemplary damages (Devenish Nutrition). See Devenish Nutrition Limited et al and Sanofi-Aventis SA et al [2007] EWHC 2394 (Ch) Mr Justice Lewison.

[5]Since the main interaction that I want to study is between public and private enforcement, I will abstract from the EC and only focus on the NCA and the local court (adding the EC would add a layer of complexity without giving any new insight).

Iossa and Palumbo (2007), in a framework of dispute resolution between two parties by a decision-maker, subject to possible appeals, show that information provision by the parties creates better monitoring (through appeals) and reduces the scope for opportunistic behaviour by the decision-maker than information provision by an external investigator appointed by the decision-maker.

Parisi (2002) – in an adjudication setting – compares the common law tradition of adversarial systems to that of civil law (inquisitorial system by an independent party). He introduces a (rent-seeking) litigation expenditure variable by parties, chosen simultaneously. Intuitively, he finds that there is excessive litigation expenditure (compared to the socially optimal level) in the adversarial system equilibrium.

Shavell (1997) provides a very clear analysis as to why and how the private incentives to sue (not necessarily in an antitrust context) will lead in equilibrium to a level of litigation that is different from the socially optimal one; this effect will be clearly present in my model. He stresses that the direction of the effect is actually ambiguous: when choosing her optimal effort, the litigant does not internalise the extra cost borne by society (through courts) or the opposing party; nor does she internalise the extra benefit to society thanks to the deterrence mechanism though. The author then discusses some possible corrective mechanisms (fees, taxes and subsidies).

Next, I briefly consider the antitrust enforcement literature. Much of the attention has been devoted to cartel detection and penalties; here, I focus on more general antitrust enforcement issues.

McAfee, Mialon and Mialon (2008) provide what is probably the closest modelling framework to mine. There, the authors assume that industry players are better informed than public enforcers, but the former can also use antitrust laws strategically. Nature endows a firm either with an anti-competitive action or with a pro-competitive one. Then the firm decides whether to take that action or none at all. Customers (downstream firms) receive a perfect signal about the action of the upstream players, whilst the NCA only gets it with probability $q > \frac{1}{2}$. As there are no fixed costs of a complaint, a cheap talk problem arises. Litigation costs are modelled to be higher for the defendants than for the plaintiff. The authors conclude that

> "[...] if the court is sufficiently accurate, adding private enforcement to public enforcement always increases social welfare, while if the court is less accurate, it increases welfare only if the government is sufficiently inefficient in litigation. Pure private enforcement is never strictly optimal. Public enforcement can achieve the social optimum with a fee for public lawsuit that induces efficient information revelation. Private enforcement can also achieve the social optimum with private damages that are efficiently multiplied and decoupled."

Bourjade, Rey and Seabright (2007) is another paper related to this strand of research. They develop a model where settling is allowed for and where the NCA can fine-tune the costs of litigation (opening a case and/or proceeding to a full trial) as well as the size of damages awarded upon success. Pre-trial settling plays an important role, which also holds true in reality. They conclude that

> "Out-of-court settlements can have a radical impact on the results of a system of private actions, and any policy proposal needs to be evaluated with this in mind."

This is because courts will embed into their own beliefs the self-selection of defendants that have preferred not to settle (these are typically innocent).

Baker (1988) investigates the effect of asymmetric private information by firm and plaintiff on likelihood of success of antitrust damage recovery. Treble damages also play a role and they may induce buyers to increase

their purchases in equilibrium. In absence of private information, antitrust laws make no difference to equilibrium allocation.

Briggs et al. (1996) look at the interaction between government suits and private suits and the signalling effects they have on each other, together with the self-selection of cases the Government drops. The possibility of settling is also considered.

On the more legal side of the antitrust enforcement literature, there have also been a number of relevant contributions.

Boege and Ost (2006) discuss the role of private antitrust enforcement in Germany and anticipate its future development. They argue that public and private antitrust enforcement are complementary and the latter clearly reduces the burden on NCAs. However, they also point out the risk that damages will reduce effectiveness of leniency programmes (relevant for cartel cases).

Diemer (2006) is sceptical about private actions leading to better antitrust enforcement due to the expected sheer number of unmeritorious claims. He nevertheless believes that public and private enforcement are complements and emphasises the (potentially strategic) role of discovery.

In addition, there is a very rich literature on the role of public vs. private enforcement of antitrust. For an excellent review the reader is directed to Segal and Whinston (2006), who provide an economic perspective on the issue. As for the more legal side of the literature, Wils (2003), for instance, originally argued quite broadly against private antitrust enforcement:

> "[F]rom the perspective of ensuring that the antitrust prohibitions are not violated, public antitrust enforcement is inherently superior to private enforcement, because of more effective investigative and sanctioning powers, because private antitrust enforcement is driven by private profit motives which fundamentally diverge from the general interest in this area, and because of the high cost of private antitrust enforcement. There is not even a case for a supplementary role for private enforcement, as the adequate level of sanctions and the adequate number and variety of prosecutions can be ensured more effectively and at a lower cost through public enforcement."

More recently though, along with the policy shift on damages by the European Commission, Wils conceded that there could still be a *compensatory* role played by damages (see Wils (2009)).

Part of the objective of this paper is therefore to bridge the information acquisition literature from law and economics to that of private and public antitrust enforcement; in doing so, I am going to build a model through which I can study the impact of different policy regimes on equilibrium outcomes and welfare. Further, and differently from most of the literature, I will study the separate effects of FOPDs and SPDs and I will show that equilibrium outcomes will differ according to market size.

3 Basic model

3.1 Setup

In this Section I introduce the various components of the basic model. On the demand side, there is a single representative buyer in each industry i and the demand schedule in industry i is $Q_i = s_i(1 - p_i)$, where s_i can be

interpreted as a parameter of market size (the representative buyer in industry i buys s_i units of the good when the price is 0).

On the supply side, in each industry i, there is a monopolist. Marginal costs of production are zero for simplicity and without loss of generality. Both buyers and sellers are risk-neutral and have discount factor $\delta = 1$. The profit-maximising price is therefore $\frac{1}{2}$ in each industry (i.e. $p_i = p = \frac{1}{2}$ $\forall i$).

3.2   The monopolist's choice ($t = 0$)

The game has three periods. At time $t = 0$, the monopolist decides whether to behave competitively (and set $p = 0$) or to commit an abuse (setting $p = \frac{1}{2}$, thus achieving the monopolistic profit).[6] If an abuse has been carried out, quantity demanded in equilibrium will be $\frac{s_i}{2}$, profits will be $\frac{s_i}{4}$, consumer surplus $\frac{s_i}{8}$ and total welfare $\frac{3s_i}{8}$ (pre-antitrust enforcement). The competitive outcome, by contrast, entails a quantity demanded of s_i, nil (supernormal) profits and consumer surplus (which coincides with total welfare in this case) of $\frac{s_i}{2}$. It is also in this sense that s is a proxy for market size: both monopoly profits and welfare are (linearly) increasing in s.

3.3   The NCA ($t = 1$)

At time $t = 1$, the NCA (due to limited resources) only investigates a fraction $0 < \lambda < 1$ of industries (markets) and this is common knowledge. The NCA receives no signal about the probability that an abuse has occurred and is bound to monitor all markets in principle, it cannot focus its efforts depending on s_i.[7]

The NCA budget is exogenously given. The main reason as to why a legislator would not want to let the NCA determine its own budget through enforcement is to avoid moral hazard. Moreover, this reflects reality.[8]

If the NCA has investigated a market, it proves guilty a firm that has charged $p = \frac{1}{2}$ with probability 1 – that is, in the basic model, there are no type II errors (false acquittals). I let $x > 0$ represent the exogenous antitrust fining policy as a proportion of the supra-competitive profit made by the firm ($\frac{s_i}{4}$).[9]

In the basic model there are no type I errors (false convictions) either: if a firm has charged the competitive price, the NCA either does not investigate it (with probability $1 - \lambda$) or investigates it (with probability λ) but correctly proves it innocent. It is common knowledge that the NCA makes no errors. Errors are introduced in Section 7 below.

3.4   The plaintiff ($t = 2$)

If the NCA has charged a firm, the buyer (whom I will also refer to as plaintiff or complainant) then decides at $t = 2$ (1) either to do nothing or (2) to ask for follow-on private damages (FOPD), at zero cost. This assumption

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[6]Whether charging $\frac{1}{2}$ actually amounts to an abuse of a dominant position is a matter that I do not want to discuss from a normative perspective. In my model, I simply take as a given that charging $\frac{1}{2}$ is a "reduced-form" abusive action yielding the monopolistic profit. I also assume (for simplicity and without loss of generality) that firms cannot charge a price in the interval $(0, \frac{1}{2})$.

[7]Under national and European legislation there could be some *de minimis* thresholds under which the NCA is not bound to intervene; I normalise these to 0 so that the NCA can in principle investigate any market in the economy.

[8]I am not aware of any significant contribution in the antitrust literature that has endogenised the NCA budget - mostly, I believe, because of the moral hazard I mentioned in the text. In addition, differently from the central banking literature where a central bank could inject several €bn into the economy, NCA "resources" in the antitrust literature simply refer to the total wage bill (or staffing requirements) at the NCA, which is a negligible fraction of the size of an economy, thus making any general equilbrium consideration rather void. In any case, I could impose a small flat tax to finance the NCA on all consumers, in all states (i.e. regardless of whether the abuse has been carried out), but this would not add any insight to the analysis.

[9]For a broad and useful discussion of antitrust fines in theory and in practice, see Wils (2006).

(a normalisation in fact) can easily be justified with reference to arguments brought forward by Wils (2009)[10].

If the plaintiff goes for FOPDs, the court will determine damages $\frac{s_i}{4}d_F$ for sure, where $0 < d_F \leq 1$ is the damage multiplier (I cap d_F at unity because the EC has clearly opted for single damages; in the US, by contrast, treble damages mean that $d_F = 3$). The latter action is clearly dominant, in that there is a positive benefit and no cost, and is thus preferable to the outside option of inaction.[11] The monopolist, in this case, would be left with a payoff of $\frac{s_i}{4}(1 - x - d_F)$.[12]

I want to study the impact of introducing stand-alone private damages (i.e. providing plaintiffs with the possibility of lodging an action with the courts absent NCA intervention) on equilibrium outcomes and welfare.[13] SPDs come with a damage multiplier $d_S \geq 0$ of abusive profits. I let courts have probability $q \leq 1$ of proving the abuse. The better relative effectiveness of the NCA vis-à-vis the local court can be justified with reference to the longer and more specialised experience that the NCA has accumulated in this area, as well as a larger volume of cases at any point in time (they are higher up on the learning curve).[14]

If a buyer wants to bring an SPD action, she incurs information acquisition (or evidence-gathering) costs. I let E be a (continuous) choice variable for the plaintiff, in the interval $[0, 1]$. E should be thought as a variety of "effort" elements, such as the opportunity cost of time for managers, legal and procedural costs, the time spent searching for old invoices and gathering time series of price offers from suppliers, etc. Exerting effort E carries a unit cost $c > 0$ and a fixed cost $F > 0$ (the latter can be seen as court fees).

I let $q = \sqrt{E}$.[15] I think of the court as a technology, with effort E by the plaintiff being unobservable to the court. More effort devoted to information acquisition by the plaintiff increases the probability that the court finds the upstream firm guilty; however, there are diminishing returns.

The most interesting case is therefore when the NCA does not investigate the abuse (with probability $1 - \lambda$). At this stage ($t = 2$), the plaintiff has to choose between lodging an SPD action before a local court (thus paying $cE + F$) and no action.

3.5 Frivolous claims

A further element of the model is the possibility for plaintiffs to "fabricate evidence" and bring a frivolous (or unmeritorious) claim, i.e. going to court even if there has been no anti-competitive action. This is not a far-fetched scenario and it has been in fact looked at in detail by the literature (see, for instance, Polinsky and Rubinfeld

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[10] "Public antitrust enforcement has a strong facilitating effect on private actions for damages. Indeed, follow-on actions for damages are much easier to bring than stand-alone actions for damages, because the public enforcement action will have established the existence of the antitrust violation, and may also have generated useful evidence as to causation and as to the harm caused to the claimant in the follow-on action."

[11] That is, throughout the paper I focus on damages that at most make the plaintiff whole. For other possible objectives that damages can attain the reader is directed to Fisher (2006).

[12] This payoff will be negative throughout my model, as I will assume that $x + d_F > 1$ to allow for the case of partial deterrence (if $x + d_F < 1$ there would be no λ that deters anti-competitive behaviour). One should not nevertheless worry about limited liability since this outcome is only due to a normalisation of competitive profits to nil. To see this, assume that a wrongdoer is caught with a probability of 0.5, the abusive profit is 100 and the competitive profit is zero. Only a 200% fine of supra-competitive profits will leave him indifferent between abusing or not. But if he commits the abuse and is caught, he will face a fine of 200 which he will not be able to pay. Now say instead that everything stays the same but the competitive profit is 50. Once again the necessary fine is 200% of supra-competitive profits, i.e. 100. So when the monopolist is caught abusing, he is now in fact solvent.

[13] Of course, in the real world, consumers can also directly complain with the NCA. However, I rule this out not just as a way of simplifying the analysis (which will become notationally cumbersome anyway) but because the current European policy is geared towards giving more scope to private enforcement and SPDs, thus bypassing the NCA by definition in this subset of cases – and it is this mechanism that I am investigating here.

[14] This has also been noted (among others) by Wils (2003, 2009).

[15] This can be generalised to any concave function with domain and range between 0 and 1. I choose this for presentational simplicity.

(1993), Hylton (2002), Diemer (2006) and Wils (2009)).[16] In any case, I remain agnostic as to whether this behaviour occurs in equilibrium. This is because I introduce a parameter $1 < a < \infty$ which multiplies the unit cost of evidence-gathering c in the case of unmeritorious suits. Hence, "fabricating evidence" is surely costlier than providing "real evidence". But the model also embeds situations in which it is far too costly (as a tends to infinity) to fabricate evidence. In this sense this is a generalisation and not a crucial assumption used to drive certain key results.[17]

3.6   Game in extensive form and social costs

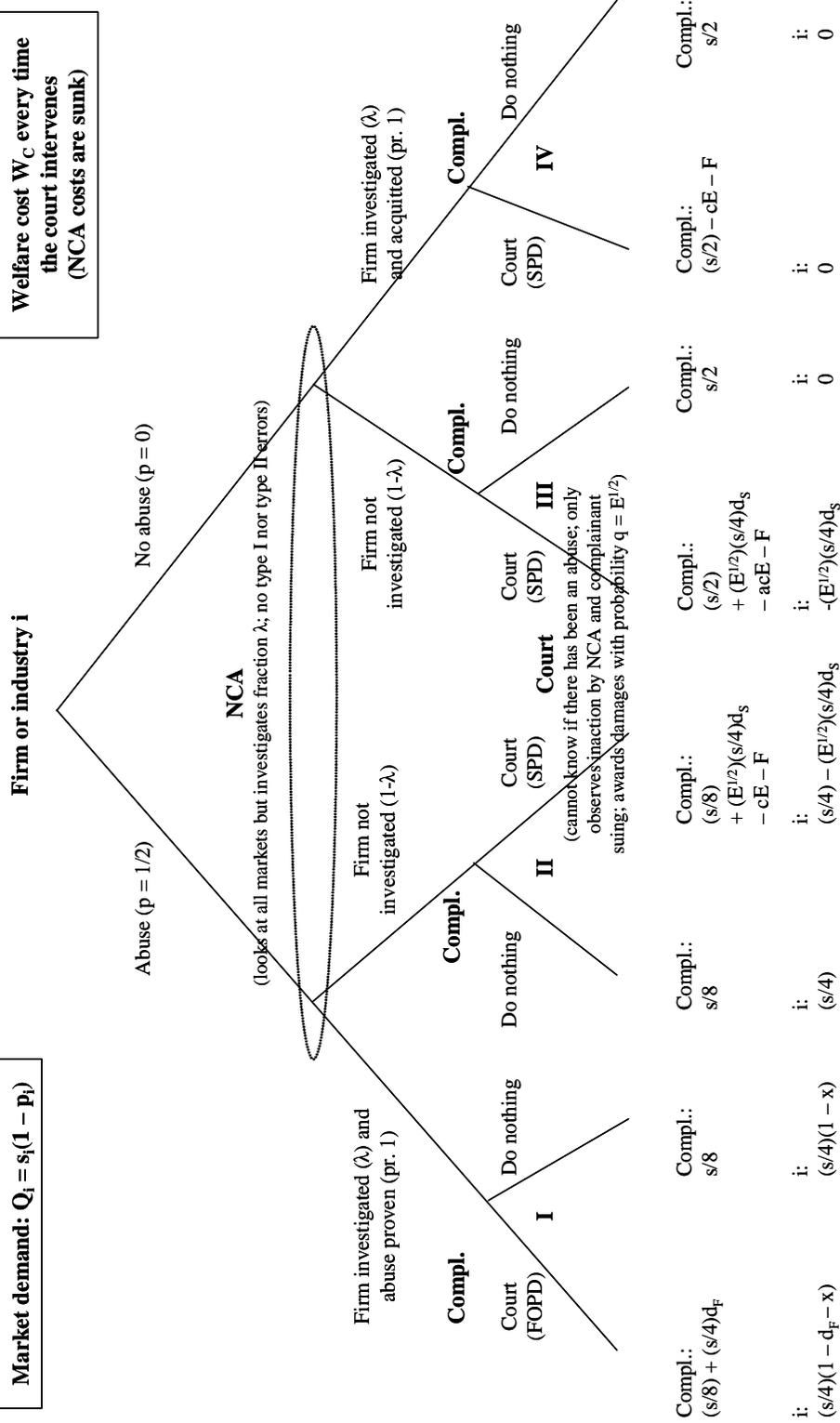Figure 1 below summarises the setting of the basic model in extensive form:

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[16]In the real world, matters are not black or white. Contracting between buyers and suppliers is often complex, occurs over several years, often includes ambiguous clauses; several markets are affected. That is, although "an abuse" might not have occurred, there may be some isolated elements that could be collected as evidence in this direction. Polinsky and Rubinfeld (1993) instead model frivolous suits as those having a lower probability of success in trial. In fact, in equilibrium, my approach is equivalent to theirs, as (given that information acquisition is costlier) less evidence will be sought and the probability of success (itself an increasing function of E) will be lower than that in a meritorious action.

[17]On the other hand, should a be arbitrarily high so that all claims are meritorious, I could give a wider interpretation to E and let it include observables (to the court) such as the number of lawyers, consultant reports or any "countable and observable" evidence. The interpretation of E has to be more restrictive in the presence of unmeritorious claims, else plaintiffs might engage in signalling games with the courts and this would lead me to a different type of game I am not considering in this paper. Many thanks to Florian Schuett for pointing out this issue.

Figure 1: Basic model in extensive form

Market demand: $Q_i = s_i(1 - p_i)$

Firm or industry i

Welfare cost W_C every time the court intervenes (NCA costs are sunk)

Abuse (p = 1/2) No abuse (p = 0)

NCA
(looks at all markets but investigates fraction λ; no type I nor type II errors)

Firm investigated (λ) and abuse proven (pr. 1) Firm not investigated (1-λ) Firm not investigated (1-λ) Firm investigated (λ) and acquitted (pr. 1)

Compl.

Compl.

Compl.

Compl.

Court (FOPD) Do nothing Court (SPD) Do nothing Court (SPD) Court (SPD) Do nothing Court (SPD) Do nothing

I **II** **Court** **III** **IV**

(cannot know if there has been an abuse; only observes \inaction by NCA and complainant suing; awards damages with probability q = $E^{1/2}$)

I
Compl.: $(s/8) + (s/4)d_F$
i: $(s/4)(1 - d_F - x)$

Do nothing:
Compl.: $s/8$
i: $(s/4)(1 - x)$

II
Compl.: $(s/8) + (E^{1/2})(s/4)d_S - cE - F$
i: $(s/4) - (E^{1/2})(s/4)d_S$

Do nothing:
Compl.: $s/8$
i: $(s/4)$

III
Compl.: $(s/2) + (E^{1/2})(s/4)d_S - acE - F$
i: $-(E^{1/2})(s/4)d_S$

Do nothing:
Compl.: $s/2$
i: 0

IV
Compl.: $(s/2) - cE - F$
i: 0

Do nothing:
Compl.: $s/2$
i: 0

The last item I want to discuss in this Section is welfare that goes beyond producer and consumer surplus. I assume that the NCA resources are sunk so $W_N = 0$ (alternatively this can be thought of as a normalisation), but $W_C > 0$ is the welfare cost faced by the court.[18] These social costs are not internalised by the plaintiff and should be thought of as all the time that judges could have devoted to other cases (in private law, contract law etc.) had no damages actions been introduced by the EC.

4 Equilibrium

To solve the basic model, one clearly needs to work backwards, first addressing the four subgames that the plaintiff can face (I, II, III and IV in the extensive form game of the basic model).[19] The full derivation is provided in Appendix A.1. Here I limit myself to stating the key results (interior solutions only), interpreting them economically and providing a graphical analysis for certain parameter values. Interior solutions are guaranteed provided the following two conditions are satisfied for the relevant ranges of s:

$$\left(0 < d_S < \sqrt{\frac{c}{aF}} \wedge 8\sqrt{\frac{cF}{d_S^2}} \leq s \leq 8\sqrt{\frac{acF}{d_S^2}}\right) \vee \left(\sqrt{\frac{c}{aF}} < d_S < \sqrt{\frac{c}{F}} \wedge 8\sqrt{\frac{cF}{d_S^2}} \leq s < 8\frac{c}{d_S^2}\right) \tag{1}$$

$$8\sqrt{\frac{acF}{d_S^2}} < s < \frac{8ac}{d_S^2(a-1)} \wedge 0 < d_S < \sqrt{\frac{ac}{(a-1)^2 F}} \tag{2}$$

Solving the four subgames the plaintiff faces, one determines three market size ranges:
$s < \frac{8}{d_S}\sqrt{cF}$, $\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$ and $s \geq \frac{8}{d_S}\sqrt{acF}$.

A) $s < \frac{8}{d_S}\sqrt{cF}$ (markets are small, complainants never go for SPDs)

Firm i sets $p = \frac{1}{2}$, i.e. commits an abuse, if:

$$\lambda \leq \frac{1}{(d_F + x)} \tag{3}$$

This is intuitive. As plaintiffs never go for SPDs (it is not worth, as the costs of court actions are too large relative to the expected benefit), the presence of SPDs exerts no deterrence effect on the upstream monopolist, whose decision to commit an abuse will only depend on the NCA's actual efficiency (λ), the fine multiplier (x) and the FOPD multiplier (d_F).

B) $\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$ (complainants only go for SPDs if there has been an abuse)

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[18]Wils (2003, 2009), for instance, is one of those who argue that the social costs of private enforcement are larger than the costs of public enforcement.

[19]For notational convenience, I drop subscript i from the market size parameter s.

Figure 2: Deterrence impact of SPDs by market size
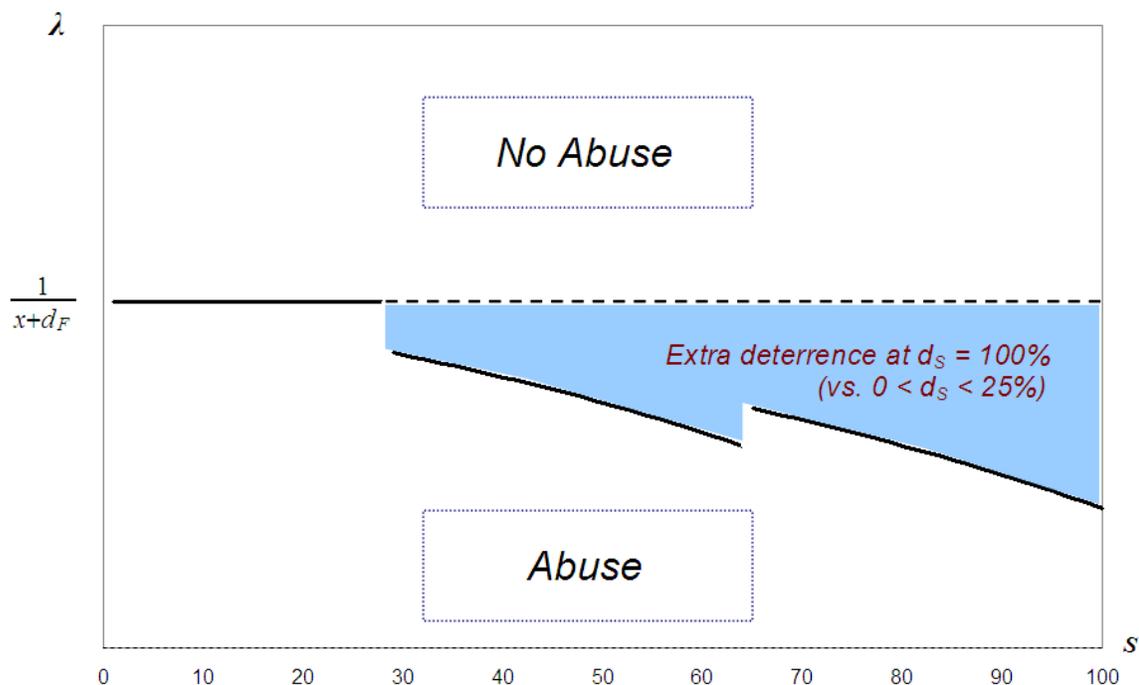
Firm i sets $p = \frac{1}{2}$ if

$$\lambda \leq \frac{8c - sd_S^2}{8c(x + d_F) - sd_S^2} \qquad (4)$$

C) $\quad s \geq \frac{8}{d_S}\sqrt{acF} \qquad$ (complainants always go for SPDs)

Firm i sets $p = \frac{1}{2}$ if:

$$\lambda \leq \frac{8ac - (a-1)sd_S^2}{8ac(x + d_F) - (a-1)sd_S^2} \qquad (5)$$

To clarify these results, I plot them into Figure 2. The main idea is to show the different impact of market size s on the threshold level of NCA efficiency that makes the monopolist indifferent between carrying out and not carrying out the abuse. I do so for two levels of SPDs: $d_S \in [0, 0.25]$ and $d_S = 1$.[20]

For $d_S \in [0, 0.25]$, SPDs play no role at all. For all market sizes consumers have too low an incentive to launch a stand-alone suit even if there is an abuse, and the monopolist anticipates this. In other words, all markets are "small" here. Hence the monopolist is only deterred by the (exogenous) levels of fine multiplier x and FOPD multiplier d_F; the relevant threshold for the abuse is the horizontal schedule at $\frac{1}{(d_F + x)}$. If the NCA is very efficient

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[20] To make the problem non-trivial, I let $x + d_F > 1$. The other parameters used in Figure 2 are set as follows: $F = 1$, $c = 13$, $a = 5$ and $0 \leq s \leq 100$.

and investigates many industries ($\lambda > \frac{1}{(d_F+x)}$), the abuse will not be carried out. This is exactly what would occur in the complete absence of SPDs.

By contrast, when $d_S = 1$, medium markets are *endogenously* found to be those with an s between roughly 29 and 65. In these markets, the monopolist knows that upon committing an abuse the consumers will go and seek SPDs, but they would not do so if the abuse is not carried out. There is therefore an extra element of deterrence for the monopolist (the shaded area), and this comes in the form of a trade-off between the efficiency level of the NCA and the market size (the downward sloping portion of the curve in the middle). Market size s matters even within the medium market interval itself because a larger s will determine larger incentives for the plaintiffs, more effort E into the court action, and a higher probability that the monopolist loses the lawsuit (and thus has to pay damages). The discrete jump in the threshold level of λ at around $s \simeq 29$ is due to the very fact that for market sizes up until that point plaintiffs place no deterrence, but from that point on all plaintiffs would go to court, and the monopolist realises that the expected cost of abusing jumps discontinuously. Finally, large markets start "biting" at market sizes larger than about $s \simeq 65$. These markets are *endogenously* defined as those where plaintiffs will always go and seek SPDs, no matter if an abuse has been carried out, because the expected private benefits of doing so exceed the costs. The initial jump in the deterrence theshold is precisely due to the fact that the monopolist, anticipating such behaviour, now faces a higher disincentive to behave competitively, since she faces higher penalties even when "playing fairly". So she might just as well commit "more abuses" (which, to be precise, means being indifferent between abusing and behaving competitively at a higher level of NCA efficiency).

It is important to note that the parameter values I have chosen were such that (1) and (2) were satisfied (for the relevant intervals of s), so that I could focus on the more interesting case. Nevertheless, even when either (1) or (2) (or both) do not hold (some sort of "corner solutions"), the same analysis can be performed and the results would still make economic sense. The only difference is that in those cases there would be full deterrence from a certain level of s, which in Figure 2 would be tantamount to the thick black line being flat at $\lambda = 0$ after some level of s. So the algebraic solutions provided in Appendix A.1 are robust to all cases. On the other hand, the more interesting case is also the likeliest to hold in the real world, as few people would be prepared to argue that antitrust is achieving (or indeed *can* achieve) full deterrence.

This completes the discussion of how agents behave in equilibrium in the basic model. Next, I look at the welfare impact of introducing the possibility of stand-alone damages actions.

5   Welfare

5.1   Total welfare

The various outcomes reviewed in Section 4 above clearly lead to different welfare levels. By "welfare" I mean total welfare in the economy, which comprises: consumer surplus (gross surplus deriving from the pricing decision of the monopolist plus possible damages minus possible cost of bringing a court action), producer surplus (profits minus possible fines and damages), possible fine revenues accruing to the NCA and (entering the welfare function with a negative sign) the opportunity cost of the courts' time. Notice that fine and damages' revenues are simply transfers between agents and will not affect total welfare.

The exercise in this Section is therefore to pin down the impact on welfare of setting SPDs at certain levels, for all values of market size (s) and NCA efficiency (λ).

To start off, I consider the benchmark example of an economy without SPDs. Total welfare will only depend on the exogenous policy parameters and x and d_F, as well as on the efficiency level of the NCA. If the NCA is relatively efficient ($\lambda > \frac{1}{x+d_F}$), there will not be an abuse, nor will there be damages. Maximum total welfare is achieved in the economy ($\frac{s}{2}$ per industry). By contrast, if the NCA is not sufficiently efficient, there will be an abuse, which will be caught with λ probability and FOPDs will be sought in those cases. Total welfare is lower ($\frac{3s}{8} - \lambda W_C$) because the sum of producer and consumer surplus is lower under monopoly than under perfect competition and total welfare ($\forall \lambda \leq \frac{1}{x+d_F}$) is actually *decreasing* in the efficiency level of the NCA. This is because a higher λ (yet less than $\frac{1}{x+d_F}$) is not sufficient to deter but at the same time it increases the number of cases brought to court for FOPDs, which causes a loss to society.

Next, in Table 1, I provide the general solutions for $\forall d_S > 0$. Small markets are those with $s < \frac{8}{d_S}\sqrt{cF}$. In these cases, the same analysis as that of an economy without SPDs applies. In medium and large markets, by contrast, at an interior solution, the threshold that leaves the monopolist indifferent between committing an abuse and behaving competitively depends on market size and the SPD multiplier themselves (in addition to the unit cost of effort, the FOPD multiplier and the fine multiplier). In the case of medium markets, maximum total welfare ($\frac{s}{2}$) can be achieved for a sufficiently efficient NCA; whereas in large market this cannot be achieved because there will be (welfare-diminishing) frivolous claims. For completeness, in Table 1 I also consider the cases where (*1*) and (*2*) do not hold and full deterrence is achieved.

Table 1 - Market outcome and total welfare as a function of the SPD multiplier
(by market size and NCA efficiency level)

	Outcome	**Welfare**
Small markets ($s < \frac{8}{d_S}\sqrt{cF}$)		
$\lambda \leq \frac{1}{x+d_F}$	Abuse, FOPDs, no SPDs	$\frac{3s}{8} - \lambda W_C$
$\lambda > \frac{1}{x+d_F}$	No abuse, no damages	$\frac{s}{2}$
Medium markets ($\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$)		
if (1) holds		
$\lambda \leq \frac{8c-sd_S^2}{8c(x+d_F)-sd_S^2}$	Abuse, FOPDs, SPDs	$\frac{3s}{8} - (1-\lambda)(\frac{s^2 d_S^2}{64c} + F) - W_C$
$\lambda > \frac{8c-sd_S^2}{8c(x+d_F)-sd_S^2}$	No abuse, no damages	$\frac{s}{2}$
if (1) does not hold		
$\forall \lambda$	No abuse, no damages	$\frac{s}{2}$
Large markets ($s \geq \frac{8}{d_S}\sqrt{acF}$)		
if (2) holds		
$\lambda \leq \frac{8ac-(a-1)sd_S^2}{8ac(x+d_F)-(a-1)sd_S^2}$	Abuse, FOPDs, SPDs	$\frac{3s}{8} - (1-\lambda)(\frac{s^2 d_S^2}{64c} + F) - W_C$
$\lambda > \frac{8ac-(a-1)sd_S^2}{8ac(x+d_F)-(a-1)sd_S^2}$	No abuse, SPDs	$\frac{s}{2} - (1-\lambda)(\frac{s^2 d_S^2}{64ac} + F + W_C)$
if (2) does not hold		
$\forall \lambda$	No abuse, SPDs	$\frac{s}{2} - (1-\lambda)(\frac{s^2 d_S^2}{64ac} + F + W_C)$

In sum, the total welfare impact of introducing SPDs ($d_S > 0$ vs. $d_S = 0$) is ambiguous: on one hand deterrence is improved, on the other there are public and private litigations costs. Moreover, there is an extra welfare loss so

long as there is a possibility of unmeritorious claims. Furthermore, by increasing the SPD multiplier, large markets materialise at a lower market size, thus magnifying the welfare loss arising from frivolous claims. For all these reasons it is far from clear what the optimal d_S level is (I consider the optimal policy in Section 8 below).

I will consider a specific parametrisation in Section 7, after introducing type I and type II errors, and then provide a graphical interpretation of what occurs to welfare.

5.2 Consumer welfare

So far I have focused on comparing *total* welfare levels, before and after the policy change. European competition policy, by contrast, focuses by and large on *consumer* welfare. If I applied this standard to my results, I could conclude that consumer welfare is either unchanged or it is improved thanks to the introduction of SPDs.

Proposition 1 *Upon the introduction of SPDs, consumer welfare cannot decrease in expected terms.*

Proof. Introducing SPDs has two sets of (gross) negative effects on total welfare: (A) the social costs of litigation (courts' opportunity cost of time) and (B) private costs of litigation. But (A) does not affect consumer welfare. And the private costs in (B) cannot be larger than the (expected) private benefit of bringing an SPD action, else the consumer will not lodge a suit. ∎

More generally, one should expect the introduction of SPDs to typically strictly increase consumer welfare, thanks to the deterrence effect (some abuses no longer occur after the introduction of SPDs) as well as the SPD monetary transfer received from the seller (which had a neutral effect on total welfare).

The only two (rather extreme) cases in which there is no change in consumer welfare upon SPD introduction are (i) if there was already full deterrence before the policy change and a is large enough to rule out all opportunistic behaviour ($a > \frac{s^2 d_S^2}{64cF} \; \forall s$); or (ii) if all markets are "small" ($d_S < \frac{8}{s}\sqrt{cF} \; \forall s$) so that no plaintiff avails herself of the SPD option (the first case I considered when I discussed Figure 2).

6 Class actions

So far I have assumed the presence of a single representative consumer in each industry. However, in the real world, industries are populated by numerous buyers and, from an antitrust enforcement perspective, class actions are a prominent feature of litigation.[21]

In this Section I therefore want to slightly modify my basic model to account for this and then draw some conclusions. To this end, I allow for the presence of n symmetric buyers in each industry i. Aggregating demands and solving the simple monopoly model one finds that the monopoly profit will now be $\frac{ns}{4}$. However, consumer surplus at individual level and the decision as to whether a buyer should bring a damages action (alone) are unaffected by the presence of n consumers.

As for the FOPD part of the game, the assumption of a class action has a bearing on social welfare. These lawsuits come at no cost to plaintiffs, so introducing class actions will not affect the incentives' structure of

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[21] For a textbook discussion of class actions, with its broad benefits and drawbacks, the reader is directed to Cooter (2008) and the references therein provided. From an actual policy perspective Rueggeberg and Schinkel (2006) have recently argued in favour of a centralised consolidation of fragmented individual antitrust damage claims in Europe.

complainants and upstream firms. By contrast, total court costs will significantly drop, since instead of accruing its opportunity cost (W_C) n times, the local court will only face it once.

Things are slightly more interesting on the SPD side of the game. The most obvious way of integrating class actions into my basic model is to assume that F, the fixed cost of filing an SPD lawsuit, is actually shared, symmetrically, by the n complainants. Moreover, the evidence put forward by complainants is additive. So the probability of the court awarding damages is still $q = \sqrt{E}$, but E now represents total evidence (effort) gathered across the whole industry (comprising n complainants).[22]

The mechanics of the solution are identical to those discussed earlier in the paper. And the precise form of the solution is also extremely close to that of the basic model, with the key (and somewhat unsurprising) difference that the threshold for a small vs. a medium market and that of a medium vs. a large market occur at a lower level of s. I summarise the old and new thresholds in Table 2 below:

Table 2 - How class actions change the thresholds for plaintiffs' decisions

Market size	Without class actions	With class actions
Small (never seek SPDs)	$s < \frac{8}{d_S}\sqrt{cF}$	$s < \frac{8}{d_S}\sqrt{\frac{cF}{(2n-1)}}$
Medium (seek SPDs only after an abuse)	$\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$	$\frac{8}{d_S}\sqrt{\frac{cF}{(2n-1)}} \leq s < \frac{8}{d_S}\sqrt{\frac{acF}{(2n-1)}}$
Large (always seek SPDs)	$s \geq \frac{8}{d_S}\sqrt{acF}$	$s \geq \frac{8}{d_S}\sqrt{\frac{acF}{(2n-1)}}$

The intuition is clear: for some marginal complainants, the costs of bringing an action exceeded the benefits; but by introducing class actions the benefits remain the same whilst the costs fall as n rises.

Hence, with class actions, there will unambiguously be more deterrence in the framework of my model.

What is ambiguous, though, is their effect on welfare, as this depends on the precise values of d_S and λ. For "high" values of λ ($\lambda > \frac{1}{x+d_F}$), class actions reduce total welfare, as the NCA was already sufficient by itself for complete deterrence and adding class actions is equivalent to increasing the number of unmeritorious suits. If λ is too low to provide deterrence even after the introduction of class actions (when $\lambda \leq \frac{8c-sd_S^2}{8c(x+d_F)-sd_S^2}$ and $\frac{8}{d_S}\sqrt{\frac{cF}{(2n-1)}} \leq s < \frac{8}{d_S}\sqrt{\frac{acF}{(2n-1)}}$), then there is no welfare improvement in medium markets (ditto for large markets when $\lambda \leq \frac{8ac-(a-1)sd_S^2}{8ac(x+d_F)-(a-1)sd_S^2}$ and $s > \frac{8}{d_S}\sqrt{\frac{acF}{(2n-1)}}$). Benefits will instead materialise in the intermediate intervals. The graphical analysis in Section 7 below will clarify these statements.

In sum, the impact of class actions on total welfare is ambiguous (once again, if one focused on the impact of class actions on *consumer* welfare, this would be non-negative).

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[22]In a single-shot game, there would be a co-ordination failure: positing that the remaining $n-1$ complainants gather evidence, I have an incentive not to, since the marginal impact on a positive court finding is minimal but I am incurring evidence-gathering costs (I would only pay my $\frac{F}{n}$ share of the fixed fee, otherwise the case would not be opened at all and I would receive no damages). Hence it cannot be an equilibrium. I side-step this issue by assuming that complainants are effectively playing a supergame, or a game where other markets are involved, and failure to co-ordinate would entail punishment such that compliance is preferable for all discount factors. (While acknowledging that this is some sort of shortcut, I believe that trying to model another game within the game I am looking at would add complexity with little benefit.)
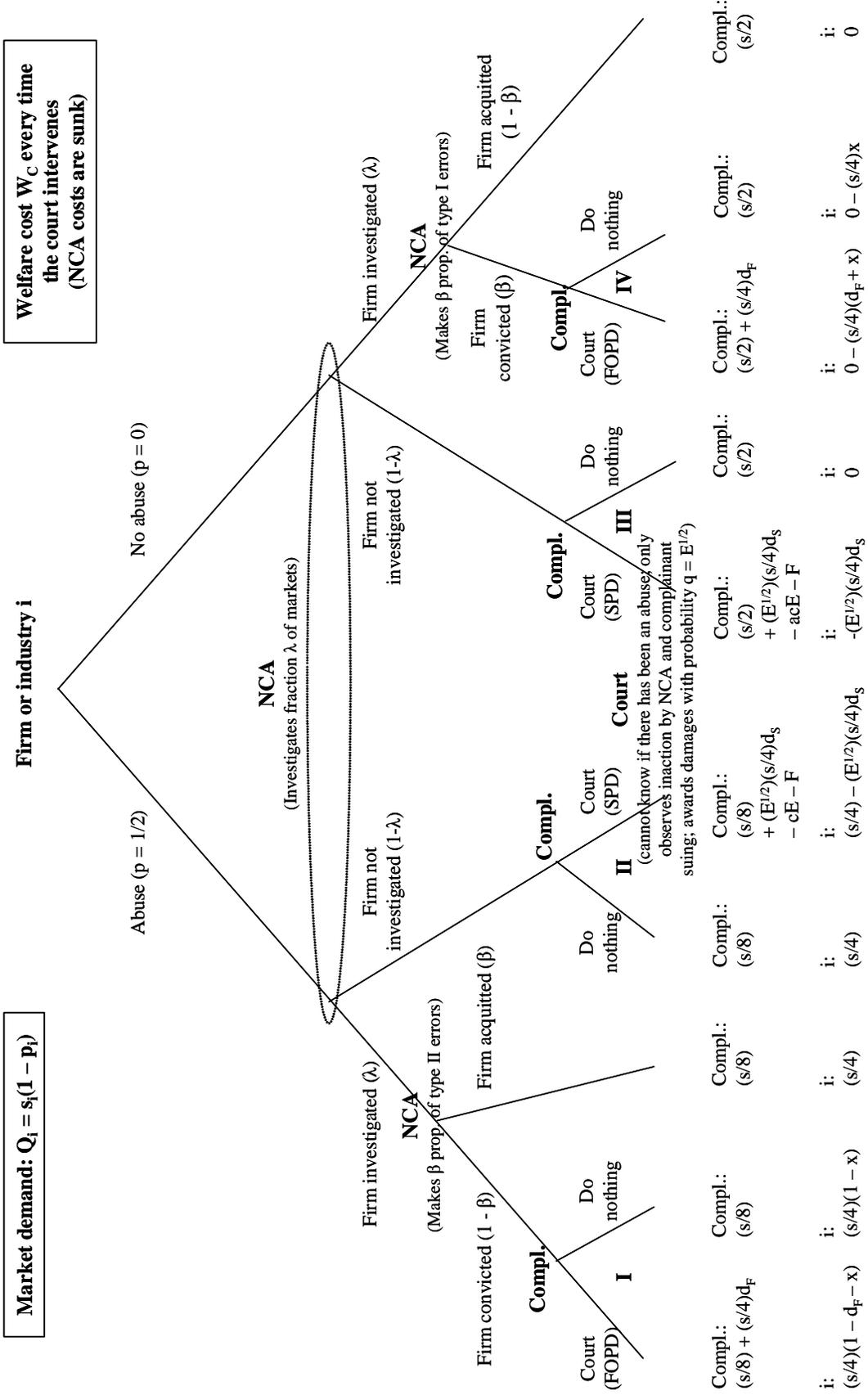
7   Introducing Type I and Type II errors by the NCA

So far I have assumed that the NCA makes no mistakes in its assessment. I now generalise the basic model precisely to account for both type I errors (false convictions) and type II errors (false acquittals), in the presence of a single representative buyer in each market.

To keep the model parsimonious I let the two error sizes be the same, and noted by β (which is common knowledge). Nevertheless, I acknowledge that the literature has typically identified false convictions as being very costly.[23] On the other hand, I can interpret β as being inversely related to amount of resources, e.g. number of staff members, placed on each case: the more staff on the case, the lower β, the higher the probability that the decision made at the end (whether a conviction or an acquittal) is the correct one. Figure 3 below depicts the extensive form of this slightly modified game:

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[23]See Easterbrook (1984). For a more recent discussion on the trade-off between type I and type II errors in antitrust see, for instance, Evans and Padilla (2004), Katsoulacos and Ulph (2008) and Immordino and Polo (2008).

Figure 3: Game with NCA errors in extensive form

Market demand: $Q_i = s_i(1 - p_i)$

Welfare cost W_C every time the court intervenes (NCA costs are sunk)

Firm or industry i

- Abuse ($p = 1/2$)
- No abuse ($p = 0$)

NCA (Investigates fraction λ of markets)

Court (cannot know if there has been an abuse; only observes inaction by NCA and complainant suing; awards damages with probability $q = E^{1/2}$)

Left branch (Abuse):

Firm investigated (λ) — **NCA** (Makes β prop. of type II errors)

- Firm convicted ($1 - \beta$) — **Compl.** — **I**
  - Court (FOPD):
    - Compl.: $(s/8) + (s/4)d_F$
    - i: $(s/4)(1 - d_F - x)$
  - Do nothing:
    - Compl.: $(s/8)$
    - i: $(s/4)(1 - x)$
- Firm acquitted (β) — Do nothing:
  - Compl.: $(s/8)$
  - i: $(s/4)$

Firm not investigated ($1-\lambda$) — **Compl.** — **II**

- Court (SPD):
  - Compl.: $(s/8) + (E^{1/2})(s/4)d_s - cE - F$
  - i: $(s/4) - (E^{1/2})(s/4)d_s$
- Do nothing:
  - Compl.: $(s/8)$
  - i: $(s/4)$

Right branch (No abuse):

Firm not investigated ($1-\lambda$) — **Compl.** — **III**

- Court (SPD):
  - Compl.: $(s/2) + (E^{1/2})(s/4)d_s - acE - F$
  - i: $-(E^{1/2})(s/4)d_s$
- Do nothing:
  - Compl.: $(s/2)$
  - i: 0

Firm investigated (λ) — **NCA** (Makes β prop. of type I errors)

- Firm convicted (β) — **Compl.** — **IV**
  - Court (FOPD):
    - Compl.: $(s/2) + (s/4)d_F$
    - i: $0 - (s/4)(d_F + x)$
  - Do nothing:
    - Compl.: $(s/2)$
    - i: $0 - (s/4)x$
- Firm acquitted ($1 - \beta$):
  - Compl.: $(s/2)$
  - i: 0

Notice that once the NCA makes a finding (and either convicts or acquits) the court cannot overrule and thus either (follow-on) damages will be awarded for sure (after a 'guilty' finding by the NCA) or they will never be awarded (after an 'innocent' finding).[24]

I can therefore solve the model again in the same fashion as for the basic model. The relevant threshold values of λ for an abuse in small, medium and large markets are, respectively[25]:

$$\lambda \leq \frac{1}{(d_F + x)(1 - 2\beta)} \tag{6}$$

$$\lambda \leq \frac{8c - sd_S^2}{8c(x + d_F)(1 - 2\beta) - sd_S^2} \tag{7}$$

$$\lambda \leq \frac{8ac - (a - 1)sd_S^2}{8ac(x + d_F)(1 - 2\beta) - (a - 1)sd_S^2} \tag{8}$$

Proposition 2 $\forall \beta \in (0, 0.5)$ *deterrence cannot improve with respect to the case of no errors by the NCA ($\beta = 0$). When (1) and (2) are satisfied, i.e. where there is partial deterrence in the absence of NCA errors, deterrence strictly worsens when allowing for NCA errors $\beta \in (0, 0.5)$.*

Proof. The second part of the statement directly follows by comparing (6), (7) and (8) to (3), (4) and (5). respectively. The first part of the statement is required to account for the cases where (1) and (2) do not hold (i.e. where the parameters are such that there is full deterrence in any case, cfr. Conditions B.ii, B.iii, C.ii and C.iii in Appendix A.1). ∎

Interestingly, therefore, the whole analysis carries through as before, with the exception of β affecting the threshold level of deterrence (more errors require a higher investigation rate to keep deterrence at the same level). This also means that Figure 2 applies equally well in the presence of β. All that happens in the (s, λ) quadrants is that the vertical intercept $(\frac{1}{(x+d_F)(1-2\beta)})$ rises with β and all the schedules shift up accordingly (the horizontal intercepts are unaffected as these are dictated by market size), as I show in Figure 4.

The intuition is the following. The error size β has a dual negative impact for the NCA: first, a larger β implies more false acquittals (type II errors), which will make abuses more appealing from the monopolist's perspective (since he "can get away with it" more often); second, a larger β implies more false convictions (type I errors), so that the monopolist will be less willing to act competitively (i.e. more prone to commit an abuse), since he would have a higher chance to be punished even when innocent.

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[24]This is not an assumption but rather how the legal system works in Europe: see Art 16(1) of Council Regulation 1/2003; Section 2.3 of the EC White Paper on Damages; or, in the UK for instance, of Section 58 of the UK Competition Act. Notice that these court proceedings (civil actions for damages) are totally different from a possible appeal that the convicted firm may bring against the NCA before a court.

[25]The parametric values of $(d_S, s, a$ and $c)$ for which the inequalities do not hold (and there is thus full deterrence) are the same as those derived in Appendix A.1.

Figure 4: Effect of NCA errors (β) on the abuse thresholds
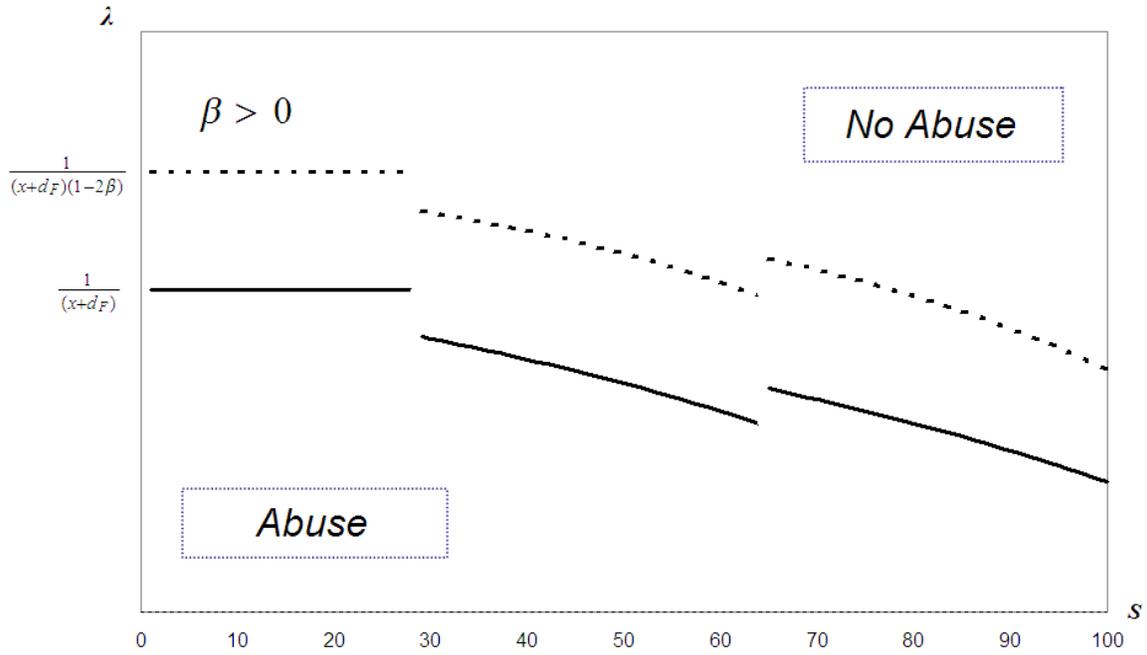
Next, in Table 3, I provide the total welfare levels in the presence of errors (and compare the case of $d_S = 1$ to that of $d_S = 0$).

Table 3 - Market outcomes and total welfare in the presence of NCA errors, with and without SPDs
(by market size and NCA efficiency level)

	$\mathbf{d_S= 1}$	
	Outcome	*Welfare*
Small markets ($\mathbf{s < 8\sqrt{cF}}$)		
$\lambda \leq \frac{1}{(x+d_F)(1-2\beta)}$	Abuse, FOPDs, no SPDs	$\frac{3s}{8} - \lambda(1-\beta)W_C$
$\lambda > \frac{1}{(x+d_F)(1-2\beta)}$	No abuse, FOPDs if NCA error	$\frac{s}{2} - \lambda\beta W_C$
Medium markets ($\mathbf{8\sqrt{cF}\leq s < 8\sqrt{acF}}$)		
if (1) holds		
$\lambda \leq \frac{8c-s}{8c(x+d_F)(1-2\beta)-s}$	Abuse, FOPDs, SPDs	$\frac{3s}{8} - (1-\lambda)(\frac{s^2}{64c} + F) - (1-\lambda\beta)W_C$
$\lambda > \frac{8c-s}{8c(x+d_F)(1-2\beta)-s}$	No abuse, FOPDs if NCA error	$\frac{s}{2} - \lambda\beta W_C$
if (1) does not hold		
$\forall\lambda$	No abuse, FOPDs if NCA error	$\frac{s}{2} - \lambda\beta W_C$
Large markets ($\mathbf{s \geq 8\sqrt{acF}}$)		
if (2) holds		
$\lambda \leq \frac{8ac-(a-1)s}{8ac(x+d_F)(1-2\beta)-(a-1)s}$	Abuse, FOPDs, SPDs	$\frac{3s}{8} - (1-\lambda)(\frac{s^2}{64c} + F) - (1-\lambda\beta)W_C$
$\lambda > \frac{8ac-(a-1)s}{8ac(x+d_F)(1-2\beta)-(a-1)s}$	No abuse, SPDs	$\frac{s}{2} - (1-\lambda)(\frac{s^2}{64ac} + F + W_C) - \lambda\beta W_C$
if (2) does not hold		
$\forall\lambda$	No abuse, SPDs	$\frac{s}{2} - (1-\lambda)(\frac{s^2}{64ac} + F + W_C) - \lambda\beta W_C$
	$\mathbf{d_S= 0}$	
	Outcome $\forall s$	*Welfare $\forall s$*
$\lambda \leq \frac{1}{(x+d_F)(1-2\beta)}$	Abuse, FOPDs, no SPDs	$\frac{3s}{8} - \lambda(1-\beta)W_C$
$\lambda > \frac{1}{(x+d_F)(1-2\beta)}$	No abuse, FOPDs if NCA error	$\frac{s}{2} - \lambda\beta W_C$

One can easily see that introducing errors by the NCA does not affect the basic trade-offs between an environment with SPDs against one without SPDs (i.e. what I showed in Table 1):

- small markets are unaffected by the introduction of SPDs;

- in medium markets, the gain in deterrence has to be traded off against sub-optimal (excessive) investments in effort E, and more court interventions;

- in large markets, there is the same trade-off as in medium markets, plus (on the negative side) the detrimental effect of unmeritorious claims.

It is now instructive to provide, by way of graphical example, a parametrisation equivalent to that made for Figure 2, but capturing error sizes too and focusing on the welfare effects of introducing SPDs. I do so in Figure 5, by comparing the settings discussed earlier in Figure 2, using the same parameter values. In addition, I assume that the NCA makes 5% of (type I and type II) errors. I also assume that the opportunity cost for the court to get involved in a case is 2.5.[26] In sum, Figure 5 adopts the following parametrisation:

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[26]I chose this value by computing one fifth of the monopolistic profit made by the the firm of median size in the example below ($W_C = 20\% * \frac{50}{4}$; the results are very robust to a very wide range of W_C anyway).

$$0 < s \leq 100 \qquad d_S = \{0, 1\} \qquad x = d_F = 1 \qquad F = 1 \qquad c = 13 \qquad a = 5 \qquad \beta = 0.05 \qquad W_C = 2.5$$

Figure 5: Welfare comparisons pre- vs. post-introduction on full (single) damages
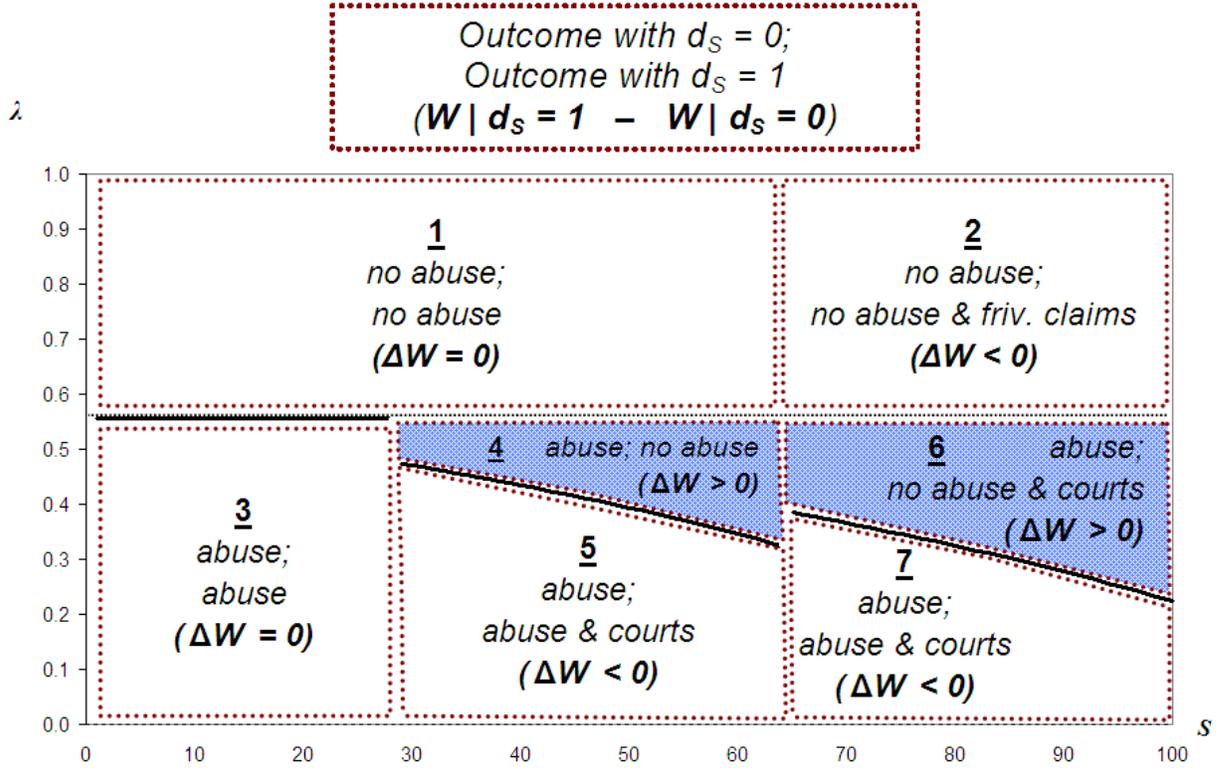
The extra deterrence achieved by SPDs is the shaded area encompassed between the horizontal line in the middle of the graph and the split curve, as already discussed in Section 4.

For small markets, there is no change in welfare when SPDs are introduced, as plaintiffs never go to court (except for FOPDs) and monopolists face no extra level of deterrence. Likewise, for $\lambda > \frac{1}{(x+d_F)(1-2\beta)}$, there is no welfare impact of SPDs on medium markets: monopolists are deterred from abusing anyway and plaintiffs do not engage in unmeritorious claims. These zero-welfare-change situations are depicted in areas 1 and 3 in Figure 5.

Areas 4 and 6 illustrate the region where welfare improves. This corresponds to the area of extra deterrence achieved by SPDs. This is intuitive, as thanks to SPDs, the abuse is deterred.[27]

However, there are two regions where welfare is reduced by the introduction of SPDs: first, in large markets, where the NCA action was sufficiently efficient to deter anti-competitive behaviour even without SPDs (area 2); but introducing SPDs generates wasteful expenditure in unmeritorious claims and court costs. Second, in medium and large markets, welfare drops where the NCA is relatively inefficient (area 5 and area 7). That is, the anti-competitive behaviour is not deterred even when SPDs are introduced and additionally there will be costly litigation

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[27]In area 6, though, there are still unmeritorious claims: so total welfare *could* fall if – keeping all parameters equal - the opportunity cost of the court was at least 60 times as large as that in my numerical simulation (which I would find unreasonable).

(and costly involvement by the courts).

The overall effect is therefore ambiguous. Notice that s is exogenously given, and that antitrust policy must (by and large) be one-size-fits-all, for legal reasons. So policy-making has to take the heterogeneous impact across market sizes into account. This is in addition to λ being unobservable, of course, in the real world, thus creating a further element of uncertainty in the context of the welfare comparisons in Figure 5.

Once again, it is important to stress that this chart is based on parametric assumptions that make (1) and (2) hold. Should this not be the case, the relevant welfare comparisons should be made according to the expressions provided in Table 3 above.

All I wanted to show in this Section was that the basic model can easily encompass errors by the NCA. The next step is to endogenise the optimal NCA policy.

8 Endogenising the NCA's optimal policy

In the previous Section I introduced the possibility that the NCA commits errors during its investigations. Now that there are two policy parameters (λ, the proportion of industries investigated and β, the error size) it is reasonable to ask what the optimal policy would be.

That NCAs are resource-constrained is no news, and the literature has already formalised the trade-off between different activities carried out by NCAs (e.g. Motta and Polo (2003) and Chang and Harrington (2008)).

I impose the following (reduced-form) constraint:[28]

$$0 < \lambda \leq \kappa\beta < 1 \tag{9}$$

To understand this constraint better, consider the following two illustrations, with $\kappa = 2$. At one limit, the NCA can investigate nearly every industry ($\lambda \approx 1$), but its precision would be very poor ($\beta \approx 0.5$); at the other extreme, the NCA could be extremely precise ($\beta \to 0$), but it would hardly have any resource left to investigate several markets ($\lambda \to 0$).

This κ-constraint can also be interpreted as spelling out the NCA technology, namely how broad its analysis can be (λ) for a *given* depth level (β).

One way of looking at the optimal policy is that the NCA therefore maximises total welfare by choosing its investigation rate and its error size, subject to its resource constraint.

So first, I want to find the optimal policy $(\widetilde{\beta}, \widetilde{\lambda})$ in absence of SPDs ($d_S = 0$), as the antitrust regime was until recently. Second, I investigate whether the adoption of an antitrust policy that also relies on SPDs changes this optimal policy. In this second case I set $d_S = 1$, as this is what the EC White Paper on Damages suggests the new policy to be, and re-optimise, to check whether the optimal policy remains $(\widetilde{\beta}, \widetilde{\lambda})$ indeed (I also consider $d_S = 0.75$ as a further example)[29].

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[28]This can be thought (although not necessarily) as deriving from the linear constraint $\lambda + \kappa(\frac{1}{\kappa} - \beta) \leq 1$, with $\kappa \geq 1$, so that $\lambda \leq \kappa\beta$; this is in addition to $0 < \lambda < 1$ (trivially) and $0 < \beta < \frac{1}{\kappa}$. In words, an error size of $\frac{100}{\kappa}\%$ comes "for free". The intuition is that should the NCA believe that it makes more then $\frac{100}{\kappa}\%$ of mistakes, it would be better off devising some simple random mechanism for assessing which behaviour is abusive. The NCA therefore has to spend resources to reduce the error size from the "default level" of $\frac{100}{\kappa}\%$. At the same time, it needs to devote resources to investigate a certain proportion of the markets.

[29]Whilst $d_S = 0.5$ would be more intuitive to consider, the numerical simulation I will use would rule that possibility out in the set of interior solutions.

When $d_S = 0$, any deterrence can only arise through public antitrust enforcement and FOPDs. Given an exogenous fine policy (x) and an exogenous FOPD multiplier (d_F), the NCA simultaneously chooses (subject to its resource constraint, (9)) both β (i.e. how high the vertical intercept in Figure 5 is); and λ (i.e. where to locate with respect to that threshold). Market size does not play a role here, so there is just a unit mass of markets to multiply the welfare function by. With insufficient deterrence, total welfare is $\frac{3}{8}$, whereas with full deterrence it becomes $\frac{1}{2}$. In essence, the total welfare function can only take two values: $\frac{1}{2}$ (no abuse in any market) or $\frac{3}{8}$ (abuse in every market). The NCA will clearly seek to choose (λ, β) so as to attain the former goal. However, this is impossible $\forall (x + d_F) \leq \frac{8}{\kappa}$.

Proposition 3 $\nexists (\lambda, \beta) \in R^+ \mid \lambda > \frac{1}{(x+d_F)(1-2\beta)} \wedge 0 < \lambda \leq \kappa\beta < 1 \wedge x + d_F \leq \frac{8}{\kappa}$

Proof. Solving the first two inequalities jointly one finds the following determinant for the quadratic inequality in β: $1 - \frac{8}{\kappa(x+d_F)}$. Hence there are no (real) solutions $\forall (x + d_F) < \frac{8}{\kappa}$. At $x + d_F = \frac{8}{\kappa}$, there would be a real (and unique) solution for β, but either the resource constraint or the deterrence constraint would be violated. ∎

In sum, in a regime which only relies on public antitrust enforcement, where the NCA can make mistakes and has a resource constraint (indicated by (9)), deterrence can only be achieved if the (exogenous) fine-cum-FOPD-multiplier is greater than $\frac{800}{\kappa}\%$.

An obvious question is what a reasonable value of κ should be.

Proposition 4 *When (1) and (2) hold (i.e. considering interior solutions), single SPDs impose zero deterrence if* $\kappa < \min\{\frac{8c-s}{\beta\{8c(x+d_F)(1-2\beta)-s\}}, \frac{8ac-(a-1)s}{\beta\{8ac(x+d_F)(1-2\beta)-(a-1)s\}}\}, \forall s \in (0, S], \forall \beta \in (0, \overline{\beta})$

Proof. It follows directly from inserting (9) into (7) and (8). ($\overline{\beta}$ is the maximum level of β such that the denominators in (7) and (8) are both positive, i.e. limiting oneself to interior solutions.) ∎

This is why, in the context of the numerical simulation I have been using, I will select $\kappa = 3$. Focusing on $\kappa \in N$ for simplicity, if I chose a lower number, I would implicitly *assume* the outcome, namely that introducing SPDs has no impact on deterrence purely because the NCA has insufficient resource by construction. By contrast, with $\kappa = 3$, *some* level of deterrence *may* be achieved according to the mechanisms that I will describe below.

Next, I consider the case where private enforcement can play a role, and in particular where the antitrust regime consists of single damages (both FOPDs and SPDs); this is indeed what the EC White Paper on damages actions seems to suggest. My aim here is to determine if the NCA should allocate its resources (λ, β) in a different fashion from a world without private enforcement through SPDs.

The program is slightly more involved and I report it in Appendix A.2. The seven programmes set in Appendix A.2, taken as a whole, are non-trivial. I therefore revert to the earlier numerical example discussed in Figure 5 and I let β vary, studying its impact on total welfare (λ will always be optimally set at $\kappa\beta$, given the resource constraint). I do so in Figure 6, for three SPD multipliers. For simplicity, I assume that s is uniformly distributed. As I stressed during this paper, the impact of SPDs on total welfare is actually ambiguous. So in Figure 6 I consider both cases. In panel a) I report the results using the values from the earlier numerical simulation and total welfare drops upon the introduction of SPDs. In panel b), by contrast, for full single damages total welfare is higher for certain values of β than in the absence of SPDs (specifically, I choose a value of a in panel b) such that there are no frivolous claims in any market).

Figure 6: Impact of β on total welfare according to SPD multiplier

a) SPDs at 100% decrease total welfare

b) SPDs at 100% can increase total welfare
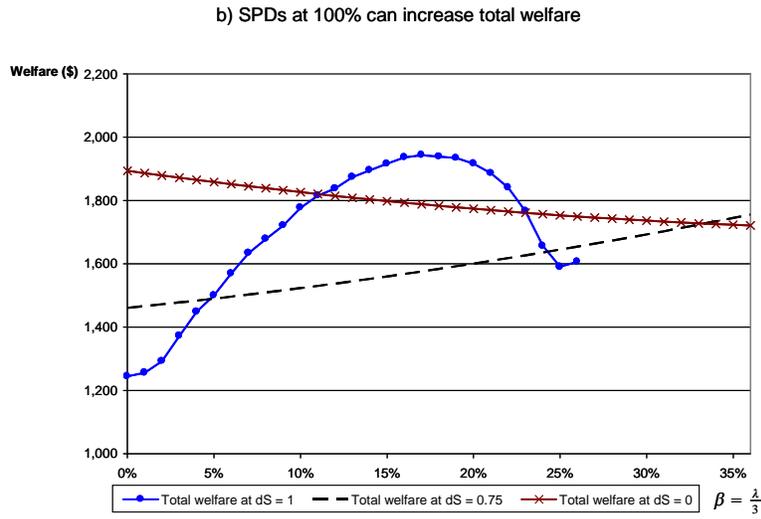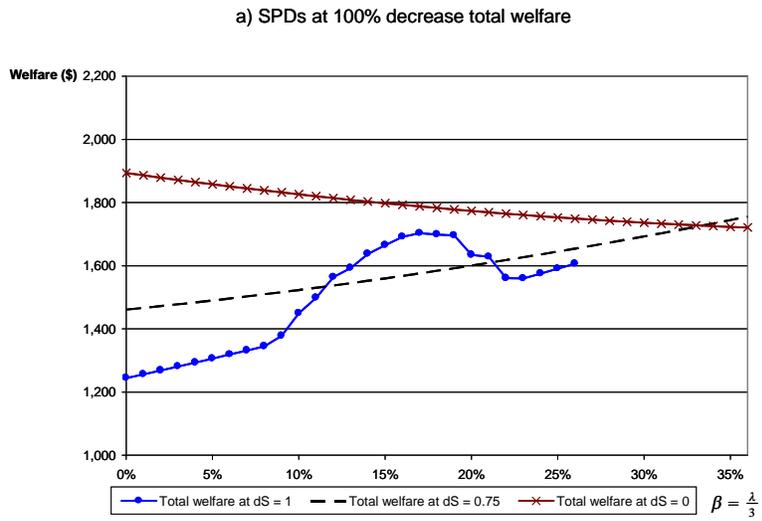
As a "control" for each panel, I consider the case of no SPDs ($d_S = 0$). Abuses occur in every market. Total welfare slightly falls as the NCA makes more mistakes (and at the same time can investigates more markets). This is purely due to the follow-on actions: when few errors are made, few markets are investigated; hence fewer abuses are caught and there are therefore fewer FOPD actions, thereby reducing the number of cases where the courts have to intervene, and increasing total welfare.

As for $d_S = 0.75$ (where I need to cap β at 36% in order to remain within the set of interior solutions), welfare levels are almost consistently below those achieved in the absence of SPDs, in both panels.

Finally, for single stand-alone damages ($d_S = 1$) - where β has to be capped at 26% - the optimal error size is around 17% (and around half of markets should be investigated) in both panels.

In sum, this simple numerical exercise has highlighted how the "breadth vs. depth" trade-off (λ, β) in NCA investigations varies depending on the the presence and level of d_S, which is typically chosen by the legislator. It has also captured the idea that *total* welfare may rise or fall upon the introduction of SPDs.

As a concluding remark to this numerical analysis, it has to be stressed that the results depended on the distribution of s. In the numerical example above I considered the discrete counterpart of a uniform distribution, with support going from 1 to 100. Intuitively, though, if I had assumed a normal distribution with mean somewhere in-between these bounds, I would have considerably increased the mass of medium-sized markets, thus increasing the proportion of consumers lodging a suit after an abuse, thereby placing a stronger constraint on the monopolists' behaviour; in other words, the deterrence effect would have been stronger.

In essence, all the results I derived analytically throughout this paper should be assessed for a given level of s. However, for any policy implication, since legislation has to be typically designed on a one-size-fits-all basis, then how s is actually distributed in the whole economy becomes relevant for welfare aggregation.

9   Conclusions, policy implications and extensions

In sum, this stylised model leads to some interesting considerations. First, although the introduction of damages actions cannot decrease deterrence (and in fact it typically increases it), the effect on *total* welfare is ambiguous. The precise outcome will depend on the specific values of the various parameters. It is clear, though, that the following forces are at stake:

- both exogenous and endogenous parameters affect which markets are "small", "medium" and "large";

- small markets are unaffected by the introduction of SPDs (since consumers never avail themselves of this possibility);

- in medium and large markets, for very low efficiency levels of the NCA, total welfare *falls* after the introduction of SPDs (deterrence is not improved but there are welfare-decreasing court suits, which involve both private and public resources);

- medium markets are those that *can* benefit most from the introduction of SPDs: deterrence is increased (for a sufficiently efficient NCA) and there are no unmeritorious claims;

- in large markets, even for a sufficiently efficient NCA, the gain in deterrence has to be weighted against wasteful expenditure in unmeritorious claims.

Second, I showed how the introduction of class actions has an ambiguous effect on total welfare.

Third, I explained why the effect (of introducing SPDs) on *consumer* welfare is positive in most cases (and neutral in a very narrow set of circumstances).

Fourth, I showed in Section 7 how the results (or outcomes dictated by the underlying forces at stake) are robust to allowing for type I and II errors by the NCA (although *actual* welfare levels will differ from the case of no errors).

Finally, I addressed the issue of devising the optimal policy by the NCA (once it faces a resource constraint and it has to choose between breadth and depth of investigations). In addition, I showed by means of a numerical example that the NCA's optimal policy (interpreted as the combination between number of investigations and depth of each) varies depending on the SPD regime chosen by the legislator.

The model can easily be extended in several directions. I indicate five below, all of which would be of particular interest from a policy perspective:

1. Replacing the upstream monopoly with an oligopoly which might engage in cartel activities and investigating the impact on equilibrium outcomes and welfare of the introduction of SPDs (as well as, perhaps, of a leniency policy towards the cartel member who first reports the cartel to the NCA);

2. Letting the NCA receive an imperfect signal about the occurrence of an abuse (cfr. McAfee et al. (2008)) and allowing it to set enforcement (investigation) priorities.

3. Enriching the litigation subgame by introducing the strategic interaction between complainants and upstream firms due to discovery issues (in *Twombly* the US Supreme Court estimated that discovery costs can amount to up to 90% of litigation costs, so this is very material).

4. Enriching the litigation subgame by introducing (cheaper) out-of-court settlements (cfr. Polinsky and Rubinfeld (1988), Briggs et al. (1996), Bourjade et al. (2007)).

5. Introducing an uncertain marginal cost (the buyer only receives a signal as in Salant (1987)).

References

[1] Baker (1988), *Private information and the deterrent effect of antitrust damage remedies*, Journal of Law, Economics, & Organization, Vol. 4(2).

[2] Becker (1968), *Crime and punishment: an economic approach*, Journal of Political Economy, Vol. 76(2).

[3] Boege and Ost (2006), *Up and running, or is it? Private enforcement – the situation in Germany and policy perspectives*, European Competition Law Review, Vol. 27(4).

[4] Bourjade, Rey and Seabright (2007), *Private antitrust enforcement in the presence of pre-trial bargaining*, IDEI Working Paper No. 499.

[5] Briggs, Huryn and McBride (1996), *Treble damages and the incentive to sue and settle*, RAND Journal of Economics Vol. 27(4).

[6] CEPS, Erasmus University and LUISS (2007), *Making antitrust damages actions more effective in the EU: welfare impact and potential scenarios*, Final Report for the European Commission.

[7] Chang and Harrington (2008), *The impact of a corporate leniency program on antitrust enforcement and cartelization*, preliminary draft presented at the 16th WZB Conference on Markets and Politics on "Deterrence in Competition Policy" (Berlin, 17-18 October 2008).

[8] Craycraft, Craycraft and Gallo (1997), *Antitrust sanctions and a firm's ability to pay*, Review of Industrial Organization, Vol. 12.

[9] Cooter, with Ulen (2008), *Law & Economics*, Pearson/Addison Wesley, 5th ed.

[10] Dewatripont and Tirole (1999), *Advocates*, Journal of Political Economy, Vol. 107 (1).

[11] Diemer (2006), *The Green Paper on Damages actions for breach of the EC antitrust rules*, European Competition Law Review, Vol. 27(6).

[12] Easterbrook (1984), *The limits of antitrust*, Texas Law Review, Vol. 63(1).

[13] European Commission (2008), *White Paper on Damages actions for breach of the EC antitrust rules*.

[14] European Commission (2008) *Staff Working Paper accompanying the White Paper on Damages actions for breach of the EC antitrust rules*.

[15] Evans and Padilla (2004), *Designing antitrust rules for assessing unilateral practices: a neo-Chicago approach*, CEPR Discussion Paper No. 4625.

[16] Fisher (2006), *Economic analysis and antitrust damages*, World Competition, Vol. 29(3).

[17] Hylton (2002), *An asymmetric-information model of litigation*, International Review of Law and Economics, Vol. 22.

[18] Immordino and Polo (2008), *Judicial errors and innovative activity*, IGIER Working Paper No. 337.

[19] Katsoulacos and Ulph (2008), On optimal legal standards for competition policy - a general analysis, ELSE Working Paper No. 308.

[20] McAfee, Mialon and Mialon (2008), *Private v. public antitrust enforcement: a strategic analysis*, Journal of Public Economics, forthcoming.

[21] Milgrom and Roberts (1986), *Relying on the information of interested parties*, The RAND Journal of Economics, Vol. 17(1).

[22] Motta and Polo (2003), *Leniency programs and cartel prosecution*, International Journal of Industrial Organization, Vol. 21.

[23] Parisi (2002), *Rent-seeking through litigation: adversarial and inquisitorial systems compared*, International Review of Law and Economics Vol. 22.

[24] Polinsky and Rubinfeld (1988), *The deterrent effect of settlements and trials*, International Review of Law and Economics, Vol. 8.

[25] Polinsky and Rubinfeld (1993), *Sanctioning Frivolous Suits: An Economic Analysis*, Georgetown Law Journal, Vol. 82.

[26] Rueggeberg and Schinkel (2006), *Consolidating antitrust damages in Europe: a proposal for standing in line with efficient private enforcement*, World Competition, Vol. 29(3).

[27] Salant (1987), *Treble damage awards in private lawsuits for price fixing*, Journal of Political Economy, Vol. 95(6).

[28] Salop and White (1986), *Economic analysis of private antitrust litigation*, Georgetown Law Journal, Vol. 74.

[29] Segal and Whinston (2006), *Public vs. private enforcement of antitrust law: a survey*, Stanford Law School Working Paper No. 335.

[30] Shavell (1997), *The fundamental divergence between the private and the social motive to use the legal system*, The Journal of Legal Studies Vol. 26(2).

[31] Wils (2003), *Should private antitrust enforcement be encouraged in Europe?*, World Competition Vol. 26(3).

[32] Wils (2006), *Optimal antitrust fines: theory and practice*, World Competition Vol. 29(2).

[33] Wils (2009), *The relationship between public antitrust enforcement and private actions for damages*, World Competition, Vol. 32(1).

A   Appendix

A.1   Deriving the equilibrium

At t = 2:

Subgame I: it is strictly dominant for the complainant to go for FOPDs. The complainant always gets $\frac{s}{8} + \frac{s}{4}d_F$ and firm i $\frac{s}{4}(1 - x - d_F)$.

Subgame II: the complainant solves:

$$\max\left[\frac{s}{8}, \frac{s}{8} + \sqrt{\widetilde{E}}\frac{s}{4}d_S - c\widetilde{E} - F\right]$$

where $\widetilde{E} = \arg\max \frac{s}{8} + \sqrt{E}\frac{s}{4}d_S - cE - F$. So $\widetilde{E} = \frac{(d_S s)^2}{64c^2}$ and the plaintiff goes to the local court to seek SPDs if $s \geq \frac{8}{d_S}\sqrt{cF}$. The plaintiff does nothing if $s < \frac{8}{d_S}\sqrt{cF}$. Throughout the paper, I concentrate on interior solutions of evidence-gathering, namely on values of c, d_S and s such that $\widetilde{E} < 1$.

Subgame III: the complainant solves:

$$\max \left[\frac{s}{2}, \frac{s}{2} + \sqrt{\tilde{E}}\frac{s}{4}d_S - ac\tilde{E} - F\right]$$

where $\tilde{E} = \arg\max \frac{s}{8} + \sqrt{E}\frac{s}{4}d_S - acE - F$ (and $a \in (1, \infty)$). So $\tilde{E} = \frac{(d_S s)^2}{(8ac)^2}$ and the plaintiff goes to the local court to seek SPDs if $s \geq \frac{8}{d_S}\sqrt{acF}$. The plaintiff does nothing if $s < \frac{8}{d_S}\sqrt{acF}$.

Subgame IV: it is strictly dominant for the complainant to go nothing. The complainant always gets $\frac{s}{2}$ and firm i nil.

At t = 1: nature plays (λ)

At t = 0:

Three relevant market size ranges were determined in the solution of the subgames at $t = 2$: $s < \frac{8}{d_S}\sqrt{cF}$, $\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$ and $s \geq \frac{8}{d_S}\sqrt{acF}$.

A) $s < \frac{8}{d_S}\sqrt{cF}$ (markets are small, complainants never go for SPDs)

Firm i sets $p = \frac{1}{2}$, i.e. commits an abuse, if $\lambda\frac{s}{4}(1 - d_F - x) + (1 - \lambda)\frac{s}{4} \geq \lambda * 0 + (1 - \lambda) * 0$, i.e. if:

$$\lambda \leq \frac{1}{(d_F + x)}$$

This is intuitive. As plaintiffs never go for SPDs (it is not worth, as the costs of court actions are too large relative to the expected benefit), the presence of SPDs exerts no deterrence effect on the upstream monopolist, whose decision to commit an abuse will only depend on the NCA's actual efficiency (λ), the fine multiplier (x) and the FOPD multiplier (d_F).

B) $\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$ (complainants only go for SPDs if there has been an abuse)

Firm i sets $p = \frac{1}{2}$, i.e. commits an abuse, when $\lambda(\frac{s}{4}(1 - x - d_F)) + (1 - \lambda)(\frac{s}{4} - \frac{(d_S s)^2}{32c}) \geq \lambda * 0 + (1 - \lambda) * 0$. To solve this inequality, I need to consider three cases:

(Condition B.i) if $s d_S^2 < 8c$
This condition can be consolidated with $\frac{8}{d_S}\sqrt{cF} \leq s < \frac{8}{d_S}\sqrt{acF}$ into:

$$\left(0 < d_S < \sqrt{\frac{c}{aF}} \wedge 8\sqrt{\frac{cF}{d_S^2}} \leq s \leq 8\sqrt{\frac{acF}{d_S^2}}\right) \vee \left(\sqrt{\frac{c}{aF}} < d_S < \sqrt{\frac{c}{F}} \wedge 8\sqrt{\frac{cF}{d_S^2}} \leq s < 8\frac{c}{d_S^2}\right) \tag{1}$$

So firm i will commit an abuse (when (1) is satisfied) if:

$$\lambda \leq \frac{8c - sd_S^2}{8c(x + d_F) - sd_S^2}$$

(Condition B.ii) if $8c < sd_S^2 < 8c(x + d_F)$: full deterrence for any λ
(since the abuse condition becomes $\lambda \leq \frac{8c - sd_S^2}{8c(x+d_F) - sd_S^2} < 0$)

(Condition B.iii) if $sd_S^2 > 8c(x + d_F)$: full deterrence for any λ
(since the abuse condition becomes $\lambda > \frac{sd_S^2 - 8c}{sd_S^2 - 8c(x+d_F)} \geq 1$)

C) $s \geq \frac{8}{d_S}\sqrt{acF}$ (complainants always go for SPDs)

Firm i sets $p = \frac{1}{2}$, i.e. commits an abuse, if $\lambda(\frac{s}{4}(1 - x - d_F)) + (1 - \lambda)(\frac{s}{4} - \frac{(d_S s)^2}{32c}) \geq \lambda * 0 + (1 - \lambda) * (-\frac{(d_S s)^2}{32ac})$.
There are again three cases:

(Condition C.i) if $(a - 1)sd_S^2 < 8ac$:
This condition can be consolidated with $s \geq \frac{8}{d_S}\sqrt{acF}$ into:

$$8\sqrt{\frac{acF}{d_S^2}} < s < \frac{8ac}{d_S^2(a - 1)} \wedge 0 < d_S < \sqrt{\frac{ac}{(a - 1)^2 F}} \tag{2}$$

So firm i will commit an abuse (when (2) is satisfied) if:

$$\lambda \leq \frac{8ac - (a - 1)sd_S^2}{8ac(x + d_F) - (a - 1)sd_S^2}$$

(Condition C.ii) if $8ac < (a - 1)sd_S^2 < 8ac(x + d_F)$: full deterrence for any λ
(since the abuse condition becomes $\lambda \leq \frac{8ac - (a-1)sd_S^2}{8ac(x+d_F) - (a-1)sd_S^2} < 0$)

(Condition C.iii) if $(a - 1)sd_S^2 > 8ac(x + d_F)$: full deterrence for any λ
(since the abuse condition becomes $\lambda > \frac{(a-1)sd_S^2 - 8ac}{(a-1)sd_S^2 - 8ac(x+d_F)} \geq 1$)

In essence, within each medium and large market size intervals (which are themselves endogenously determined by s and d_S) there will be two different threshold levels of λ: either zero (full deterrence) or λ as a continuous function of the other paramaters. As for small market sizes, there is a unique threshold to commit an abuse, which only depends on exogenous parameters (x and d_F).

A.2   Computing the optimal policy analytically

Welfare aggregation across markets of different sizes entails a number of considerations. First I need to integrate over s, as the actual welfare levels are different for different market sizes, and yet the policy has to be set symmetrically for all markets. Second, I need to assign probability weights to each relevant range of s (this is where the uniform distribution assumption of s simplifies the computation). Third, welfare levels will clearly differ across different

combinations of (λ, β). Fourth, as illustrated in Table 3, the optimal policy will depend on whether (1) and (2) hold. These conditions represent the intersection of the market size constraint (from the plaintiffs ' perspective, which I will take into account into the integral limits) and the interior solution constraints for the monopolist. These latter constraints can be traced back to Condition B.i and Condition C.i in Appendix A.1. So, recalling that here I am looking at the case of $d_S = 1$, I will distinguish two cases: $s < 8c$, which implies $(a-1)s < 8ac$; and $(a-1)s > 8ac$, which implies $s > 8c$:[30]

I) $s < 8c$, $\forall s$ (interior solution, i.e. partial deterrence)

The NCA has to compare the total welfare levels achieved in five possible cases and then choose the (λ, β) feasible pair which yields the highest welfare.

    i) Firms commit an abuse in every market

$$\max_{\beta, \lambda} \quad \frac{8\sqrt{cF}}{S} \int_0^{8\sqrt{cF}} (\tfrac{3s}{8} - \lambda(1-\beta)W_C)\ ds + \frac{S - 8\sqrt{cF}}{S} \int_{8\sqrt{cF}}^{S} \tfrac{3s}{8} - (1-\lambda)(\tfrac{s^2}{64c} + F) - (1-\lambda\beta)W_C\ ds$$

$$if \quad \lambda \le \frac{8ac - (a-1)s}{8ac(x+d_F)(1-2\beta) - (a-1)s} \wedge \lambda \le \frac{8c - s}{8c(x+d_F)(1-2\beta) - s} \quad \forall s \quad s.t. \quad (9)$$

    ii) Full deterrence (no firm commits an abuse):

$$\max_{\beta, \lambda} \quad \frac{8\sqrt{acF}}{S} \int_0^{8\sqrt{acF}} (\tfrac{s}{2} - \lambda\beta W_C)\ ds + \frac{S - 8\sqrt{acF}}{S} \int_{8\sqrt{acF}}^{S} \tfrac{s}{2} - (1-\lambda)(\tfrac{s^2}{64ac} + F + W_C) - \lambda\beta W_C\ ds$$

$$if \quad \lambda > \frac{1}{(x+d_F)(1-2\beta)} \quad s.t. \quad (9)$$

This can be ruled out $\forall x + d_F \le \tfrac{8}{\kappa}$ (applying Proposition 3).

    iii) Abuse in small and medium markets only:

$$\max_{\beta, \lambda} \quad \frac{8\sqrt{cF}}{S} \int_0^{8\sqrt{cF}} (\tfrac{3s}{8} - \lambda(1-\beta)W_C)ds + \frac{8\sqrt{acF} - 8\sqrt{cF}}{S} \int_{8\sqrt{cF}}^{8\sqrt{acF}} \left(\tfrac{3s}{8} - (1-\lambda)(\tfrac{s^2}{64c} + F) - (1-\lambda\beta)W_C\right)ds +$$

$$+ \frac{S - 8\sqrt{acF}}{S} \int_{8\sqrt{acF}}^{S} \left(\tfrac{s}{2} - (1-\lambda)(\tfrac{s^2}{64ac} + F + W_C) - \lambda\beta W_C\right)ds$$

$$if \quad \lambda \le \frac{1}{(x+d_F)(1-2\beta)} \wedge \frac{8c - s}{8c(x+d_F)(1-2\beta) - s} \ge \lambda > \frac{8ac - (a-1)s}{8ac(x+d_F)(1-2\beta) - (a-1)s} \ (\forall s \ge 8\sqrt{cF}) \quad s.t. \quad (9)$$

---

[30] I do not consider the intermediate interval, i.e. "corner solutions" in medium markets only, for brevity reasons.

iv) Abuse in small and large markets only:

$$\max_{\beta,\lambda} \quad \frac{8\sqrt{cF}}{S} \int_0^{8\sqrt{cF}} (\tfrac{3s}{8} - \lambda(1-\beta)W_C)ds + \frac{8\sqrt{acF}-8\sqrt{cF}}{S} \int_{8\sqrt{cF}}^{8\sqrt{acF}} \left(\tfrac{s}{2} - \lambda\beta W_C\right) ds+$$

$$+ \frac{S-8\sqrt{acF}}{S} \int_{8\sqrt{acF}}^{S} \left(\tfrac{3s}{8} - (1-\lambda)(\tfrac{s^2}{64c}+F) - (1-\lambda\beta)W_C\right) ds$$

$$if \quad \lambda \le \tfrac{1}{(x+d_F)(1-2\beta)} \wedge \tfrac{8c-s}{8c(x+d_F)(1-2\beta)-s} < \lambda \le \tfrac{8ac-(a-1)s}{8ac(x+d_F)(1-2\beta)-(a-1)s} \ (\forall s \ge 8\sqrt{cF}) \qquad s.t. \qquad (9)$$

v) Abuse in small markets only:

$$\max_{\beta,\lambda} \quad \frac{8\sqrt{cF}}{S} \int_0^{8\sqrt{cF}} (\tfrac{3s}{8} - \lambda(1-\beta)W_C)ds + \frac{8\sqrt{acF}-8\sqrt{cF}}{S} \int_{8\sqrt{cF}}^{8\sqrt{acF}} \left(\tfrac{s}{2} - \lambda\beta W_C\right) ds+$$

$$+ \frac{S-8\sqrt{acF}}{S} \int_{8\sqrt{acF}}^{S} \left(\tfrac{s}{2} - (1-\lambda)(\tfrac{s^2}{64ac}+F+W_C) - \lambda\beta W_C\right) ds$$

$$if \quad \lambda \le \tfrac{1}{(x+d_F)(1-2\beta)} \wedge \lambda > \tfrac{8c-s}{8c(x+d_F)(1-2\beta)-s} \ (\forall s \ge 8\sqrt{acF}) \wedge$$
$$\lambda > \tfrac{8ac-(a-1)s}{8ac(x+d_F)(1-2\beta)-(a-1)s} \ (\forall \ 8\sqrt{cF} \le s < 8\sqrt{acF}) \qquad s.t. \qquad (9)$$

(II) $(a-1)s > 8ac, \forall s > 8\sqrt{cF}$ (corner solution, i.e. full deterrence in medium and large markets)

The NCA will implicitly solve the following two programs and choose the one that yields higher welfare. Deterrence in medium and large markets is guaranteed by the "corner condition" just assumed; deterrence in the small markets, as ever, only depends on the efficiency level of the NCA.

(i) $\quad \max_{\beta,\lambda} \quad \frac{8\sqrt{acF}}{S} \int_0^{8\sqrt{acF}} \left(\tfrac{s}{2} - \lambda\beta W_C\right) ds + \frac{S-8\sqrt{acF}}{S} \int_{8\sqrt{acF}}^{S} \left(\tfrac{s}{2} - (1-\lambda)(\tfrac{s^2}{64ac}+F+W_C) - \lambda\beta W_C\right) ds$

$$if \quad \lambda > \tfrac{1}{(x+d_F)(1-2\beta)} \qquad s.t. \qquad (9)$$

(ii) $\quad \max_{\beta,\lambda} \quad \frac{8\sqrt{cF}}{S} \int_0^{8\sqrt{cF}} \tfrac{3s}{8} - \lambda(1-\beta)W_C \ ds + \frac{8\sqrt{acF}-8\sqrt{cF}}{S} \int_{8\sqrt{cF}}^{8\sqrt{acF}} \tfrac{s}{2} - \lambda\beta W_C \ ds+$

$$+ \frac{1}{S-8\sqrt{acF}} \int_{8\sqrt{acF}}^{S} \tfrac{s}{2} - (1-\lambda)(\tfrac{s^2}{64ac}+F+W_C) - \lambda\beta W_C \ ds$$

$$if \quad \lambda \le \tfrac{1}{(x+d_F)(1-2\beta)} \qquad s.t. \qquad (9)$$