Horizontal Mergers, Information Sharing, Uncertainty, and Risk Aversion in Cournot competition

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19th April 2010

Working Draft

The traditional horizontal merger literature has asserted that the merger incentives are highly limited under Cournot competition, thereby reverting empirical evidence. In order to give possible explanations for the increase in merger activities in the last 20 years, I relax on the restrictive assumptions applied by this literature by integrating uncertainty, risk aversion and information sharing mechanisms into the horizontal merger literature. I find that merger profitability increases with efficiency gains and with the level of risk and risk aversion; furthermore, in contrast to previous literature, I find that insiders have incentives not to reveal private information about efficiency gains, when the outsiders are risk averse. From a regulatory point of view, I find that consumer surplus is increasing with uncertainty and risk aversion.

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1. Introduction

Merger and acquisition activities experienced substantial growth until 2007. Grimpe (2007), for example, states that the worldwide transaction value in M&A deals amounted to 7.7bn € in the second and third quarter of 2007, an increase of 100 percent within two quarters. This empirical evidence of merger incentives could though not be explained by the early models of the horizontal merger literature. These assert that horizontal mergers are rarely profitable and therefore underestimate the incentives to merge. In recent times, the theoretical merger literature has made efforts to explain the growth of merger activities. One strand of literature has focused on the cost structure of the merged entity. The cost structure changes post-merger as the merged entity can produce more efficiently than previously and the effect of the change in the cost structure on the incentives to merge has been analyzed in the horizontal merger literature (e.g. Farrell & Shapiro, 1990). However, this change in the capital structure (i.e. the efficiency gains) might still be uncertain at the time of production and only the merged entity might be aware of the realized efficiency gains. A further strand of the merger literature has started to investigate the effects of private information pooling between the insiders on the incentives to merge (e.g. Zhou, 2008; Banal-Estañol, 2007). As discussed below, many obstacles still have to be analyzed in order to fully assess the incentives to share private information about uncertain parameters evolving from the merger. Thirdly, the effect of risk aversion on the incentives to merge has only been investigated numerically thoroughly (Kao & Hughes, 1993).

I relax on the restrictive assumptions applied by the horizontal merger literature by integrating uncertainty, risk aversion and information sharing mechanisms in order to give possible explanations for the increase in merger activities. In particular, I assume a merger to take place in a Cournot oligopoly of risk-averse firms with differentiated products, where the merging firms are able to realize stochastic efficiency gains. The paper focuses on analysing the impact of uncertainty, risk aversion, and information sharing on the merger incentives as well as on consumer surplus.

I analyze the optimal decision rules for the insiders and find that absent risk aversion and under complete information, insiders reduce output post-merger as in the traditional literature; this effect however diminishes as efficiency gains increase, resulting in higher merger profitability. When insiders and outsiders are uncertain about the future efficiency gains, insiders increase output and profit as risk aversion increases, similar to findings by Banal-Estañol & Ottaviani (2006); this effect is even enhanced by a decrease in marginal costs and an increase in the industry size, oppositely to findings by Salant, Switzer & Reynolds (1983), and further influenced by the substitutability of the products. Furthermore, unlike in previous literature in e.g. Raith (1996), which asserts that it is always profitable for firms to share private cost information in Cournot competition, I find that insiders have the incentive not to share their full or noisy private information, which is increasing in the outsiders' level of risk aversion, and that merger incentives decrease the more firms merge ("merger scale") in this setting. From a welfare point of view, consumers' surplus is maximized as aggregate output is maximized. Aggregate output, in turn, increases with uncertainty and risk aversion.

1.1. Horizontal Merger literature

Upon a horizontal merger, unilateral effects are the effects resulting from the internalization of competition between the merging firms by exerting market power when operating in the same economic relevant market. Coordinated effects, oppositely, result from firms engaging in tacit collusion, as a result of which their behaviour may approximate that of a single dominant firm (e.g. Ivaldi, Jullien, Rey, Seabright & Tirole, 2003). The colluding firms may be able to maintain higher prices by tacitly agreeing that any deviation from the collusive path would trigger some retaliation. This paper focuses on an analysis of changes in the unilateral effects resulting from uncertainty, risk aversion and information sharing mechanisms in horizontal mergers. First, post-merger the merged entity as well as the outsiders pursue their profit-maximizing self-interest by alternating their output decision in Cournot competition, leading to the notion of "unilateral" effects. The merging firms decrease their output, thereby increasing price, as part of the sales which they would have lost pre-merger, now is taken up by a partnering merging entity. Second, as well-known under Cournot competition (e.g. Dixit, 1986), the rivals or outsiders react on the output decrease of the merging firms by raising their output as customers might switch from the (merged) group ("insiders") to the outsiders due to the higher price set by the insiders. As a result, the merger profitability for the insiders depends on the extent and interdependence of these two effects, the possibility of the insiders to raise price and the outsiders' (in-)ability to take advantage thereof.

Salant, Switzer & Reynolds (1983) have shown that merger profitability in Cournot markets crucially depends on the industry size, i.e. the number of firms in the relevant economic market, and the number of insiders, i.e. firms which merge. They show that first the more firms are in the industry pre-merger, the lower is the chance that the merger is privately profitable for the merging firms (i.e. incentives to merge exist for the insiders). This is, the higher the number of opportunistic firms, i.e. firms taking advantage of the price increase of the merging firms, the lower the chance that the merger is privately profitable. Second, they show that merger profitability increases in the number of insiders. This is, fewer firms can react tough on the strategic price increase of the merging firms. As a conclusion of both effects, they find that 80% of the industry must merge for a merger to be privately profitable.

This result has long been challenged by the horizontal merger literature. Perry & Porter (1985) find that the 80% benchmark decreases to 50% if increasing instead of constant marginal costs are considered. The underlying reason for this finding is that the merged firm's cost function alters upon the merger: The capital endowment of the merged entity increases due to the merger and as marginal costs decrease with capital endowment, the marginal cost curve of the merged entity is less steep than in the pre-merger setting. As rivals' capital endowment does not change post-merger, their incentives to increase production are limited, since marginal costs increase with output.

A further point of interest has been the influence of the concavity of the demand function on merger profitability. Cheung (1992), and partly Levin (1990), generalize the linear demand assumption of Salant, Switzer & Reynolds (1983) and show that for all demand functions satisfying $2p'(x) + p''(x)X < 0$, i.e. allowing for strictly convex demand by assuming that industry revenue is concave in industry output, horizontal mergers increase the market power of all firms, while not necessarily profitably for the merged firm. In particular, the merged firm can exploit its market power profitably only if its market share exceeds 50%, thereby lowering the 80% benchmark of Salant, Switzer & Reynolds (1983). Faulí-Oller (1997) finds that merger profitability depends on the degree of the demand function's concavity. Explicitly, the greater the concavity of the demand function, the lower the merger profitability as the market share is higher the lower the degree of concavity.[1]

The effect of product differentiation on merger profitability has been examined by Leahy (2002), who finds that horizontal mergers are profitable if product differentiation is high enough. If the insiders increase their price post-merger, the customers' willingness to switch from the to the outsiders is limited due to the existing product differentiation. Similar to the effect of increasing marginal costs, outsiders cannot take advantage of the insiders' actions.

Further points of interest have been market size and asymmetry of costs. While in the previous literature, merger profitability did not depend on market size and symmetric firms were assumed, Faulí-Oller (2002) shows that for linear demand mergers are only profitable, if the efficiency gains imposed by the merger are large enough. Furthermore, he finds that if market size decreases, price decreases and the cost differential becomes relatively greater: Under Cournot competition, differences in the firms' size are explained by differences in price margins. As the market size decreases, the equilibrium price decreases and the size differences increases for given costs. As laid out before, profitability increases as the cost differential increases and therefore, as the market size decreases.

1.2. Uncertainty and Risk Aversion literature

A standard assumption in oligopoly theory is that firms are risk-neutral. However, there are several reasons why firms may act as if they were risk-averse or actually are risk-averse. Firms may act as if they are risk-averse in cases of non-diversified owners (i.e. firms' payoff should take into account the level of risk aversion of the undiversified shareholders.), liquidity constraints (i.e. firms may refrain from risky investments and may want to avoid liquidity squeezes when liquidity is scarce), costly financial distress (i.e. if financial turmoil is costly, firms may wish to avoid such turmoil by foregoing risky situations), non-linear tax systems (i.e. the higher the tax rates, the greater is the amount of risk a firm is willing to take; see e.g. Domar & Musgrave (1944); Feldstein (1969); Waterson (1985)); furthermore, firms may act risk-averse due to the delegation of control to risk-averse managers or managers with an incentive structure linked to the profits of the organization; specifically, the degree of risk aversion amongst decision-makers, who maximize their lifetime incomes, is likely to be higher as they avoid risky situations and prefer steady income growth if owners are risk-averse (Monsen & Downs, 1965). As a further reason for the assumption of risk aversion, the recent financial crisis has given plenty of evidence

[1] Hennessy (2000) even shows that certain demand functions exist under which horizontal mergers are privately profitable for the merging firms independent of the industry size and the number of merging firms.

of potential differences in the degree of risk aversion of firms, explicitly banks.

Altering the assumption of risk-neutrality has several implications for product market competition. To analyse these implications, risk aversion has to be separated into its two single components, *risk / uncertainty*, expressed as σ, and the degree of *aversion*, expressed in the form of a risk aversion coefficient R, (Diamond & Stiglitz, 1974). This implies, only once uncertainty has been introduced, the firm's attitude towards risk becomes important. As shown by Baron (1970), while uncertainty is a necessary condition, any associated effect or profit is caused by risk aversion rather than by the uncertainty. The key intuition behind the effects of risk aversion is that firms give relatively greater weight to realisations with low profits. The risk-averse firm acts as though its marginal costs are higher / demand is lower than the expected values (e.g. Hirshleifer & Riley, 1992; Laffont, 1989) as illustrated below.

Risk / Uncertainty

While uncertainty can persist with respect to numerous business decisions (e.g. R&D success, price uncertainty), two general cases of uncertainty, demand uncertainty and cost / efficiency gains uncertainty, are in the focus of this analysis. In the first case, demand uncertainty, (all) market participants are uncertain about the (common) demand intercept, while in the second case, cost information, the uncertainty is focused on the private cost parameter of (each individual) firm. While the firms' best response strategy itself, i.e. good performance in low-profit-states, is principally the same irrespective of the prevailing type of uncertainty, *pursuing* the best response strategy becomes more difficult when analysing the underlying information structure and the rivals' best response to new information in an oligopoly framework as shown in the next section.

Important to note is the interdependence of demand and cost uncertainty: If one firm is uncertain about its costs and, therefore about its profit and output, its rivals are uncertain about their residual demand as they are uncertain about the output of the cost-uncertain firm, which can be analysed analytically in quadratic preference models (Kühn & Vives, 1995).

The impact of the *degree of uncertainty* on the firms' utility has drawn special attention in previous literature. Rothschild & Stiglitz (1970, Theorem 2) illustrate that for mean preserving increases in risk (i.e. the mean of the random variable is kept constant), the expected utility decreases (increases) if the utility function is strictly concave (convex), where concave utility functions are associated with the exposure to risk aversion. However, Rothschild and Stiglitz showed that this definition yields only a partial ordering over the set of cumulative distributions in two ways. First, only cumulative distributions of the same mean can be ordered. Second, not all distributions with the same mean can be ordered, e.g. distributions with the same mean but different cumulative distribution functions. Thus, a necessary, but not sufficient condition for one distribution to be riskier than another is that their means are equal.

Diamond & Stiglitz (1974) obtain a stronger result by specifically modeling riskier distributions of the utility with mean utility preserving increases in risk. This implies, they extend Rothschild & Stiglitz (1970)'s definition of increasing risk as $G(x)$ is at least as risky as $F(x)$ if $G(x)$ can be obtained from $F(x)$ by a sequence of steps, each of which shifting weight from the center to its tails while keeping the expectation of the utility function constant. Put differently, if one increasing utility function is more concave (i.e. more risk averse) than another, then any change in the distribution of random income which constitutes a mean utility preserving increase in risk for the second utility function will lower the expectation of the first (Ito & Machina, 1983). As a response to greater (mean utility preserving) risk, firms adjust the control variable so as to make the utility function show less risk aversion.

Applying these findings to demand uncertainty, both types of uncertainty increases, mean preserving and mean utility preserving, lead to output reductions of the competitive firm as illustrated by the respective authors and Sandmo (1971) for the case of mean preserving risk increases.

A further strand of research has focused on the dependence of the degree of risk aversion on wealth or initial capital endowment (Arrow, 1965; Pratt, 1964). Risk aversion can be independent (i.e. constant), increasing or decreasing in the level of wealth or capital endowment.[2]

Degree of risk aversion

As noted above, while uncertainty is a necessary condition for effects on profit to materialize, the degree of risk aversion specifies the impact on profit. A pioneering analysis by Baron (1970) shows that in perfectly competitive markets increased price risk-aversion lowers the quantity produced. In a monopoly framework, Baron (1971) and Leland (1972) derived similar results. If risk aversion is prevalent, prices are higher and output lower than if firms were indifferent to risk. The higher prices and lower output are caused by the risk premium which increases as firms become more risk-averse as discussed below.

Asplund (2002) has shown that in Cournot competition more risk-averse firms set lower quantities, irrespective of the type of uncertainty (i.e. cost uncertainty or demand uncertainty). This follows from the firms' intuition to perform well, when uncertainty realization might decrease profits: Firms reduce output from the level that maximizes profit in order to reduce the variance of its risk. The firm is willing to sacrifice an amount of expected profits, the *risk premium*, in order to eliminate the risk and obtain the expected profit with certainty.

Pratt (1964) showed that for all utility functions, assuming small risks, the respective risk premium can be modeled as half time the risk aversion coefficient times the variance of the risk involved, i.e. $R \cdot \sigma^2$.[3]

Banal-Estañol & Ottaviani (2006) have introduced the analysis of risk aversion in a horizontal merger setting. However, unlike my analysis, they focus on the best division of ownership (i.e. takeover versus equal division of ownership) in cases of risk-averse

[2] I focus on constant absolute risk aversion, which implies that the higher the curvature of the utility function, the higher the risk aversion, whereas relative risk aversion weights the measure of risk aversion by the level of wealth (Arrow, 1965; Pratt, 1964).

[3] For a discussion on the risk aversion coefficient, please be referred to e.g. Pratt (1964).

firm mergers. Related to my analysis, they find that risk aversion stipulates a higher probability for the insiders to increase production post-merger due to the higher risk bearing potential of the (larger) merged firm in contrast to the pre-merger set of (smaller) firms, leading to an increase in merger profitability.

1.3. Information Sharing literature

The information sharing literature has centered on the question whether the sharing of private information is profit-enhancing for the revealing party (i.e. whether incentives to share private information about a random variable exist). Focusing on the revelation incentives in Cournot competition, the information sharing literature generally distinguishes between uncertainty about a common variable (i.e. uncertainty affecting all participants, e.g. demand) and a private variable (i.e. uncertainty affecting only one firm or each firm individually, e.g. cost). Furthermore, the literature distinguishes between the case, in which one signal about the realization of one firm's uncertainty (i.e. the random variable) does or does not infer information about other firms' realization of their random variable (Raith, 1996).[4]

In an industry of $i, j \dots n$ competing firms, the sharing of information has principally three effects. From a standpoint of firm i, the firm is better informed due to the new information it receives from its competitors, $j \dots n$, and can react optimally on the new information. Second, the competitors react on the new information they receive from firm i. Third, the reaction of the competitors on the new information obtained from firm i might alter the best response of firm i. Important to note for my further analysis is that the first effect does not occur if only unilateral information revelation persists, i.e. only one firm reveals information to its competitors and does not obtain new information from its competitors.

It has long been debated which effect incentivizes the firm to reveal its private information (e.g. Raith, 1996; Vives, 2002; Gal-Or, 1986, 1985). While the first effect is always positive (new information is always good), this channel is shut off when considering unilateral information revelation. The effects of the second and third channel crucially depend on the type of uncertainty under consideration.

In the case of uncertainty about a private variable, the correlation of decision strategies is reduced and firms have incentives to reveal their private information: If one firm observes e.g. a lower cost signal, it expands output, while the other firm, if informed, reduces output. The reverse holds for the observation of a high cost signal. Gal-Or (1986) and Shapiro (1986) show that the firm's profit increases from the sharing of a low cost signal always exceed the negative profit decreasing effects from the revelation of a high cost signal: According to Shapiro (1986), the variability of industry output (i.e. the probability that firms over- or underproduce due to missing information) decreases the firms' expected profit. The firm receives lower profits when they produce more output,

[4] Raith (1996) reclassifies the notion of private variables in that he differentiates between models, in which the shocks are independent, i.e. information about one shock does not information about another shock, and models, in which the information revealer is perfectly informed about the uncertain parameter.

so the expected price per unit is reduced when output is variable. This variability of output is reduced upon information revelation as firms' mistakes are eliminated. As a conclusion, firms would reveal information they have about the realization of their private random variable.

In the case of a common (demand) variable (see Gal-Or, 1985, for homogeneous products and Vives, 1984, for differentiated products), the correlation of decision strategies is increased. For Cournot competition, when demand is high and this information is shared, all firms (i.e. the revealing firm and its competitors) increase their output. The benefit or loss from concealing information is derived by multiplying the additional output produced from revealing information by the market price. Since prices are higher when demand is high, the gain accrued from not revealing information more than compensates for the loss when demand is low, hence the firm would choose to conceal its private information (Gal-Or, 1985). However, this result is reversed, if the products are sufficiently differentiated (Raith, 1996).[5]

Several authors have already started to analyse the impact of information in horizontal merger settings. However, these contributions only focus on the information *pooling between the merging firms* upon a merger (i.e. the insiders are informed about the partners' private costs, while outsiders are not). Put differently, no information sharing mechanism between insiders and outsiders and no change in the information structure is modeled by these authors, but rather the information availability after the merger incentivizes mergers, independent of the information structure in place.

For the case of uncertainty about private costs, incentives to merge are increased as Banal-Estañol (2007) argues for increasing marginal costs and Zhou (2008) argues for constant marginal costs in most cases[6]. Drawing on the effects of information sharing above, their main argument is that when merging firms have more information about each other (i.e. the other insiders) post-merger, which they do not have in a pre-merger setting, they are able to rationalize their production. When considering increasing marginal costs the incentives are even higher than for constant marginal cost due to the factors discussed above, i.e. the outsiders' lower possibilities of adverse reaction (Perry & Porter, 1985).

For the case of demand uncertainty (Gal-Or, 1988), the merged firms estimate demand more accurately by pooling their private signals, making the merger more profitable, which is in line with the effect of being better informed in the information sharing literature. To avoid competition, however, the merging firms respond to market signals less aggressively (i.e. the merging firms reduce production upon the observation of a favou-

[5]With high product differentiation, the gain from specifying output according to the new information about the realization of demand outweighs the opportunistic decisions by competitors due to the higher production differentiation and thereby the less intense competition. This effect resembles the effect of increasing marginal costs discussed by Kirby (1988) for information sharing and by Perry & Porter (1985) in a merger setting.

[6]Zhou (2008, Proposition 4) finds that the incentives to merge are increased only if the cost uncertainty is large enough in a two-firm-merger and merger profitability largely depends on the number of merging firms, k. When k is very large, few firms are left outside to take advantage of the reduced competition, so the merger is profitable. When k is very small, although the merger is unprofitable due to the reaction of non-merged firms, the loss to the merged firms is small because only a few outlets are shut down.

rable signal in order to 'accommodate the remaining firms' that merge (Gal-Or, 1988, p. 640)). If the outsiders are able to anticipate the merged firm's actions, the outsiders are induced to be more aggressive, lowering the incentives to merge [7]. Gal-Or shows that the net effect can go either way and may finally expose the merging firm to an informational disadvantage, giving rise to her conclusion that in Cournot competition uncertainty can provide an additional incentive to merge, only if in the absence of the uncertainty, such incentives exist anyway.

It is important to note that my paper differs substantially from these contributions as their analyses focus on the effect of *information pooling* between the insiders, while my analysis focuses on *information sharing* from the insiders to the outsiders.

2. The Model

In order to model and assess the effects of risk aversion, uncertainty, and information sharing on the incentives to merge, I analyse four different cases, each differing in their information structure, as discussed below.

Insiders and outsiders play a five step game: In the first step, in an industry of n risk-averse firms producing differentiated products, k firms exogenously decide to merge; subsequently they form the insiders, which generate stochastic efficiency gains. $n - k$ firms form the outsiders post-merger. In the second step, insiders decide and commit on their information revelation strategy, i.e. whether to conceal, reveal or partially reveal the private information they receive in a third step. In a fourth step, the insiders conceal, reveal or partially reveal the private information they received in the third step according to the information revelation strategy they determined in step two. At last, in step five, the firms compete under Cournot competition.

This game structure assumes that the insiders commit to an information revelation strategy prior to obtaining private information about their efficiency gains. This assumption roars back to the fact that in reality merging firms also have to decide on their information revelation strategy with the competition authority, when they only have expectations about these and before knowing them precisely (even prior to the merger!), in order to obtain a merger clearance. Also, this assumption is well-established in the information sharing literature as well as in the horizontal merger literature (e.g. Gal-Or, 1985, 1986).

To model this five step game, I construct and analyse four cases in sections 3.2 and 3.3. In the first case, neither the merging firms ("insiders") nor the non-merging firms ("outsiders") are informed ex ante about the efficiency gains. In the second case, only the merging firms are perfectly informed, while in the third case, the merging firms share their private information about their efficiency gains with the outsiders. In the fourth case, the merging firms ex ante only receive a noisy signal about their efficiency gains, which they can further dilute when informing outsiders (e.g. Gal-Or, 1985, 1986).

[7] Since an outsider knows only its own signal, the estimation is possible only when signals are correlated, specifically if correlation is high and the number of merging firms, k, is low.

The firms optimize their utility, following their utility function $U(\pi) = -e^{-R\pi}$. According to this utility function, firms are exposed to constant absolute risk aversion (CARA). Constant absolute risk aversion assumes that the firms' degree of risk aversion is constant with regards to its wealth or initial endowment. R is the risk aversion coefficient, defined as $R = -\frac{U''}{U'}$. According to Pratt (1964), the effect of risk aversion can be measured in the form of a risk premium, modeled as half time the risk aversion coefficient times the risk involved, i.e. the variance of the random profit function: $\frac{1}{2} \cdot R \cdot Var(f(\pi[\phi]))$, where ϕ denotes the random variable under consideration, i.e. uncertain efficiency gains for the insiders and an uncertain demand shock for the outsiders. Intuitively, firms are willing to sacrifice an amount of expected profits in order to eliminate the risk and obtain the expected profit with certainty. The variance of the random profit function with respect to the random variable under consideration (i.e. demand shock for outsiders, efficiency gains for insiders) can be computed in each case using a Taylor expansion series ("Delta Method") leading to $\frac{1}{2} \cdot R \cdot Var(\pi[\phi]) \approx \frac{1}{2} \cdot R \cdot (f'(\pi(E[\phi])))^2 \cdot Var[\phi]$. As a result the firms' expected utility becomes,

$$EU(\pi) = -e^{-R\left(E(\pi)-\frac{1}{2}\cdot R\cdot\left(f'\left(\pi(E[\phi])\right)\right)^2 Var[\phi]\right)} \tag{1}$$

where,

$$EU(\pi) = U\big(E(\pi) - \Delta\pi\big)$$

To maximize expected utility, firms maximize the bracket term in the exponent, where $\Delta\pi$ denotes the risk premium(e.g. Baron, 1970), leading to,

$$max \ \ f\big(\pi[\phi]\big) - \frac{1}{2} \cdot R \cdot \Big(f'\big(\pi(E[\phi])\big)\Big)^2 \cdot Var[\phi] \tag{2}$$

The firm's profit function, $f(\pi)$, can be derived from the consumer utility function, $U(x)$, and the implied inverse demand function, $p(x)$, as follows.

Consumers follow a usual linear-quadratic utility function (e.g. Vives, 1985; Friedman, 1977),

$$U(x) = a\sum_{i=0}^{n} x_i - \frac{1}{2}\left(b\sum_{i}^{n} x_i^2 - 2d\sum_{j\neq i} x_i x_j\right) \tag{3}$$

where $b, d > 0, b > d, b + (n-1)d > 0$ to ensure that $U(x)$ is strictly concave (Vives, 2001).

This leads to the inverse demand function,

$$p(x) = a - bx_i - dX_{-i} \tag{4}$$

As a result, the firms' pre-merger profit function becomes, assuming constant marginal costs c[8]:

[8]Important to note, the effects of marginal cost, c, and efficiency gains, δ, are analysed separately in the further analysis. Accordingly, symmetric marginal costs do not infer any information about efficiency gains achieved by the insiders, but only about the level of marginal costs, excluding efficiency gains.

$$\pi = \big(a - bx_i - dX_{-i}\big) \cdot x - c \cdot x \tag{5}$$

Post-merger, the insiders generate (random) efficiency gains $\tilde{\delta}$, which directly affect marginal costs, $(1-\tilde{\delta})c_t$, where c_t denotes the insiders' level of marginal costs. Important to note is that the uncertainty about the efficiency gains to be achieved by the merging firms and induced by the horizontal merger translates into a residual demand uncertainty of the outsiders, i.e. the outsiders are unaware of the production output of the insiders and accordingly about their residual demand. Accordingly, the outsiders are exposed to an endogenous random demand shock, θ_i. For each stochastic shock, efficiency gains and demand, the distributional properties coincide as $Var = \sigma^2$ and $Cov(\phi_i, \phi_j) = \rho\sigma^2$ for $i \neq j$[9]. The shocks are positively, independently or negatively correlated depending on whether $\rho \gtreqless 0$. Accordingly, the insiders' and outsiders' expected profit functions become,

$$E(\pi_t) = \sum_{t=1}^{k} \Big(a - bx_t - bX_{-t} - (1-\tilde{\delta})c_t\Big)x_t \tag{6}$$

$$E(\pi_i) = \sum_{i=1}^{n-k} \Big(a - \theta_i - bx_i - bX_{-i} - c_i\Big)x_i \tag{7}$$

Each firms's reaction function given by differentiating (5) pre-merger, and (6) and (7) post-merger, slopes downward. While the effect of the uncertain insiders' efficiency gains on the outsiders is shown as an endogenous demand shock explicitly as θ_i in (7) as outsiders are uncertain about their residual demand due to the uncertainty of the insiders' output, the outsiders' profit function is affected by the change in the insiders output, x_t, post-merger. As shown by Dixit (1986), each firm's marginal revenue is lowered by an increase in rivals' output.

$$p'(X) + x_i p''(X) < 0 \tag{8}$$

Furthermore, a weak stability condition is imposed as each firm's residual demand curve intersects its marginal cost curve from above.

As discussed above, differing underlying information structures are assumed to analyse the effects of uncertainty and risk aversion. Depending on the information the participants possess, the random term drops out in the respective profit function and, likewise, the risk premium does as σ^2, i.e. the risk variance, becomes zero.

[9] These distributional properties are assumed in section 3.2.1 and 3.2.2 and alternated and precised for partial information sharing in section 3.2.3 and Appendix C to take account of the insiders' influence on the signal shared with the outsiders.

3. Model analysis

3.1. Pre-merger equilibria

Prior to the merger, neither the insiders nor the outsiders are exposed to stochastic efficiency gains or (residual) demand. To maximize (1), consequently, the general maximization problem in (2) simplifies to a maximization of the deterministic profit function (5) for all firms since no uncertainty, and thereby no stochastic variable, gives rise to a risk premium. The equilibrium output and profit become under a symmetric level of marginal costs, i.e. $c_t = c_i = c$,

$$x = \frac{a-c}{2b+d(n-1)} \tag{9}$$

$$\pi = b(\frac{a-c}{2b+d(n-1)})^2 \tag{10}$$

3.2. Insiders' merger incentives and merger profitability

3.2.1. Insiders and Outsiders are uncertain about the insiders' efficiency gains

In this setting, in which insiders and outsiders are unaware about the insiders' efficiency gains[10] and, accordingly, about the outsiders' residual demand, both parties compete under uncertainty.

Given this underlying information structure, insiders and outsiders maximize (2), i.e. $E(\pi) - \frac{R}{2}Var(\pi)$, in order to maximize their expected utility function (1),

$$max \sum_{t=1}^{k} \Big(a - bx_t - bX_{-t} - (1-\delta)c_t\Big)x_t - \frac{R}{2}\sigma^2\Big(\big(\sum_{t=1}^{k} x_t c_t\big)^2 + \rho \sum_{i,ji\neq j} x_t c_t x_j c_j\Big) \tag{11}$$

$$max \sum_{i=1}^{n-k} \Big(a - bx_i - bX_{-i} - c_i\Big)x_i - \frac{R}{2}\sigma^2 x_i^2 \tag{12}$$

The maximization problem (2) yields for insiders and outsiders, respectively,

$$\big(b + d(k-1) + \frac{R}{2}\sigma^2 c_t^2(1+(k-1)\rho)\big)x_t^2 \tag{13}$$

$$(b + \frac{R}{2}\sigma^2)x_i^2 \tag{14}$$

From (13) it therefore becomes apparent that expected utility (1) increases with production output. This implies, merger incentives increase with the merging firms' output.

[10]Insiders have either not received a signal about the realization of their efficiency gains or the signal did not convey any information. As outlined in section 3.2.3, this would be achieved when $e_t = \infty$.

As a consequence, insiders maximize their output in order to maximize their utility! Therefore, to investigate merger profitability in this case, where neither insiders nor outsiders are aware of the insiders' efficiency gains, insiders' output pre- and post-merger has to be compared.

According to Appendix A, the insiders equilibrium output yields

$$x_t = \frac{a(2b - d + R\sigma^2) - (1-\delta)c_t(2b + d(n-k-1) + R\sigma^2) + d(n-k)c_i}{(2b + d(n-k-1) + R\sigma^2)(2b + 2d(k-1) + R\sigma^2 c_t^2(1 + (k-1)\rho)) - d^2 k(n-k)} \tag{15}$$

Comparing (9) and (15) and assuming a symmetric level of marginal costs, i.e. $c_t = c_i = c$, insiders reduce output post-merger, if they are not risk averse. This finding confirms the findings of the existing traditional horizontal merger literature (e.g. Salant, Switzer & Reynolds (1983)). The output reduction, however, diminishes as efficiency gains increase, resulting in higher merger profitability.

When insiders and outsiders are uncertain about the future efficiency gains, i.e. $\sigma^2 > 0$,and are risk averse, insiders increase output and profit as risk aversion increases, similar to findings by Banal-Estañol & Ottaviani (2006); this effect is even enhanced by a decrease in marginal costs and, oppositely to the findings by Salant, Switzer & Reynolds (1983), by an increase in the industry size. Generally, compared to the risk-neutral case, outsiders react less opportunistically than in the pre-merger setting due to their risk-aversion ("risk-averse behaviour effect"). This implies they do not take advantage of the price increase by the merging firms (if insiders were to reduce output) as in the case of Salant, Switzer & Reynolds (1983), or, if insiders increase output, the merging firms oppositely can expand output even more due to the larger industry size and as more outsiders decrease output even more than they would if they had been risk-neutral. A further influencing factor is the substitutability of the products. A decrease in the substitutability of the products leads to lower merger profitability, if $d((2b + d(n-k-1) + R\sigma^2)((a-c)(k-1) - \delta c(n-1)) - (a-c)) > 0$. Drawing on the arguments above, due to the risk aversion, an increase in the insiders' output is most profitable if products are homogeneous and outsiders need to decrease output is therefore highest.

Furthermore, as risk aversion increases, the insiders' expected utility increases as seen in (13) as long as the shocks are not too negatively correlated, which can be excluded given that the shock stems from the same source, i.e. the insiders' efficiency gains.

3.2.2. Perfectly informed insiders

In this case, it is explored whether perfectly informed insiders' merger incentives increase when they conceal or reveal the perfect private information which they receive about the realization of their stochastic efficiency gains. Explicitly, it is assumed that after the merger insiders receive a perfect signal about the realization of their efficiency gains imposed by the merger. Subsequently, the merging firms conceal or reveal their private information prior to their and the outsiders' production decision.

The information revelation decision directly infers the uncertainty structure. If insiders do not share their private information, asymmetric uncertainty persists (i.e. insiders are informed, while outsiders are not as in section 3.2.1), while if the insiders reveal their information, insiders and outsiders are not exposed to uncertainty and compete in a deterministic setting.

As before, given this underlying information structure, insiders and outsiders maximize (2), i.e. $E(\pi)-\frac{R}{2}Var(\pi)$, in order to maximize their expected utility function (1). Insiders are perfectly informed about their efficiency gains and therefore maximize their expected profit as follows (i.e. they do not account for a risk premium).

$$max \sum_{t=1}^{k} \Big(a - bx_t - bX_{-t} - (1-\delta)c_t\Big)x_t \tag{16}$$

If outsiders are not informed about the realization of the insiders efficiency gains, their maximization problem is identical to (12). Otherwise, their maximization becomes,

$$max \sum_{i=1}^{n-k} \Big(a - bx_i - bX_{-i} - c_i\Big)x_i \tag{17}$$

In the course of the analysis of this scenario, it will be shown that insiders especially consider two effects, when deciding upon their merger, the outsiders' risk-averse behaviour effect and the efficiency gains effect, both of which enhancing insiders' expected utility. The first effect stems from the outsiders uncertainty and risk-aversion, while the second effect captures the extent of efficiency gains achieved by the insiders upon the merger. As shown in Appendix B.1, the insiders' equilibrium profit solves to $\pi_t = (b + d(k-1))x_t^2$, which in this deterministic setting for the insiders already maximizes expected utility according to (16), when $\Delta\pi = 0$. According to Appendix B.1, the insiders equilibrium output yields

$$x_t = \frac{a(2b - d + R\sigma^2) - (1-\delta)c_t(2b + d(n-k-1) + R\sigma^2) + d(n-k)c_i}{(2b + d(n-k-1) + R\sigma^2)(2b + 2d(k-1)) - d^2k(n-k)} \tag{18}$$

if outsiders are not informed about the insiders' efficiency gains and according to Appendix B.2

$$x_t = \frac{a(2b - d) - (1-\delta)c_t(2b + d(n-k-1)) + d(n-k)c_i}{(2b + d(n-k-1))(2b + 2d(k-1)) - d^2k(n-k)} \tag{19}$$

if outsiders are informed.

As expected utility increases -and therewith merger profitability and merger incentives- with production output as shown in Appendix B.3, comparing (18) and (19) sheds light on whether insiders should conceal or reveal their private information in order to maximize merger profitability and merger incentives.

As shown in Appendix B.3 under the assumption of a symmetric level of marginal costs, the merging firms should largely conceal their private information. This finding is in sharp contrast to the findings by Gal-Or (1986) and Shapiro (1986), who find that insiders have incentives to share private information if uncertainties exist about private values (e.g. costs). My finding is increasing in the degree of risk aversion, R, the extent of uncertainty, σ^2, as well as product differentiation, d. Similar to the reasoning, when insiders and outsiders are exposed to uncertainty, outsiders produce less (i.e. act as if demand is lower) due to their risk-aversion ("risk-averse behaviour effect"). While under the setting of symmetric uncertainty both, insiders and outsiders, were exposed to risk-aversion, here insiders have an influence on the outsiders' exposure to risk aversion due to their inforomation sharing strategy. Given the nature of strategic substitutes, the outsiders' lower output due to their risk-averse behaviour enhances merger incentives as insiders can take further advantage of the risk-averse behaviour of the outsiders, leading to the optimal information sharing strategy of concealing private information.

In order to finally decide on whether the merger itself is (privately) profitable, the equilibrium output pre-merger and under the asymmetric information structure has to be compared as insiders have incentives not to share their private information about the realization of their efficiency gains as shown above. According to Appendix B.3, the merging firms' output (and, thereby its expected utility) is higher, if the merger takes place, and increases in the efficiency gains, δ, since the merging firms increase output as costs decrease as illustrated above ("efficiency gains effect"), as well as the uncertainty and risk aversion, due to the higher risk bearing potential of the merged firm, provoking the outsiders risk-averse behaviour effect. However, as the merger scale, i.e. the number of insiders, increases, merger incentives decrease, since the insiders can take less advantage of the outsiders risk-averse behaviour. This is also in sharp contrast to the findings by Salant, Switzer & Reynolds (1983) and the traditional horizontal merger literature, who find that merger incentives increase with the number of insiders. My effect is specifically caused by the insiders' lower possibilities to take advantage of the outsiders' risk-averse behaviour effect as the merger scale increases as less outsiders remain in the industry if the merger scale increases.

3.2.3. Partially informed insiders

Partial Information Sharing structure

In this model setting, insiders receive a noisy signal about their efficiency gains before they make their production decision and reveal this signal partially, fully or not at all. The following analysis sheds light on whether insiders should dilute the information they have when communicating with the outsiders (or a competition authority) and what the optimal level of dilution is.

It is assumed that $\tilde{\delta}$ is a random variable that is normally distributed with mean δ[11] and variance η. Before choosing its output strategy the merging firms observe a signal for their efficiency gains as follows:

[11] Without loss of generality, the mean of δ could also be normalized to zero.

$$z_t = \tilde{\delta} + e_t \tag{20}$$

where $e_t \sim N(0, m)$ and where e_t and $\tilde{\delta}$ are independently distributed.

When revealing information to the outsiders, the insiders themselves convey a signal in the form of:

$$\hat{z}_t = z_t + f_t \tag{21}$$

where $f_t \sim N(0, s_t)$.

Prior to receiving their own signal, though, the insiders choose an information revelation strategy in accordance with the five stage game. In this case, they choose the amount of garbling, s_t, to which any report they make e.g. to competition authorities is subject[12]. The amount of garbling by the insiders is denoted by s_t. If $s_t = 0$, insiders perfectly reveal their private information, if $s = \infty$ insiders generate infinite noise in their signal so that the informational content is worthless. If $0 < s_t < \infty$, insiders partially reveal their private information.

As discussed previously, the insiders' efficiency gain uncertainty translates into a (residual) demand uncertainty for the outsiders. Accordingly, insiders signal equivalently conveys information about the realization of the residual demand illustrated in (7). The outsiders' uncertainty might differ, depending on the individual signal they receive. However, given that the uncertainty stems from the same source, i.e. the insiders' efficiency gains, it is assumed that this information about the realization of the insiders' efficiency gains received by one outsider may be positively correlated with the information received by another outsider. The parameter h determines this correlation. When $h = 0$, private signals are completely uncorrelated[13]. When $h = \eta$, the coefficient of correlation between the demand shocks observed by the outsiders is one.[14]

After insiders report their signal to the competition authority, the competition authority makes the information available to the outsiders. The reported information is subsequently used by each firm to select its output strategy. This information set for the insiders is denoted by $t_t = (z_t, \hat{z}_t, s_t)$ and for the outsiders $t_i = (\hat{z}_t, s_t)$. Hence, each firm, insiders and outsiders, can condition its output strategy on the basis of its information set.

Determination of Insiders' Partial Information revelation strategy

Following Gal-Or (1985, 1986), I consider only subgame perfect equilibria. Hence it has to be guaranteed that the strategy choice made by each firm initially remains optimal at the time of their production decision. This problem can be solved through backward induction, starting with the optimal production decision for a given level of garbling s_t.

[12] The assumption is that the insideres have to inform the competition authorities on achievable efficiency gains in order to receive merger clearance: Efficiency gains increase social welfare (Farrell & Shapiro, 1990) and thereby increase the probability of merger clearance.

[13] This condition coincides with Raith (1996)'s notion of independent values.

[14] Since the source of the uncertainty is the same for all demand shocks, i.e. the insiders' signal about the realization of its efficiency gains, this case is of particular interest.

Since both, insiders and outsiders, are still exposed to uncertainty at the time of setting their production strategy, insiders and outsiders follow (2), where the output strategy though specifically depends on the private information set, t_t and t_i.

Proposition: For given s_t, the following equilibrium outputs are the unique Nash equilibrium under Cournot competition of the maximization subgame,

$x_t = C_0 + C_1 z_t + C_2 \hat{z}_i$

and

$x_i = B_0 + B_1 \hat{z}_i$

Proof. Provided in Appendix C

As shown in Appendix C, the equilibrium output of the merging firms becomes

$$\begin{aligned} C_0 &= \frac{a(2b - d + R\sigma^2) + d(n-k)c_i}{y_1 y_2 - d^2(n-k)k} \\ C_1 &= -\frac{d(n-k)((\eta + m)dkmc_t)}{y_1((\eta + m + (k-1)h)(\eta + m + s_t)(y_1 y_2 - d^2(n-k)k))} \\ C_2 &= -\frac{m}{(\eta + m + (k-1)h)y_1} c_t \end{aligned} \tag{22}$$

where,

$y_1 = 2b + 2d(k-1) + R\sigma^2 c_t^2 (1 - (k-1)\rho)$

$y_2 = 2b + d(n-k-1) + R\sigma^2$

Since the insiders are still exposed to uncertainty as they may not receive a perfect signal of the realization of their efficiency gains, the maximization problem is identical to the maximization problem in section 3.2.1, (11). According to (13), insiders maximize output in order to maximize expected utility, which yields the equilibrium of the output decision subgame.

To evaluate whether the insiders should fully, partially or not at all reveal their private information signal, it therefore has to be analysed when the insiders' equilibrium output is highest for any given s_t. From (22) it is obvious that only, C_1, i.e. the insiders' reaction to their private signal, depends on the garbling term. Specifically, if C_1 decreases, output and thereby merger profitability increases. C_1 is a strictly decreasing function of s_t. This implies, as s_t increases, C_1 decreases. Hence, as $s \to \infty$, merger profitability increases and $s_t = \infty$, i.e. conveying no information and -in line with our findings above- concealing the private signal, is a dominant strategy for each merging firm at the Cournot equilibrium with uncertainty about private efficiency gains.

3.3. Consumer Surplus

Consumer surplus, i.e. the net gain to consumers for a particular amount of a good, is a primary measure for the competition authorities of the (anti-)competitive effects imposed on consumers resulting from a (horizontal) merger and is computed as the area under the demand curve between 0 and the amount of the good (X), i.e. the equilibrium output of the firms, minus the monetary cost of acquiring that amount of the good ($p \cdot X$). This may be expressed in terms of the integral of the inverse demand function $p(x)$ as follows

$$CS = \int_0^X p(x)dx - pX = \int_0^X (p(x) - p)dx \tag{23}$$

Using the inverse demand function,

$$CS = \frac{1}{2}\big(a - p(x)\big)x_i = \frac{1}{2}\Big(\sum_{i=1}^{n} bx_i^2 + \sum_{j=1, j\neq i}^{n} dx_i x_j\Big) \tag{24}$$

This implies that consumer surplus increases with industry output as shown by Dixit (1986) in an duopoly setting and extended by Farrell & Shapiro (1990) to an oligopoly setting under Cournot competition with horizontal mergers.[15] Upon the output change of one party in the industry, the other parties reaction is to change their output in the opposite direction. Importantly, the reaction of the other parties does not offset the change in the industry output induced by the initial change in output of the first party.

As shown above, mergers private profitability increases when insiders increase their output post-merger. As output, and thereby merger profitability, increases with a higher probability under the assumptions of risk aversion and uncertainty as shown above, these mergers also induce a positive change in the consumer surplus: The higher output of the merging firms leads to a higher industry output resulting in a higher consumer surplus. Therefore, not only the merger incentives have been underestimated by the previous traditional merger literature as discussed, but also the positive impact of risk aversion and uncertainty on consumer surplus has previously been neglected by foregoing a holistic analysis of the effects of risk aversion and uncertainty on the incentives to merge.

4. Results and Implications of the five stage game

The results for the five stage game outlined in section 2 can be summarized as follows. As outlined, after the exogenous merger decision, the merging entities have to decide on their information revelation strategy about the information they receive subsequently. The insiders might receive one out of three types of signal containing either no, partial, or full information about the efficiency gains they achieve upon the merger. As shown, assuming risk-averse firms, the insiders' optimal revelation strategy is to conceal any information they have as shown in the preceding sections if the efficiency gains are not too high[16], in order to maximize their expected utility when competing under Cournot competition in the last stage of the game.

When insiders increase their output post-merger, mergers (private) profitability increases. As shown in the previous literature, this is largely not the case when firms

[15] Dixit (1986) and Farrell & Shapiro (1990) also assume that demand curves slope downward and that each firm's residual demand curve intersects its marginal cost curve from above

[16] To see the opposing effect of efficiency gains, please be referred to (50) in Appendix B.3.

operate in the vNM framework with risk-neutral firms and under disrespect of efficiency gains. I have shown though that the output reduction of risk-neutral merging firms diminishes as efficiency gains increase, resulting in higher merger profitability.

Introducing uncertainty and risk aversion into the framework changes merger incentives and optimal information sharing strategies considerably. If the private signal to the insiders did not convey any information about the realization of the stochastic efficiency gains or if the insiders did not receive a signal before competition takes place, insiders increase output and profit as risk aversion or efficiency gains increase. Insiders can expand output induced by the efficiency gains and insiders risk-bearing potential further due to the risk-averse behaviour of the outsiders, as these behave softer than under risk-neutrality shown in the traditional horizontal merger literature. Thereby, merger profitability is higher in a setting of risk aversion. Whether horizontal mergers are privately profitable in a setting of complete uncertainty depends on the degree as well as the extent of the expected efficiency gains.

If the private signal to the insiders conveyed full or partial information about the efficiency gains, the merging firms have incentives to conceal their private information. This finding is in sharp contrast to the findings by Gal-Or (1986) and Shapiro (1986), who find that insiders should share private information if uncertainties exist about private values (e.g. costs). The effect of concealing private information is triggered and increasing by the degree of risk aversion, R, the extent of uncertainty, σ, as well product differentiation, d. Given the nature of strategic substitutes, the outsiders' lower output due to risk-averse behaviour enhances merger incentives as insiders can take advantage of the risk-averse behaviour of the outsiders, leading to the optimal information sharing strategy of concealing private information.

5. Conclusion

I have shown in this paper that merger profitability, and thereby the incentives to merge, have been underestimated by the traditional horizontal merger literature (e.g. Salant, Switzer & Reynolds (1983)) by foregoing a holistic analysis of the effects of risk aversion and uncertainty on the incentives to merge. Furthermore, I find that the information sharing incentives about private variables are reversed if risk aversion is taken into account.

From a regulatory point of view, these mergers have not been to the consumers' disadvantage. Rather oppositely, horizontal mergers in an industry of firms, which are or act as if they are risk-averse, favour consumers. Finally, competition authorities should underline the importance of uncertainty and risk aversion in their guidelines and should put particular emphasis on the industries' information structure when assessing antitrust cases.

A. Insiders and Outsiders are uncertain about the insiders' efficiency gains

If k insiders decide to merge, the merging firms' and the outsiders' general maximization problems in (2) become,

$$max\sum_{t=1}^{k}\Big(a-bx_t-bX_{-t}-(1-\delta)c_t\Big)x_t-\frac{R}{2}\sigma^2\Big(\big(\sum_{t=1}^{k}x_t\cdot c_t\big)^2+\rho\sum_{i,ji\neq j}x_t\cdot c_t\cdot x_j\cdot c_j\Big) \quad (25)$$

$$max\sum_{i=1}^{n-k}\Big(a-bx_i-bX_{-i}-c_i\Big)x_i-\frac{R}{2}\sigma^2\cdot x_i^2 \quad (26)$$

Solving the resulting first order conditions for the insiders' and outsiders' equilibrium output yields,

$$x_t=\frac{a(2b-d+R\sigma^2)-(1-\delta)c_t(2b+d(n-k-1)+R\sigma^2)+d(n-k)c_i}{(2b+d(n-k-1)+R\sigma^2)(2b+2d(k-1)+R\sigma^2c_t^2(1+(k-1)\rho))-d^2k(n-k)} \quad (27)$$

$$x_i=\frac{(a-c_i)(2b+2d(k-1)+R\sigma^2c_t^2(1+(k-1)\rho))-dk(a-(1-\delta)c_t}{(2b+d(n-k-1)+R\sigma^2)(2b+2d(k-1)+R\sigma^2c_t^2(1+(k-1)\rho))-d^2k(n-k)} \quad (28)$$

Substituting into the inverse demand functions and solving for the equilibrium profits $\pi_t=(p(x_t)-(1-\delta)c_t)x_t$ and $\pi_i=(p(x_i)-c_i)x_i$ yields,

$$\pi_t=(b+d(k-1)+R\sigma^2c_t^2(1+(k-1)\rho))x_t^2 \quad (29)$$

$$\pi_i=(b+R\sigma^2)x_i^2 \quad (30)$$

Assuming symmetric marginal costs, i.e. $c_t=c_i=c$, insiders increase output if,

$$\frac{(a-(1-\delta)c)(2b+d(n-k-1)+R\sigma^2)-d(n-k)(a-c)}{(2b+d(n-k-1)+R\sigma^2)(2b+2d(k-1)+R\sigma^2c^2(1+(k-1)\rho))-d^2(n-k)k} > \frac{a-c}{2b+d(n-1)} \quad (31)$$

This simplifies to,

$$(a-c)\Big(\big(1-c^2(1+(k-1)\rho)\big)-\frac{2b-d+R\sigma^2}{2b+d(n-k-i)+R\sigma^2}\Big)R\sigma^2 > (a-c)d(k-1)-\delta c(2b+d(n-1)) \quad (32)$$

B. Perfectly informed informed insiders

B.1. Perfectly informed informed insiders do not reveal their information

If k insiders decide to merge and are perfectly informed about their efficiency gains, the merging firms' and the outsiders' maximization problems become

$$max \sum_{t=1}^{k} \Big(a - bx_t - bX_{-t} - (1-\delta)c_t\Big)x_t \tag{33}$$

$$max \sum_{i=1}^{n-k} \Big(a - bx_i - bX_{-i} - c_i\Big)x_i - \frac{R}{2}\sigma^2 \cdot x_i^2 \tag{34}$$

Solving the resulting first order conditions for the insiders' and outsiders' output yields,

$$x_t = \frac{a(2b - d + R\sigma^2) - (1-\delta)c_t(2b + d(n-k-1) + R\sigma^2) + d(n-k)c_i}{(2b + d(n-k-1) + R\sigma^2)(2b + 2d(k-1)) - d^2k(n-k)} \tag{35}$$

$$x_i = \frac{(a - c_i)(2b + 2d(k-1)) - dk(a - (1-\delta)c_t}{(2b + d(n-k-1) + R\sigma^2)(2b + 2d(k-1)) - d^2k(n-k)} \tag{36}$$

Substituting into the inverse demand functions and solving for the equilibrium profits $\pi_t = (p(x_t) - (1-\delta)c_t)x_t$ and $\pi_i = (p(x_i) - c_i)x_i$ yields,

$$\pi_t = (b + d(k-1))x_t^2 \tag{37}$$

$$\pi_i = (b + R\sigma^2)x_i^2 \tag{38}$$

To finally maximize expected utility, the firms maximize $E(\pi) - \frac{R}{2}Var(\pi)$ as shown above. The maximization problem therefore yields for the insiders, where $Var(\pi) = 0$, and the outsiders,

$$(b + d(k-1))x_t^2 \tag{39}$$

$$(b + \frac{R}{2}\sigma^2)x_i^2 \tag{40}$$

B.2. Insiders and Outsiders are perfectly informed about insiders' efficiency gains

If k insiders decide to merge and both, insiders and outsiders, are perfectly informed about the insiders' efficiency gains prior to their production decision, the merging firms' and the outsiders' maximization problems become

$$max \sum_{t=1}^{k} \Big(a - bx_t - bX_{-t} - (1-\delta)c_t\Big)x_t \tag{41}$$

$$max \sum_{i=1}^{n-k} \Big(a - bx_i - bX_{-i} - c_i\Big) x_i \tag{42}$$

Solving the resulting first order conditions for insiders' and outsiders' output yields,

$$x_t = \frac{a(2b-d) - (1-\delta)c_t(2b + d(n-k-1)) + d(n-k)c_i}{(2b + d(n-k-1))(2b + 2d(k-1)) - d^2k(n-k)} \tag{43}$$

$$x_i = \frac{(a-c_i)(2b + 2d(k-1)) - dk(a - (1-\delta)c_t)}{(2b + d(n-k-1))(2b + 2d(k-1)) - d^2k(n-k)} \tag{44}$$

Substituting into the inverse demand functions (4) and solving for the equilibrium profits $\pi_t = (p(x_t) - (1-\delta)c_t)x_t)$ and $\pi_i = (p(x_i) - c_i)x_i)$ yields,

$$\pi_t = (b + d(k-1))x_t^2 \tag{45}$$

$$\pi_i = bx_i^2 \tag{46}$$

To finally maximize expected utility, the firms maximize $E(\pi) - \frac{R}{2}Var(\pi)$ as shown above. Since $Var(\pi) = 0$ for insiders and outsiders, the maximization problem solves to,

$$(b + d(k-1))x_t^2 \tag{47}$$

$$bx_i^2 \tag{48}$$

B.3. Should insiders share their private information?

This proof follows in two steps. In the first step, it is analysed, whether the expected utility of the merging firms is higher when the merging firms share their information they possess or not. Analytically, insiders will conceal their information, if (39) > (47) and reveal their private information about their efficiency gains otherwise.

In accordance to the arguments made that expected utility increases with production output and assuming symmetric marginal costs, i.e. $c_t = c_i = c$, insiders utility is higher if they conceal their information, if

$$\frac{a(2b - d + R\sigma^2) - (1-\delta)c_t(2b + d(n-k-1) + R\sigma^2) + d(n-k)c_i}{(2b + d(n-k-1) + R\sigma^2)(2b + 2d(k-1)) - d^2k(n-k)} > \frac{a(2b-d) - (1-\delta)c_t(2b + d(n-k-1)) + d(n-k)c_i}{(2b + d(n-k-1))(2b + 2d(k-1)) - d^2k(n-k)} \tag{49}$$

This simplifies to,

$$d(n-k)R\sigma^2\Big((a-c)(2b + d(k-2)) - dk\delta c\Big) > 0 \tag{50}$$

$R\sigma^2$ in (50) relates to the outsiders' risk aversion. As d and the outsiders' risk aversion increase, the area in which the insiders should conceal their private information increases, since the brackets term in (50) is mostly positive[17]. As a conclusion, insiders have incentives to conceal their private information about their efficiency gains.

In a second step, it is necessary to investigate, under which conditions the expected utility received when concealing information results in a privately profitable merger.

To conclude on this question, the equilibrium output when insiders are perfectly informed and conceal their information and the pre-merger setting have to be compared. Specifically, the merger is profitable, if

$$\frac{a(2b-d+R\sigma^2)-(1-\delta)c_t(2b+d(n-k-1)+R\sigma^2)+d(n-k)c_i}{(2b+d(n-k-1)+R\sigma^2)(2b+2d(k-1))-d^2k(n-k)} > \frac{a-c}{2b+d(n-1)} \quad (51)$$

Therefore, insiders output, and therefore the expected utility, is larger post-merger if,

$$((a-c)(d(n-k)-d(k-1))(2b+d(n-k-1)+R\sigma^2)+\delta(2b+d(n-1))(2b+d(n-k-1)+R\sigma^2) > (a-c)d(n-k)(2b+d(n-k-1)) \quad (52)$$

This implies, under this setting, where insiders conceal their private information about the realization of their stochastic efficiency gains from the risk-averse outsiders, a merger is privately profitable, but this effect diminishes in the number of insiders. As the number of insiders increases, these can only take advantage of fewer outsiders' risk-averse behaviour, who forgo on profits according to (1). Specifically, if $k \to n$, merger incentives persist in the extreme as long as the efficiency gains δ, which also increase merger profitability, are high enough, i.e. $\delta(2b+d(n-1)) > (a-c)d(n-1)$, and thereby counter-effect the lower merger profitability caused by the higher merger scale.

C. Partially informed insiders

Derivation of the posterior expected values

The insiders receive the signal

$$z_t = \tilde{\delta}_t + e_t \quad (53)$$

$\tilde{\delta}_t \sim N(\delta_t, \eta)$
$e_t \sim N(0, m)$

[17]It is easy to show that if asymmetric marginal costs are considered, the extent of marginal cost asymmetry enhances information revelation incentives if $c_t < c_i$, as efficiency gains do as to be seen in (50). Furthermore, the information revelation strategy would be alternated towards sharing private information, if the efficiency gains were very high.

and produce the signal

$$\hat{z}_t = z_t + f_t \tag{54}$$

$z_t \sim N(\delta_t, \eta + m)$
$f_t \sim N(0, s_t)$

The outsiders receive this signal, which equivalently conveys information about the realization of their residual demand. Specifically, if the outsiders receive information about the efficiency gains of the insiders, this signal of the insiders, $\hat{z}_t$, conveys diluted information about insiders' efficiency gains as well as outsiders' random demand.

Since the distribution of priors and private signals is normal and demand is linear, each firm follows a linear decision rule (Radner, 1962) of the form

$$\begin{aligned} x_t &= C_0 + C_1 \hat{z}_t + C_2 z_t \\ x_i &= B_0 + B_1 \hat{z}_i \end{aligned} \tag{55}$$

Since both, insiders and outsiders, are still uncertain about the insiders' efficiency gains and their residual demand respectively, the first order conditions of the insiders and the outsiders, respectively, follow from (11) and (12),

$$E\Big(a - 2b\Big(C_0 + \sum_{t=1}^{k} C_1 \hat{z}_t + \sum_{t=1}^{k} C_2 z_t\Big) - d \sum_{i=k+1}^{n} \Big(B_0 + B_1 \hat{z}_i\Big) - \sum_{t=1}^{k} \Big(1 - \delta_t\Big) c_t \\ - R\sigma^2 c_t^2 \Big(1 + (k-1)\rho\Big(C_0 + \sum_{t=1}^{k} C_1 \hat{z}_t + \sum_{t=1}^{k} C_2 z_t\Big) = 0 \tag{56}$$

$$E\Big(a - d(C_0 + \sum_{t=1}^{k} C_1 \hat{z}_t + \sum_{t=1}^{k} C_2 z_t) - 2b\Big(B_0 + B_1 \hat{z}_i\Big) - d \sum_{i=k+1//i\neq j}^{n} \Big(B_0 + B_1 \hat{z}_i\Big) \\ - c_i - R\sigma^2 \Big(B_0 + B_1 \hat{z}_i\Big)\Big) = 0 \tag{57}$$

Using the distributional properties of $\hat{z}_t$, z_t, and δ_t, the posterior expected values for the unobserved variables δ_t for the insiders and z_t for the outsiders can be derived as follows:

1) $E(\delta_t \mid z_1 \cdots z_k)$

The signal the insiders receive can be decomposed to,

$$z_t = \delta_t + e_t \rightarrow \frac{\sum_{t=1}^{k} z_t}{k} = \frac{\sum_{t=1}^{k} \delta_t}{k} + \frac{\sum_{t=1}^{k} e_t}{k} \tag{58}$$

Using expected operators on the unknown variables yields,

$$E\Big(\frac{\sum_{t=1}^{k}\delta_t}{k} \mid z_1\cdots z_k\Big) = \frac{\sum_{t=1}^{k} z_t}{k} - E\Big(\frac{\sum_{t=1}^{k} e_t}{k} \mid z_1\cdots z_k\Big) \tag{59}$$

Accordingly, to find the posterior expected value of δ_t, since z_t is observed by the insiders, only the expected value of e_t has to be computed using the respective Variance-Covariance Matrix,

$$\begin{array}{c} \\ e_t \\ z_1 \\ \vdots \\ z \\ \vdots \\ z_n \end{array}\begin{array}{c} \begin{array}{cccccc} e_t & z_1 & \cdots & z & \cdots & z_n \end{array} \\ \begin{pmatrix} m & 0 & \cdots & m & \cdots & 0 \\ 0 & \eta+m & \cdots & h & \cdots & h \\ \vdots & & \ddots & \vdots & & \vdots \\ m & h & \cdots & \ddots & & \vdots \\ \vdots & & & & \ddots & \vdots \\ 0 & h & \cdots & \cdots & & \eta+m \end{pmatrix} \end{array} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

Based upon DeGroot (1970),

$$E(e_t \mid z_1\cdots z_k) = S_{12}S_{22}^{-1}\begin{pmatrix} z_1 \\ \vdots \\ z_k \end{pmatrix}$$

Substituting in yields,

$$\begin{pmatrix} 0 & \cdots & m & \cdots & 0 \end{pmatrix}\begin{pmatrix} \eta+m & h & \cdots & h \\ & & & \vdots \\ h & \eta+m & & \vdots \\ \vdots & & \ddots & \vdots \\ h & & & \eta+m \end{pmatrix}\begin{pmatrix} z_1 \\ \vdots \\ \vdots \\ \vdots \\ z_k \end{pmatrix}$$

Therefore,

$$E(\frac{\sum_{t=1}^{k} e_t}{k}) = \frac{m\sum_{i=1}^{k} z_t}{(\eta+m+(k-1)h)k} \tag{60}$$

Substituting in (58) results in the posterior expected value for δ_t,

$$E\Big(\delta_t \mid z_1\cdots z_k\Big) = \frac{\sum_{i=1}^{k} z_t}{k} - \frac{m\sum_{i=1}^{k} z_t}{(\eta+m+(k-1)h)k} = \frac{\eta+(k-1)h}{\eta+m+(k-1)h}z_t \tag{61}$$

2) $E(z_t \mid \hat{z}_i)$

Using the Variance-Covariance Matrix,

$$\begin{pmatrix} \eta + m + s_t & \eta + m \\ \eta + m & \eta + m + s_t \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \tag{62}$$

and DeGroot's rule, $S_{12}S_{22}^{-1}\hat{z}_i$, the expected posterior value for $E(z_t \mid \hat{z}_i)$ becomes,

$$E(z_t \mid \hat{z}_i) = \frac{\eta + m}{\eta + m + s_t}\hat{z}_t \tag{63}$$

since $\hat{z}_i = \hat{z}_t$.

Substituting the posterior expected values into the first order conditions (56) and (57) yields a system of five equations with five unknowns,

$$\begin{aligned}
&(1)\ \ a - (2b + 2d(k-1))C_0 - d(n-k)B_0 - R\sigma^2 c_t^2(1 + (k-1)\rho)C_0 = 0 \\
&(2)\ \ a - 2bB_0 - dkC_0 - d(n-k-1)B_0 - c_i - R\sigma^2 B_0 = 0 \\
&(3)\ \ (-2b - 2d(k-1))C_3 - (1 - \frac{\eta + (k-1)h}{\eta + m + (k-1)h})c_t - R\sigma^2 c_t^2(1 + (k-1)\rho)C_3 = 0 \\
&(4)\ \ (-2b - 2d(k-1))C_2 - d(n-k)B_1 - R\sigma^2 c_t^2(1 + (k-1)\rho)C_2 = 0 \\
&(5)\ \ -dkC_2 - 2bB1 - d(n-k-1)B_1 - dkC_3\frac{\eta + m}{\eta + m + s_t} = 0
\end{aligned} \tag{64}$$

Solving for the five unknowns yields,

$$\begin{aligned}
B_0 &= \frac{(a - c_i)y_1y_2 - dka}{y_1y_2 - d^2(n-k)k} \\
B_1 &= \frac{(\eta + m)dkmc_t}{(\eta + m + (k-1)h)(\eta + m + s_t)(y_1y_2 - d^2(n-k)k)} \\
C_0 &= \frac{a(2b - d + R\sigma^2) + d(n-k)c_i}{y_1y_2 - d^2(n-k)k} \\
C_1 &= -\frac{d(n-k)(\eta + m)dkmc_t}{y_1((\eta + m + (k-1)h)(\eta + m + s_t)(y_1y_2 - d^2(n-k)k))} \\
C_2 &= -\frac{m}{(\eta + m + (k-1)h)y_1}c_t
\end{aligned} \tag{65}$$

where,

$y_1 = 2b + 2d(k-1) + R\sigma^2 c_t^2(1 - (k-1)\rho)$

$y_2 = 2b + d(n-k-1) + R\sigma^2$

References

Arrow, K. J. (1965). Aspects of the theory of risk-bearing. Essay.

Asplund, M. (2002). Risk-averse firms in oligopoly. *International Journal of Industrial Organization*, *20*, 995–1012.

Banal-Estañol, A. (2007). Information-sharing implications of horizontal mergers. *International Journal of Industrial Organization*, *25*, 31–49.

Banal-Estañol, A. & Ottaviani, M. (2006). Mergers with product market risk. *Journal of Economics & Management Strategy*, *15*, 577–608.

Baron, D. P. (1970). Price uncertainty, utility, and industry equilibrium in pure competition. *International Economic Review*, *11*, 463–480.

Baron, D. P. (1971). Demand uncertainty in imperfect competition. *International Economic Review*, *12*, 196–208.

Cheung, F. K. (1992). Two remarks on the equilibrium analysis of horizontal merger. *Economics Letters*, *40*, 119–123.

DeGroot, M. H. (1970). *Optimal Statistical Decisions*. McGraw Hill, New York.

Diamond, P. A. & Stiglitz, J. E. (1974). Increases in risk and in risk aversion. *Journal of Economic Theory*, *8*, 337–360.

Dixit, A. (1986). Comparative statics for oligopoly. *International Economic Review*, *27*, 107–122.

Domar, E. D. & Musgrave, R. A. (1944). Proportional income taxation and risk taking. *The Quaterly Journal of Economics*, *58*, 388–422.

Farrell, J. & Shapiro, C. (1990). Horizontal mergers: An equilibrium analysis. *American Economic Review*, *80*, 107–126.

Faulí-Oller, R. (1997). On merger profitability in a cournot setting. *Economics Letters*, *54*, 75–79.

Faulí-Oller, R. (2002). Assessing the anticompetitive effect of mergers for market power. *International Game Theory Review*, *4*, 449–458.

Feldstein, M. S. (1969). The effects of taxation on risk taking. *Journal of Political Economy*, *77*, 755–764.

Friedman, J. W. (1977). *Oligopoly and the Theory of Games*. North-Holland, Amsterdam.

Gal-Or, E. (1985). Information sharing in oligopoly. *Econometrica*, *53*, 329–343.

Gal-Or, E. (1986). Information transmission cournot and bertrand equilibria. *Review of Economic Studies*, *43*, 85–92.

Gal-Or, E. (1988). The informational advantages or disadvantages of horizontal mergers. *International Economic Review*, *29*, 639–661.

Grimpe, C. (2007). Transaktionswerte deutlich gestiegen. M&a report, Zentrum für Europäische Wirtschaftsforschung.

Hennessy, D. A. (2000). Cournot oligopoly conditions under which any horizontal merger is profitable. *Review of Industrial Organization*, *17*, 277–284.

Hirshleifer, J. & Riley, J. G. (1992). *The Analytics of Uncertainty and Information*. Cambridge University Press.

Ito, T. & Machina, M. J. (1983). The incentive implications of incomplete insurance: The multiplicative case. *Economics Letters*, *13*, 319–323.

Ivaldi, M., Jullien, B., Rey, P., Seabright, P., & Tirole, J. (2003). The economics of tacit collusion. Technical report, DG Competition, European Commission.

Kao, J. L. & Hughes, J. S. (1993). Note on risk aversion and sharing of firm-specific information in duopolies. *The Journal of Industrial Economics*, *41*, 103–112.

Kühn, K.-U. & Vives, X. (1995). Information exchanges among firms and their impact on competition. Technical report, European Commission.

Kirby, A. J. (1988). Trade associations as information exchange mechanisms. *RAND Journal of Economics*, *19*, 138–146.

Laffont, J.-J. (1989). *The Economics of Uncertainty and Information*. MIT Press, Cambridge, USA.

Leahy, D. (2002). The profitability of horizontal mergers: A general model. University College Dublin.

Leland, H. E. (1972). Theory of the firm facing uncertain demand. *The American Economic Review*, *62*, 278–291.

Levin, D. (1990). Horizontal mergers: The 50-percent benchmark. *The American Economic Review*, *80*, 1238–1245.

Monsen, J. R. & Downs, A. (1965). A theory of large managerial firms. *The Journal of Political Economy*, *73*, 221–236.

Perry, M. K. & Porter, R. H. (1985). Oligopoly and the incentive for horizontal merger. *The American Economic Review*, *75*, 219–227.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, *32*, 122–36.

Radner, R. (1962). Team decision problems. *The Annals of Mathematical Statistics*, *33*, 857–881.

Raith, M. (1996). A general model of information sharing in oligopoly. *Journal of Economic Theory*, *71*, 260–288.

Rothschild, M. & Stiglitz, J. E. (1970). Increasing risk i: A definition. *Journal of Economic Theory*, *2*, 225–243.

Salant, S. W., Switzer, S., & Reynolds, R. J. (1983). Losses from horizontal merger: The effects of an exogenous change in industry structure on cournot-nash equilibrium. *The Quarterly Journal of Economics*, *98*, 185–199.

Sandmo, A. (1971). On the theory of the competitive firm under price uncertainty. *American Economic Review*, *61*, 65–73.

Shapiro, C. (1986). Exchange of cost information in oligopoly. *Review of Economic Studies*, *43*, 433–446.

Vives, X. (1984). Duopoly information equilibrium: Cournot and bertrand. *Journal of Economic Theory*, *34*, 71–94.

Vives, X. (1985). On the efficiency of bertrand and cournot equilibria with product differentation. *Journal of Economic Theory*, *36*, 166–175.

Vives, X. (2001). *Oligopoly Pricing old ideas and new tools*. Massachusetts Institute of Technology Press, Cambridge, USA.

Vives, X. (2002). Private information, strategic behavior, and efficiency in cournot markets. *RAND Journal of Economics*, *33*, 361–376.

Waterson, M. (1985). On progressive taxation and risk-taking. *Oxford Economic Papers*, *37*, 510–519.

Zhou, W. (2008). Large is beautiful: Horizontal mergers for better exploitation of production shocks. *The Journal of Industrial Economics*, *56*, 68–93.