Cooperative Customer Data Acquisition and Sharing Among Rivals

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Abstract

We present a duopoly model of price discrimination with two-dimensional consumer heterogeneity to analyze two types of cooperation between competitors involving customer data: joint information acquisition and information sharing. We find that incentives for both cooperation types depend on the willingness to switch brands of consumers. Firms are unlikely to jointly acquire customer data when consumers are mobile between brands. If consumers are less willing to switch brands, firms have incentives to cooperate in collecting data on the transportation cost parameters. Incentives to share information depend on the portfolio of data firms hold. Information sharing arises with both mobile and immobile consumers, and it benefits both firms and consumers in the former case, but reduces consumer welfare in the latter. Competition authorities ought to scrutinize such cooperation agreements on a case-by-case basis devoting special attention to consumer switching behavior.

JEL-Classification: D43; L13; L15; O30.

Keywords: Information sharing, data acquisition, price discrimination.

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1 Introduction

Recent advances in information technologies allow firms to collect, analyze and share detailed information on their customers. The collected information is often used to segment markets based upon consumer attributes such as purchase history, brand loyalty and other personal characteristics. The use of detailed customer databases to price discriminate is of potential interest for competition policy, especially where it involves cooperation between rival firms.

This paper focuses on the effects of two types of cooperation between rivals involving customer data: joint data acquisition and sharing. There are several industries where competitors cooperate in collecting data on customers. Industry associations may provide a forum for developing the rules for such data collection, as observed among others in the airline industry, telecommunications, health care, university education, mail order and banking. Apart from cooperating in acquiring data, the possibility of sharing customer databases among competitors has also been widely discussed in many of these industries. While cooperation between rival firms based on the acquisition and sharing of customer data has initiated a heated debate among consumer privacy advocates, business groups and regulators, theoretical work on the topic is still scarce. We aim to make a contribution to fill this gap. We address the incentives of rival firms to cooperate in customer data acquisition and in sharing parts or all of their customer databases and evaluate the welfare effects of these practices in the context of a model of competitive price discrimination.

We use a duopoly model of price discrimination with two-dimensional heterogeneity of consumer preferences to analyze the incentives of firms to jointly acquire and share customer information. We allow firms to hold two different datasets on consumers, reflecting brand preferences and preference strengths (transportation cost parameters). Firms may obtain data additional to their existing datasets and exchange their data either partially or entirely with the rival. Depending on the data a firm has, it may offer uniform prices to all consumers, target specific consumer groups with its prices or set individual prices.

We are interested in three main questions: First, what type of data will be acquired by firms when they cooperatively decide on data collection? Second, under which conditions is a firm holding a particular dataset willing to provide the competitor with access to it? Third, how does cooperation in data acquisition and sharing affect competition and welfare? Although we are aware that cooperation based on customer data may raise concerns about privacy as well as collusion, we leave these issues aside in this paper. To focus on the competitive effects of joint information acquisition and sharing, we assume that firms use data solely for price discrimination purposes, and do not consider collusion incentives and consumer privacy issues.

Our results highlight the importance of the consumers' willingness to switch brands influencing the incentives of firms to acquire additional data or to engage in information sharing. If a small price decrease can motivate a relatively large share of consumers to switch brands, cooperation between firms (having access to the same customer data) in acquiring additional data is unlikely. However, there is potential for information sharing, if one firm has more information than its rival, which benefits consumers and is increasing welfare. If on the other hand consumers are generally loyal to their firms and price changes induce relatively little switching, cooperation on data acquisition and sharing can be profitable to firms. If such cooperation takes place, it is harmful to consumers.

The main intuition behind our results is as follows. If consumers are mobile, a cooperation aimed at increasing the firms' ability to target specific groups or individuals is more likely to induce competition, which gives little scope for using the data for extracting rents. As we show, information sharing may still be profitable for the firms under particular conditions. The driver of information sharing in this case is allocative efficiency, arising from the even allocation of consumers between firms when information is shared.

However, with immobile consumers, a cooperation aiming at increasing the firms' ability to price discriminate has a very different impact. Since price changes induce little switching, firms can use their customer data to extract rents from consumers, while the competition intensifying effect of additional data is weak. Under these circumstances, consumers are likely to be harmed by such a cooperation between firms.

Competition authorities should therefore scrutinize agreements of rival firms to cooperate in customer data acquisition and sharing on a case-by-case basis. Apart from the question whether intensified information flows may facilitate collusion, a critical question to analyze is whether consumers are mobile enough such that the cooperation may actually result in increased competition. Our motivation is to better understand the incentives of firms to gather and share consumer information in a competitive environment. There is an ongoing debate in several industries on how to regulate the use of customer data by firms. An increasingly important interplay of competition policy and consumer privacy seems to be emerging from these discussions. Examples include telecommunication firms gathering Customer Proprietary Network Information on numbers dialled, timing of calls, and services used. This information allows the targeting of special offers to customers, who are about to switch providers. Airlines exchange detailed data on personal characteristics and travel details of passengers (Passenger Name Records).[1] Technologies exist to enable airlines to make passenger-specific offers.[2] Other examples include the retail industry, where firms join database cooperatives. These are platforms through which catalogues exchange their consumer information with other catalogues. Examples of such platforms are the Abacus Alliance, Experian's Z-24 Catalogue Database as well as I-Behavior, among others. Firms exchange personal identifiers (names, addresses, birth dates, SSNs) of individuals through these coops as well as consumer preference information.

At the same time, the liberalization of many industries where customer data is intensively collected (telecommunication, airlines) creates situations where competing firms differ significantly in the amount of customer data at their disposal, where incumbents often possess much more detailed information on customers than entrants. Our model applies well to such a situation and allows us to look at the incumbent's incentives to share its information with the entrant.

2 Related Literature

Despite the increasing importance of information acquisition and sharing, few theoretical papers have directly addressed these questions.[3] Two exceptions are Liu and Serfes (2006) and Chen et al. (2001), who focus on information sharing among rivals. Liu and Serfes employ a two-period duopoly model with horizontally and vertically differentiated firms. In the first period, firms set uniform prices and collect information on their customers. In the second period, they use the information to make personalized offers. The authors show that information sharing takes place,

[1] Competing airlines usually store their passenger data in the database of a Computerized Reservation System (CRS), such as Amadeus, Galileo, Sabre or Worldspan. The data can also be used for marketing purposes.

[2] Sweney, M. (2009). BA to run ads on boarding passes, The Guardian, accessed 21.3.2010, www.guardian.co.uk/media/2009/aug/14/british-airways-advertising-boarding-passes

[3] The question of sharing of customer data has been adressed in the banking and finance literature, but this strand focuses on customer default risks, whereas we consider data on consumer preferences.

if firms are sufficiently asymmetric in customer bases. In the case of asymmetry, the smaller firm has an incentive to share its customer information with the larger one. We take a different approach to model information exchange. By allowing firms to distinguish between consumer brand preferences and transportation cost parameters, we are able to address the question of partial information sharing, i.e. the exchange of only one type of information. Somewhat in contrast to the results of Liu and Serfes is the paper by Chen et al. (2001), who show that firms engage in the sharing of customer information only when they are not too asymmetric and the level of targetability is low. Liu and Serfes (2006) as well as Chen et al. (2001) argue that it is the market shares of firms, which drives information sharing. By introducing additional heterogeneity in consumer transportation cost parameters, we conclude instead that it is the willingness of consumers to switch brands together with the portfolio of data firms hold, which determine whether information sharing takes place. In fact, we find that information sharing may occur even with firms having perfectly symmetric market shares, depending on the knowledge they have on consumers.

Similar to our analysis, Esteves (2009) considers price discrimination where firms have access to partial information on the brand and product preferences of consumers. The author presents a two-dimensional Hotelling model with consumers located on a unit square, where the axes represent the two dimensions of consumer preferences. With partial information firms can observe a consumer's location only in one of two dimensions and discriminate accordingly. Her main result is that price discrimination increases industry profits, if firms have information about the location of consumers in the less differentiated dimension (and ignore information about the more differentiated one). Our model differs from Esteves' setting because we allow consumers to be heterogenous in their transportation cost parameters and allow firms to hold asymmetric information sets. In contrast to Esteves, we also explicitly address information sharing.

Our paper is also related to the literature on competitive price discrimination. Earlier papers in this strand of literature focused on the question whether competition eliminates price discrimination. Borenstein (1985) presents a spatial model of monopolistic competition and shows that price discrimination prevails in a duopoly environment. He treats consumers as being heterogenous along three dimensions: their reservation prices and brand preferences as well as the strength of the latter. The author relies upon numerical simulation to compare which sorting strategy is more profitable: price discrimination based upon reservation prices or strength of brand preferences. In contrast, we consider the discrimination based upon brand preference and transportation cost parameters and the exchange of these information types. We also analyze cases, where firms differ in their knowledge concerning these.

Thisse and Vives (1988) compare pricing decisions both with symmetric and asymmetric information between firms. They apply a standard Hotelling model where firms may or may not observe the location of each consumer in the market. The authors show that price discrimination tends to intensify competition for each consumer and discriminatory prices are usually lower than uniform prices. A similar view emerges from a model of competitive couponing by Bester and Petrakis (1996) who analyze the sellers' incentives to offer rebates to their customers in two distinct regions. They find that offering rebates to consumers with coupons tends to intensify competition leading to lower prices and profits. Our model differs from the Thisse-Vives setting by introducing additional heterogeneity in the transportation cost parameter. Diverging from Bester-Petrakis, firms in our model may hold more information, which allows for first-degree price discrimination. Moreover, firms may have asymmetric information sets.

In their survey on price discrimination Armstrong (2006) and Stole (2007) provide a useful summary on the competitive effects of price discrimination and use the notion of best-response symmetry and asymmetry originally introduced by Corts (1998). We will rely on this concept in the explanation of our results and recap it in more detail in Section 4.

The rest of the paper is organized as follows. Section 3 presents the model. In Section 4, we investigate the incentives of firms to cooperate in acquiring information on consumers. In Section 5, we turn to the analysis of information sharing. Section 6 concludes.

3 The Model

We present a duopoly pricing game between two firms, A and B, situated at the two ends of a Hotelling line of unit length. Firm A is located at point 0, firm B at point 1. The firms have equal marginal costs which are normalized to zero. Firms may price discriminate if they have the necessary information about consumer preferences, otherwise they set uniform prices.

A consumer's position on the interval, denoted by $x \in [0, 1]$, reflects his brand preference for the ideal product. If buying from a firm which does not provide a consumer with his ideal product he incurs linear transportation costs. Consumers are heterogenous with respect to their transportation cost per unit distance, which we denote by $t \in (\underline{t}, \bar{t}]$. We distinguish between two distributions of transportation cost parameters. Consumers are *mobile* if $\underline{t} = 0$. They are *immobile* if $\underline{t} > 0$ and $\bar{t}/\underline{t} \leq \sqrt{e}$. The values of x and t are assumed to be distributed independently and uniformly. We denote the density function of the transportation cost parameters with $f(t) = 1/(\bar{t} - \underline{t})$. A consumer's utility from buying a product of firm $i \in \{A, B\}$ is given by

$$U_i(p_i, t, x) = v - t\,|x - x_i| - p_i,$$

where v is a basic utility from the product, which is the same across all consumers and x_i is firm i's address with $x_A = 0$ and $x_B = 1$. Consumer preferences can then be fully described by a pair (x, t). A consumer buys from the firm delivering a higher utility and visits firm A if the following condition holds:

$$t(1 - 2x) + p_B > p_A, \tag{1}$$

and firm B otherwise.

Firms set prices $p_i(x, t)$ to maximize their profits,

$$\pi_i = \int_{X_i}\int_{T_i} p_i(x, t)dtdx$$

with X_i and T_i denoting the sets of locations and transportation cost parameters of consumers who buy from firm i. Depending on the information available to the firms, they can engage in price discrimination or set uniform prices. If a firm has information on the consumer locations and transportation cost parameters, it can set individual prices to each consumer. With partial information on either consumer locations or transportation cost parameters, the firms are able to price discriminate across groups of consumers. Before we proceed with the analysis, we clarify the way we think of how firms may hold, acquire and share customer data, and introduce some useful definitions.

3.1 Customer Data and Timing

We refer to the sets X and T as *datasets*. Datasets X and T contain information about the brand preferences and transportation cost parameters of *all* consumers, respectively. We define the union of datasets firm i holds as firm i's *information set* and denote it by I_i. Each firm may either hold information only about transportation cost parameters ($I_i = T$), only about locations ($I_i = X$), complete information about consumer preferences ($I_i = T \cup X$), or no information ($I_i = \varnothing$). We use the term *information scenario* to describe the datasets held by both firms in a pricing game. We refer to the cases, where $I_A = I_B$ as *symmetric information scenarios*. Cases, where $I_A \neq I_B$ will be referred to as *asymmetric information scenarios*. We say a consumer $\{x, t\}$ is on *firm i's turf* if he would choose firm i over firm j for equal prices.

Throughout this paper we assume that firms can acquire and exchange datasets X or T in their entirety, i.e. containing information on brand preferences or transportation cost parameters of *all* consumers, respectively. With this approach to model transactions of customer data we depart from the existing literature, which usually assumes that firms may only hold data on a subset of consumers, usually based on their previous purchases. For example, Liu and Serfes (2007) assume that firms learn only the brand preferences of consumers visiting them in period 1. They can sell this dataset to the competitor and use this information to price discriminate in period 2. Similarly, Chen et al. (2001) assume that firms only know the preferences of their loyal consumers and of those who are extremely price sensible ("switchers"), but not of the loyal consumers of the competitor.

Our approach fits particularly well to two real life situations. First, to data acquisition practices of industry associations acquiring data on all consumers in a market, which then becomes available to each member of the association. Such cooperative data acquisition practices will be the focus of Section 4. Second, our model applies well to the case of newly liberalized markets, where the incumbent may have detailed information on all consumers in the market and entrants either have no customer data at all, or are able to gather partial information on all consumers from external sources. Such information may include addresses and demographic characteristics, but not a detailed purchase history. The incumbent may then consider to share parts or all of its customer dataset with the entrant. This question is dealt with in Section 5. All proofs are left for the Appendix.

Finally, we make the following assumptions regarding price ties and the timing of pricing decisions.

Assumption 1: *In case both firms have the same information sets and offer equal utilities, i.e.*

$$t(1-2x)+p_B=p_A, \tag{2}$$

the consumer chooses the firm closer in the brand preference space (if $x=1/2$, then w.l.o.g. the consumer visits firm A).

Assumption 2: *In symmetric information scenarios firms set prices simultaneously. In asymmetric information scenarios we assume w.l.o.g. that firm A (B) is the firm with more (less) information. The firm with more information moves first and the other firm follows.*

Assumption 3: *In symmetric information scenarios firms use all of their available data to price discriminate.*

Assumption 1 states that in case of a price tie in symmetric information scenarios, consumers behave in the socially optimal manner and choose the nearest firm. Assumption 2 relates to the timing of pricing decisions, and is in line with much of the literature on competitive price discrimination, where firms choose their targeted offers after they have set uniform prices (e.g. Thisse and Vives 1988; Shaffer and Zhang 2000). It corresponds furthermore to the observation that prices can be adjusted slower if they are applied to a larger group of consumers. In particular, it is more difficult to adjust a firm's regular (uniform) price than changing discounts made by coupons and targeted offers. Assumption 3 is in line with Thisse and Vives (1988) and Liu and Serfes (2004), who discuss the question whether firms use all of their data to set prices in a one-dimensional setting and find that it is a dominant strategy for firms to use their ability to price discriminate. Lemma 1 shows that this behavior is optimal in asymmetric information scenarios.

Lemma 1: *In asymmetric information scenarios firms use all of their available data for price discrimination.*

Lemma 1 implies that in only subgame perfect Nash equilibrium in asymmetric information scenarios the firms always use all the data they have, and never throw away or disregard data. Using all the available data is no surprise for the firm with more information (firm A), which moves after observing the competitor's prices. Doing so simply increases the degrees of freedom in its pricing. Perhaps less obviously, it is also the same reason why the firm with less data (firm B) uses all of its knowledge on customers to price discriminate. The only asymmetric information scenarios where firm B has to decide whether or not to use all of its data are those where firm A has full information. We will show that in these cases firm A's strategy is to match its competitors prices wherever it can. Firm B cannot influence this behavior by firm A, hence using all of its data is optimal for firm B as it allows more freedom in tailoring its prices.

We solve for the pure strategy Nash equilibrium of a game with the following timing: In stage 1 firms decide whether or not to engage in cooperative data acquisition (Section 4) or information sharing (Section 5). In stage 2, firms set prices and consumers make purchasing decisions.

4 Cooperative Acquisition of Customer Data

In this section, we analyze the incentives of firms to cooperatively acquire customer information for price discrimination. Joint acquisition of customer data can be observed in a range of industries, including telecommunications, health care, and airlines. Industry associations often provide a forum for designing data acquisition standards for the members. For example, some national medical associations provide uniform software solutions to their members to manage patient medical records, in effect standardizing customer data the doctors acquire.[4] Similarly, the airline industry collects several customer related data in its Passenger Name Records (PNR). The content of these records is standardized by the International Air Transport Association (IATA),

[4] The American Medical Association provides a web platform for this purpose to its members (see "AMA announces more partners in Web portal project," American Medical News, March 16, 2010). Medical associations in Europe have similar initiatives, such as the ELGA project in Austria.

the global industry association.[5] While such cooperative data acquisition aims at increasing the quality of service (and standardization), the collected data can also provide an excellent basis for price discrimination. Airlines, for example, actively disclose their PNR databases to so called Computerized Reservation Systems such as Amadeus and Sabre, which can be used for booking, but also promotional activities. Another example for cooperative data acquisition is the case of U.S. American colleges, where education institutions cooperate in the College Board to jointly collect information on students for awarding institutional aid funds.[6]

In the following, we show that with mobile consumers (i.e. with low transportation cost parameters) and in the case where firms hold the same information on consumers, they do not cooperate on acquiring more data. If consumers are immobile, then firms jointly acquire the dataset with transportation cost parameters, regardless whether they already hold data on brand preferences. Generally, our results indicate that price discrimination may provide sufficient incentives for joint information acquisition. Information on brand preferences is never acquired. Whether cooperative acquisition of information on transportation cost parameters takes place depends on the consumer willingness to switch brands. Although more information on consumers potentially allows firms to extract more rents from consumers, intensified price competition may reduce prices and profits. The competition effect dominates if consumer mobility is relatively high. If consumers are relatively loyal to their brands (i.e. transportation cost parameters are rather high) acquiring data on transportation cost parameters induces little additional competition, but allows better rent extraction through improved targeting.

We focus on information scenarios with firms holding identical datasets and analyze the incentives of firms to cooperatively acquire the same information on consumer preferences. We restrict our attention to symmetric pure strategy Nash equilibria and compare firms' profits under all symmetric information scenarios. Lemma 2 shows that the existence of a symmetric Nash equilibrium in pure strategies depends on $\bar{t}/\underline{t}$, the ratio of the highest to the lowest transportation cost parameters. An equilibrium in pure strategies exists if consumers are not too different regarding their transportation cost parameters and $\bar{t}/\underline{t}$ is small (i.e., $\bar{t}/\underline{t} \leq \sqrt{e}$).

[5]See e.g. IATA, "Security Fact Sheet", Updated: February 2010, retrieved: March 19, 2010; http://www.iata.org/whatwedo/safety_security/security/data-exchange.htm, IATA, "Data Exchange", retrieved: March 19, 2010; http://www.iata.org/pressroom/facts_figures/fact_sheets/security.htm.

[6]The internet platform PROFILE serves this purpose (https://profileonline.collegeboard.com/prf/index.jsp).

Lemma 2. *With firms having no information about consumers* ($I_i = I_j = \varnothing$)

i) no symmetric pure strategy Nash equilibrium exists if $t \in (0, \bar{t}]$,

ii) a sufficient condition for the existence of a unique pure strategies equilibrium is $t \in [\underline{t}, \bar{t}]$ *with* $\underline{t} > 0$ and $\bar{t}/\underline{t} \leq \sqrt{e}$. *In this case both firms' prices equal the harmonic mean of the range of transportation cost parameters.*

Proposition 1 summarizes our insights on cooperative data acquisition incentives and Table 1 presents the profits of the firms in the symmetric information scenarios. To compute the profits in Table 1 we used the values $\underline{t} = 0$ and $\bar{t} = 1$.

Proposition 1. *Firms' incentives to jointly acquire information on consumer preferences depend on the distribution of transportation cost parameters.*

i) If consumers are mobile and firms have partial information on consumers (either $I_i = I_j = X$ *or* $I_i = I_j = T$*), firms have no incentives to jointly acquire further information for price discrimination purposes. Profits across symmetric information scenarios are ranked as* $\Pi_i^{XT|XT} < \Pi_i^{X|X} < \Pi_i^{T|T}$.

ii) If consumers are immobile, firms do not jointly acquire dataset X *and acquire dataset* T. *Profits across information scenarios are ranked as* $\Pi_i^{X|X} < \Pi_i^{XT|XT} < \Pi_i^{\varnothing|\varnothing} < \Pi_i^{T|T}$.

To understand the incentives for obtaining different types of information, it is useful to recall the concepts of *best-response symmetry* and *best-response asymmetry* discussed by Corts (1998). He refers to models, where both firms set higher prices for the same group of consumers as exhibiting best-response symmetry. In contrast, best-response asymmetry exists where one firm sets lower (higher) prices for those consumers who have a higher (lower) willingness to pay for the other firm.

We have best-response symmetry in the scenario, where firms only know transportation cost parameters. All other symmetric information scenarios give rise to best-response asymmetry. When firms only hold information on consumer transportation cost parameters, both set higher prices for those consumers, who are less willing to switch brands (i.e. those with higher values for t) and lower prices to those, who are ready to switch brands. In our case, best-response functions take the form

$$p_i^{T|T}(p_j) = (p_j^{T|T} + t)/2 \text{ for } i, j \in \{A, B\} \text{ and } i \neq j,$$

Table 1: Cooperative Information Acquisition Incentives

Before Data Acquisition					Data Acquired	After Data Acquisition					Cooperation?
I_A	I_B	Π_A	Π_B	Π_A+Π_B		I_A	I_B	Π_A	Π_B	Π_A+Π_B	
Mobile Consumers ($\underline{t}$= 0 and $\bar{t}$ = 1)											
X	X	.14	.14	.28	T	$X \cup T$	$X \cup T$	.13	.13	.25	No
T	T	.25	.25	.50	X	$X \cup T$	$X \cup T$	.13	.13	.25	No
Immobile Consumers ($\underline{t}$ = 1 and $\bar{t} = \sqrt{e}$)											
$\varnothing$	$\varnothing$	.65	.65	1.3	X	X	X	.20	.20	.41	No
$\varnothing$	$\varnothing$	.65	.65	1.3	T	T	T	.66	.66	1.32	Yes
$\varnothing$	$\varnothing$	.65	.65	1.3	$X \cup T$	$X \cup T$	$X \cup T$	.33	.33	.66	No
X	X	.20	.20	.41	T	$X \cup T$	$X \cup T$	.33	.33	.66	Yes
T	T	.66	.66	1.32	X	$X \cup T$	$X \cup T$	.33	.33	.66	No

which both increase in t. In contrast, if firms have information only on brand preferences and consumers are mobile, the best-response functions are

$$p_A^{X|X}(p_B) = \begin{cases} p_B/2 + \bar{t}(1-2x)/2 & \forall x \leq 1/2 \\ p_B/2 - \underline{t}(2x-1)/2 & \forall x > 1/2 \end{cases}$$

$$p_B^{X|X}(p_A) = \begin{cases} p_A/2 - \underline{t}(1-2x)/2 & \forall x \leq 1/2 \\ p_A/2 + \bar{t}(2x-1)/2 & \forall x > 1/2. \end{cases}$$

Clearly, in the latter case both firms set higher prices for consumers who prefer their brand and lower ones for those who like the competitor more. Since different groups of consumers prefer the two brands, the best-response functions imply best-response asymmetry. Formally, $p_A(p_B|x < 1/2) > p_A(p_B|x > 1/2)$ while $p_B(p_A|x < 1/2) < p_B(p_A|x > 1/2)$.

If both types of information are jointly available to both firms then only best-response asymmetry is preserved. The best-response functions in this case are

$$p_A^{XT|XT}(p_B) = \begin{cases} p_B + t(1-2x) & \forall x \leq 1/2 \\ \max\{0, p_B + t(1-2x)\} & \forall x > 1/2 \end{cases}$$

$$p_B^{XT|XT}(p_A) = \begin{cases} \max\{0, p_A - t(1-2x)\} & \forall x \leq 1/2 \\ p_A - t(1-2x) & \forall x > 1/2. \end{cases}$$

It is easily verified that $p_A(p_B|x < 1/2) > p_A(p_B|x > 1/2)$ while $p_B(p_A|x < 1/2) < p_B(p_A|x > 1/2)$, hence the reaction functions imply best-response asymmetry when both types of information are available. Since additional information on consumer brand preferences always induces best-response asymmetry, firms do not want to cooperatively acquire dataset X. If firms initially have no information on consumers and acquire dataset T they switch to best-response symmetry, which increases industry profits.

Our results make clear that the concepts of best-response symmetry and asymmetry do not fully explain incentives to cooperatively acquire customer data. In particular, they do not explain why joint information acquisition may or may not take place if the market exhibits the same best-response property both before and after the acquisition of additional information. This is the case when firms initially have data only on consumer brand preferences and cooperate on gathering data on transportation cost parameters. As mentioned above, both scenarios $I_i = I_j = X$ and $I_i = I_j = X \cup T$ exhibit best-response asymmetry.

Proposition 1 states that it depends on the distribution of the transportation cost parameters whether acquiring dataset T to the data on brand preferences raises industry profits. If consumers do not differ much in terms of the strength of their brand preferences (i.e., $\bar{t}/\underline{t} \leq \sqrt{e}$) acquiring dataset T is profitable. If however consumer mobility is relatively high, then complementing dataset X with T reduces industry profits.

An open question is why firms do not acquire dataset T in addition to their brand preference data with mobile consumers and why they do acquire it, if consumer mobility is low. Here a closer look at the two main effects at work is necessary. First, the *rent extraction effect*: more information on consumers enables firms to better target and segment consumers. Second, the *competition effect* takes account for the change in the strength of price competition between firms. Whether firms have incentives to acquire additional information on consumers depends on the sum of these two effects.

If consumers are immobile, they visit the closest firm in both information scenarios $I_i = I_j = X$ and $I_i = I_j = X \cup T$, as shown in Figure 1. Additional information on transportation cost parameters allows firms to better target consumers. Although with the firms having both datasets X and T each consumer receives individual offers from both firms, since consumers are immobile, the better targeting induces little competition and the rent extraction effect dominates.

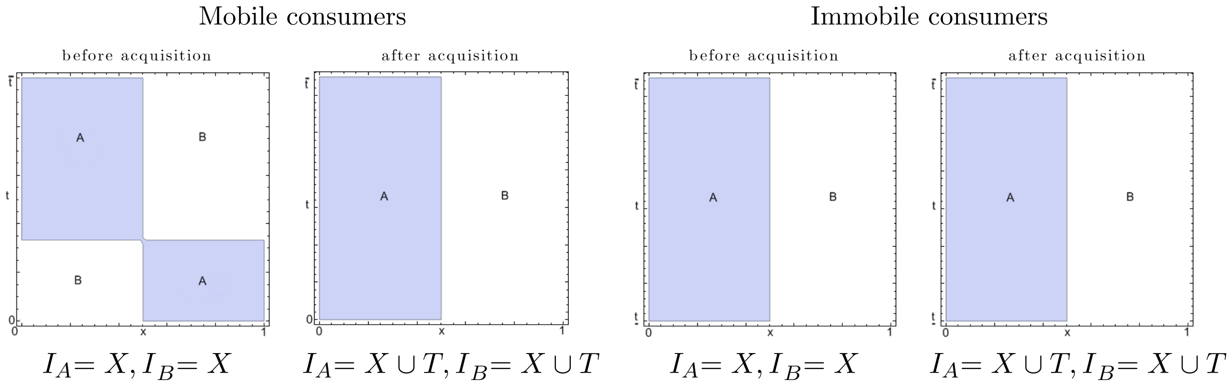
If however consumers are mobile, firms will not complement their existing data on brand preferences with information on consumer transportation cost parameters. Note that pricing strategies and hence equilibrium prices in the scenario where both firms have full information do not depend on the distribution of transportation cost parameters. The reason for the altered incentives to acquire dataset T is that firms' pricing decisions in the information scenario $I_i = I_j = X$ change depending on the mobility of consumers. Let us take a closer look at firms' strategies in this information scenario. Due to the symmetry of firms it is sufficient to focus on the region with $x \leq 1/2$ and analyze competition on firm A's turf.

If consumer mobility is low, for any given price by firm B to a group of consumers with brand preference $x \leq 1/2$, firm A can keep all consumer of this group, without having to significantly decrease its price offered to it. Firm A's optimal strategy is to set a price for a group x, which allows it to attract all members, even those with low transportation cost parameters. The low willingness of consumers to switch brands and firm A's strategy to hold them all in turn induces firm B to price very aggressively on A's turf, and decrease its price till zero, putting a downward pressure on firm A's prices. In the end, firm A holds all consumers on its own turf, but can charge every group x a relatively low price. The same forces are at work on firm B's turf. With industry profits being relatively low, moving into the information scenario with full customer data is attractive.

If consumer mobility is high, it is expensive for firm A to hold all consumers with a given x. To do so, firm A must reduce its prices to prevent consumers with the lowest transportation costs from switching to firm B. It is more profitable for firm A to give up the most mobile consumers, and set a price for every group x which targets the consumers with higher values of t. Firm B is hence able to capture the most mobile consumers on A's turf, even with a relatively high price. In the emerging equilibrium firm A sets prices to every group x on its turf to target consumers with higher transportation cost parameters, while firm B targets those with lower values of t. With industry profits being relatively high in the information scenario $I_i = I_j = X$, firms do not want to acquire data on consumer transportation costs.

Our results make clear that best-response symmetry and asymmetry are not anchored in a particular type of information. The same type of information can exhibit both best-response

Figure 1: Demand Regions with Mobile and Immobile Consumers in X|X and XT|XT



symmetry and asymmetry depending on the additional data firms have. In particular, information on transportation cost parameters may induce different strategies, either best-response symmetry (if only dataset T is available) and best-response asymmetry (if dataset T is combined with dataset X).

Our results extend the analysis in Armstrong (2006), who emphasized that firms have an incentive to acquire information on their consumers, if they can discriminate consumers according to their transportation cost parameters. As we show, this is not always the case. It holds that industry profits are higher, if firms can only discriminate based on T compared to the case when they do not hold any consumer data. However, depending on the distribution of transportation cost parameters, industry profits may either increase or decrease when firms have access to both sets of information compared to the case when firms can only discriminate based on X.

It is useful to inspect the change of consumer surplus and social welfare across information scenarios. The next proposition summarizes our results.

Proposition 2. *The ranking of consumer surplus (CS) and social welfare (SW) in symmetric information scenarios with simultaneous pricing decisions depends on the distribution of the transportation cost parameters.*

i) If $\underline{t} = 0$, *then consumer surplus and social welfare are ranked as* $CS^{T|T} < CS^{X|X} < CS^{XT|XT}$ *and* $W^{X|X} < W^{T|T} = W^{XT|XT}$.

ii) If $\underline{t} > 0$ *and* $\bar{t}/\underline{t} \leq \sqrt{e}$, *then consumer surplus is ranked as* $CS^{T|T} < CS^{\varnothing|\varnothing} < CS^{XT|XT} < CS^{X|X}$ *and social welfare is same in all the symmetric information scenarios.*

Two effects determine the ranking of consumer surplus along information scenarios: first, a competition effect capturing consumer payments. Second, there is also an allocative effect arising from the distribution of consumers between firms. Allocative efficiency requires that consumers choose the nearest firm. The only case where allocative efficiency is distorted is the scenario where firms have access only to information on brand preferences and consumers are mobile: consumers with the lowest transportation cost parameters ($t < \bar{t}/3$) then visit the firm further away, giving rise to allocative inefficiencies. When allocative efficiency is preserved, the ranking of consumer surplus is the opposite of the ranking of industry profits.

We conclude, that price discrimination may provide sufficient incentives for firms to cooperatively acquire information on consumer transportation costs. With mobile consumers firms do not acquire additional data if they already hold some, although doing so would be socially beneficial. With immobile consumers firms cooperate to acquire data on consumer transportation cost parameters, regardless what data they already have. This is neutral to social welfare and decreases consumer surplus.

5 Sharing of Customer Data

In this section, we analyze the incentives of firms to share their datasets with the competitor. We introduce an initial stage into our game in which the firm with more data considers selling dataset X or T or both together to the competitor. Modelling information sharing this way allows us to investigate how the exchange of different types of data influences pricing decisions. We furthermore assume that datasets are verifiable, and hence exclude the possibility to deliver false information on consumer preferences to the competitor.

For the analysis of information sharing incentives only the asymmetric information scenarios are relevant. As stated in Assumption 2, in asymmetric information scenarios the firm with less data moves first and the other firm follows.

By deriving equilibria in a broad range of asymmetric information scenarios, we are able to analyze the incentives of firms to exchange information with each other. Our main question is under which conditions a firm possessing a particular dataset is willing to provide the competitor with access to it. Either information on brand preferences and/or on transportation cost parameters of *all* consumers can be given to the rival. Thus, information exchange is partial, if a firm has access to both datasets, but shares only one of them with its competitor. Gains from trading customer information arise when the joint profits of the firms increase.

We again distinguish between the cases of mobile and immobile consumers. Proposition 3 summarizes our results on information sharing and the profits of the firms are stated in Table 2. For this table we used the values $\underline{t} = 1$ and $\bar{t} = \sqrt{e}$.

Proposition 3. *Incentives to share information depend on the distribution of consumer transportation cost parameters and the portfolio of data both firms hold.*

i) With mobile consumers (i.e. $\underline{t} = 0$) a firm with full information on consumers shares its private data on transportation cost parameters with the competitor if the latter holds data on customer brand preferences.

ii) If consumers are immobile (i.e. $\underline{t} > 0$ and $\bar{t}/\underline{t} \leq \sqrt{e}$), then data on consumer transportation cost parameters is shared in two cases: first, if one firm has full information on consumers while the other holds data on customer brand preferences. Second, if one firm has full information on consumers while the other has no data.

Table 2: Joint Profits and Incentives for Information Sharing

Before Data Sharing					Data Shared	After Data Sharing					Sharing?
I_A	I_B	Π_A	Π_B	Π_A+Π_B		I_A	I_B	Π_A	Π_B	Π_A+Π_B	
Mobile Consumers ($\bar{t} = 1$)											
X	$\varnothing$	.28	.12	.40	X	X	X	.14	.14	.28	No
T	$\varnothing$	.47	.23	.70	T	T	T	.25	.25	.50	No
$X \cup T$	$\varnothing$	.32	.05	.37	X	$X \cup T$	X	.16	.06	.22	No
$X \cup T$	$\varnothing$	.32	.05	.37	T	$X \cup T$	T	.28	.06	.34	No
$X \cup T$	X	.16	.06	.22	T	$X \cup T$	$X \cup T$	.13	.13	.25	Yes
$X \cup T$	T	.28	.06	.34	X	$X \cup T$	$X \cup T$	.13	.13	.25	No
Immobile Consumers ($\underline{t} = 1$ and $\bar{t} = \sqrt{e}$)											
X	$\varnothing$	.47	.19	.66	X	X	X	.20	.20	.41	No
T	$\varnothing$	1.02	.73	1.75	T	T	T	.66	.66	1.32	No
$X \cup T$	$\varnothing$	.74	.16	.90	X	$X \cup T$	X	.33	.21	.54	No
$X \cup T$	$\varnothing$	.74	.16	.90	T	$X \cup T$	T	.74	.17	.91	Yes
$X \cup T$	X	.33	.21	.54	T	$X \cup T$	$X \cup T$	.33	.33	.66	Yes
$X \cup T$	T	.74	.17	.91	X	$X \cup T$	$X \cup T$	.33	.33	.66	No

Conventional explanation for the incentives of firms to share information is whether doing so induces best-response symmetry in the market (Armstrong 2006). Our results confirm that firms generally do not wish to engage in information sharing, if this leads to a shift from best-response symmetry to asymmetry. However, such a shift rarely occurs in our analysis. It is more common, that the market exhibits best-response asymmetry both before and after (potential) information sharing. For instance, data on consumer brand preferences is never shared in our model. The reason for this is that dataset X induces best-response asymmetry (and hence stronger competition) if both firms have it. This offsets any benefits arising from the possibility to better target consumers. However, we show that although dataset X is never shared, it plays a decisive role for the incentives of firms whether to share the dataset on transportation cost parameters of consumers. We call this interplay between the datasets X and T the *portfolio effect*. With this label we refer to the observation that the incentives to share a particular dataset depend on what other data both firms already hold. The same dataset may or may not be shared with the competitor depending on what additional data firms already hold. In particular, the necessary condition for sharing dataset T is that the firm with more information also holds dataset X. If one firm has data only on consumer transportation cost parameters (while the other has no data at all), information sharing does not take place.

Another important driver of information sharing is the timing of the pricing decisions. Our model does not predict information sharing if one firm only has data on consumer transportation cost parameters while the other has no data at all. This is in contrast to the result in Armstrong (2006), who shows that transportation cost data will be shared in a similar scenario assuming simultaneous pricing decisions. Also in our setting, it is easy to check that in the same situation with simultaneous pricing decisions the firm possessing the dataset on transportation cost parameters shares it with its competitor even without transfers.

Our results imply that best-response symmetry is not a sufficient condition for information sharing. Regardless of the timing of pricing decisions, if both firms know only the consumer transportation cost parameters, pricing decisions correspond to best-response symmetry. Nevertheless, with sequential moves firms do not engage in information sharing. The reason for this is that with sequential moves equilibrium prices are relatively high without information sharing compared to prices with simultaneous moves, as shown in Table 2. With sequential moves, the firm setting a uniform price (firm B) takes into account the strategic effect of its price on the price of the competitor, which creates additional incentives to price high. If due to information sharing firms move to the scenario, where both know (only) consumer transportation cost parameters, firm A's best-response function remains unchanged, but firm B's pricing changes: it price discriminates by setting higher (lower) prices for consumers with higher (lower) transportation cost parameter values. Both firms gain by being able to price high for slack consumers (with higher transportation cost parameters) but lose on having to set lower prices to those who are eager to switch brands and have lower transportation cost parameters. With sequential moves information sharing induces higher prices for all consumers with $t > 0.57\bar{t}$ and lower prices for all other consumers. With simultaneous moves firms are able to set higher prices for a larger set of consumers (due to lower prices without information sharing), especially for those with $t > 0.45\bar{t}$. The benefits of information exchange are larger with simultaneous pricing decisions (see Table 3).

Table 3: Pricing Decisions with Mobile Consumers

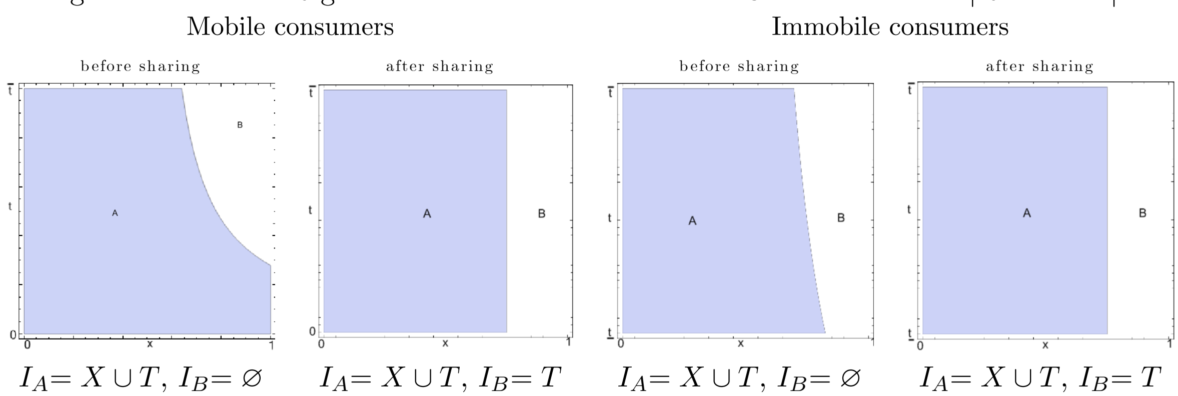
Pricing Decisions	$I_A = T$ and $I_B = \varnothing$		$I_A = T$ and $I_B = T$	
Simultaneous	$p_A^* = (t + p_B^*)/2$	$\Pi_A^{T\|\varnothing} \approx 0.24\bar{t}$	$p_A^* = t$	$\Pi_A^{T\|\varnothing} = 0.25\bar{t}$
	$p_B^* \approx 0.45\bar{t}$	$\Pi_B^{T\|\varnothing} \approx 0.19\bar{t}$	$p_B^* = t$	$\Pi_B^{T\|\varnothing} = 0.25\bar{t}$
Sequential	$p_A^* = (t + p_B^*)/2$	$\Pi_A^{T\|\varnothing} \approx 0.47\bar{t}$	$p_A^* = 5t/4$	$\Pi_A^{T\|\varnothing} \approx 0,30\bar{t}$
	$p_B^* \approx 0.85\bar{t}$	$\Pi_B^{T\|\varnothing} \approx 0.23\bar{t}$	$p_B^* = 3t/2$	$\Pi_B^{T\|\varnothing} \approx 0.28\bar{t}$

Our results highlight the importance of consumer transportation cost parameters. Figure 2 presents the demand regions with mobile and immobile consumers for the information scenarios $XT|\varnothing$ and $XT|T$. With mobile consumers a firm with full information does not share its dataset T with the competitor who does not hold any data, while in the same scenario with immobile consumers this data will be shared, even without monetary transfers. The fact that incentives for sharing dataset T differ in the scenario with $I_i = X \cup T$ and $I_j = \varnothing$ with mobile and immobile consumers originate from the differences in pricing strategies of the firm with less information (firm B) before potential information sharing. In the scenario after information sharing (i.e. with $I_i = X \cup T$ and $I_j = T$) regardless of the distribution of transportation cost parameters, firm B sets $p_B = t/2$ and firm A matches this price to leave consumers indifferent whenever it can do so with a non-negative price. Firm A pursues the same strategy in the information scenario before potential information sharing with $I_i = X \cup T$ and $I_j = \varnothing$: it matches the price of the competitor and leaves consumers indifferent whenever it can with a non-negative price.

The strategy of Firm B, however, depends on the level of consumer mobility. Having to set a uniform price, firm B trades off attracting more mobile consumers at the expense of pricing lower. If all consumers are mobile, firm B solves this trade-off by tailoring its price to target only its most loyal consumers (i.e. those who are close to it and have high transportation costs). This relatively high price serves as basis for firm A as well, resulting in high overall industry profits. In contrast, with immobile consumers (given firm A's strategy) it is optimal for firm B to set a uniform price which allows it to attract some of the consumers even with the lowest transportation costs, close to firm B. To do so, firm B must decrease its price to avoid being undercut by firm A, resulting in a relatively low uniform price by firm B. As firm A bases its prices on firm B's uniform price, all prices in the market are relatively low.

What changes, if firm B gets database T? By being able to identify groups of consumers with the same transportation cost parameters, firm B will set lower (higher) prices to those with lower (higher) values of t. With mobile consumers, firm B's uniform price is targeted at consumers with higher values of t. In this case the improved ability to price discriminate allows firm B to increase the price only for a few consumers (with nearly maximal values of transportation cost parameters), while it reduces the price for all consumers with lower t values. As firm A acts similarly, the additional information generally leads to a price decrease in the market.

Figure 2: Demand Regions with Mobile and Immobile Consumers in XT| 0 and XT|T



With immobile consumers the price of firm B is aimed to appeal even to consumers with low values of t. With additional data on transportation cost parameters firm B can increase the price for most consumers, which drives up firm A's prices as well. Hence, with immobile consumers both firms profit from sharing dataset T. Finally, we turn to the welfare implications of customer information sharing. Proposition 4 summarizes our insights.

Proposition 4. *Welfare implications of customer data sharing depend on the distribution of the transportations cost parameters among consumers:*

i) With mobile consumers (i.e. $\underline{t} = 0$*) information sharing is neutral for consumer surplus and enhances social welfare.*

ii) With immobile consumers (i.e. $\underline{t} > 0$ *and* $\overline{t}/\underline{t} < \sqrt{e}$*) information sharing always decreases consumer surplus and social welfare either decreases or does not change.*

Proposition 4 highlights the importance of the distribution of consumer transportation cost parameters for the welfare effects of information sharing. When consumers are mobile, information sharing is Pareto-optimal: it increases consumer surplus and industry profits. With immobile consumers, however, information sharing harms consumers and is at best neutral to social welfare. In our setup social welfare can only decrease due to the misallocation of consumers, i.e. when consumer do not visit their closest firm.

When consumers are mobile and a firm with full information shares its dataset T with the rival holding dataset X, social welfare increases because it leads to a more efficient allocation of consumers among the firms. In the resulting equilibrium all consumers are served by their most preferred firm. Consumers on firm B's turf with high transportation costs lose because firm B will use its knowledge on extracting higher rents from them. However, consumers on firm B's turf with low transportation cost parameters gain because they are served by their preferred firm. In our setting, these two effects cancel each other out, which renders information sharing neutral for consumer surplus.

When consumers are immobile between brands, information sharing takes place in two cases: a firm with full information shares its dataset T with the rival either holding dataset X or no information. In the former case, sharing customer data does not affect social welfare, since consumers choose the closest firm both before and after the transaction. Information sharing here leads solely to a redistribution of rents from consumers to firms, due to the improved targeting ability.

If consumers are immobile and the firm with full information shares dataset T with the rival, who initially holds no data, social welfare decreases. This result is driven by the increased misallocation of consumers between firms: Some consumers with high values of t (which previously visited their most preferred firm, B) now choose firm A. This negative effect is not compensated by the improved allocation of some consumers with low values of t, which previously visited their less preferred firm, A. Since industry profits increase due to data sharing, consumer surplus declines.

6 Conclusions

We have presented a duopoly model of price discrimination between horizontally differentiated firms possessing different sets of information on consumer preferences. Of particular interest to us are two kinds of agreements among rivals with respect to customer data: cooperative information acquisition and information sharing.

We provide a novel approach to model cooperation with regard to customer data. In our model, we distinguish between two datasets firms may acquire and share: the brand preferences and transportation cost parameters of *all* consumers. A firm holding both datasets can decide to share only one or both.

With mobile consumers, firms will not cooperate to acquire customer data, if they already hold any of the two datasets. When consumers are immobile, firms cooperate to obtain dataset on transportation cost parameters regardless of whether they possess data on brand preferences. In this case, information acquisition is neutral to social welfare and reduces consumer surplus.

Incentives to share information depend on the portfolio of data the firms hold and the distribution of consumers with respect to transportation cost parameters. Information sharing may arise with both mobile and immobile consumers, and it benefits both firms as well as consumers in the first case, but reduces consumer welfare in the later. Competition authorities ought to scrutinize such agreements on a case-by-case basis and devote special attention to consumer preferences with respect to the firms.

Appendix

Definitions and Notation. Before we proceed with the proofs we introduce some useful definitions and notation. Let $t^c(p_A, p_B, x)$ denote the transportation cost parameters of consumers with brand preference x who are indifferent between firms A and B for given prices p_A and p_B, i.e., $U_A(p_A, t^c, x) = U_B(p_B, t^c, x)$: $t^c(p_A, p_B, x) = (p_B - p_A)/(2x - 1)$. Since firms' equilibrium strategies may differ on the intervals $x \leq 1/2$ and $x > 1/2$, it is useful to distinguish between $\underline{t}^c := t^c(x \leq 1/2)$ and $\overline{t}^c := t^c(x > 1/2)$.

Similarly, let $x^c(p_A, p_B, t)$ denote the brand preference of consumers with transportation cost parameter t indifferent between firms A and B for given prices p_A and p_B, i.e., $U_A(p_A, t, x^c) = U_B(p_B, t, x^c)$: $x^c(p_A, p_B, t) = 1/2 - (p_A - p_B)/2t$. Note that $x^c(p_A, p_B, t)$ is the inverse function of $t^c(p_A, p_B, x)$. The notations $\underline{x}$ and $\overline{x}$ stand for the brand preferences of the indifferent consumers for given prices p_A and p_B with the lowest and highest transportation cost parameters, respectively, i.e., $t^c(p_A, p_B, \underline{x}) = \underline{t}$ and $t^c(p_A, p_B, \overline{x}) = \overline{t}$.

We will use the notations $A(\underline{t}, \overline{t}) := (\overline{t} + \underline{t})/2$ and $H(\underline{t}, \overline{t}) = (\overline{t} - \underline{t})/\ln(\overline{t}/\underline{t})$ to denote the arithmetic and the harmonic mean of the transportation cost parameters $t \in (\underline{t}, \overline{t})$ when $\underline{t} > 0$, respectively. It is also useful to introduce parameter $k := \overline{t}/\underline{t}$, provided that $\underline{t} > 0$.

We will omit the notation of information scenarios for best-response functions and equilibrium prices, which should be clear from the context.

Proof of Lemma 1. It is easy to see that in every asymmetric information scenario firm A always finds it optimal to use all of its data. Since it moves after firm B it cannot influence firm B's pricing strategy. Using all of its data to price discriminate simply allows firm A greater freedom to customize its prices. In the scenarios with $\{I_A, I_B\} = \{X \cup T, X\}$ and $\{I_A, I_B\} = \{X \cup T, T\}$ firm B can also choose between uniform and discriminatory (group) prices. By comparing $\Pi_B^{XT|\varnothing}$ with $\Pi_B^{XT|X}$ and $\Pi_B^{XT|\varnothing}$ with $\Pi_B^{XT|T}$ in the proof of Proposition 3 we find that firm B is better off using its available data.

Proof of Lemma 2. We first prove part *i)* of Lemma 2. We show first that a small deviation downwards from the competitor's price is always profitable. If firms set equal prices, they capture half of the consumers and realize profits $\Pi_i^{\varnothing|\varnothing}(p_j, p_j) = p_j/2$. If $p_i < p_j$, firm i captures all consumers on its own turf and some consumers with low transportation cost parameters on the competitor's turf. Without loss of generality we focus on the case where

$p_A < p_B$. Solving $t^c = \bar{t}$ for x we get $\overline{x} = (p_B - p_A)/(2\bar{t}) + 1/2$. Firm A's profits if it sets a price lower than its competitor are $\Pi_A|_{p_A<p_B} = \int_0^{\overline{x}}\int_0^{\bar{t}} f(t)p_A dtdx + \int_{\overline{x}}^{1}\int_0^{t^c} f(t)p_A dtdx = p_A\left[(p_B - p_A)/\left(2\bar{t}\right) + 1/2 - (p_B - p_A)\ln\left((p_B - p_A)/\bar{t}\right)/2\right]$. It is helpful to introduce $\Delta = p_B - p_A$ with $\Delta \in (0, p_B]$ as the magnitude of firm A's downward deviation from firm B's price. Comparing profits with and without deviation from $p_B > 0$ we get that deviation is not profitable if $p_B < \Delta + \bar{t}/\left[1 - \bar{t}\ln(\Delta/\bar{t})\right]$ for any $\Delta \in (0, \bar{t}]$. We now show that there is no such price p_B, which fulfills the latter condition. Note that the RHS of this condition is increasing in Δ, hence, it is fulfilled for any $\Delta \in (0, \bar{t}]$ if and only if it holds for the lowest possible value of Δ. As $\lim_{\Delta \longrightarrow 0}[\Delta + \bar{t}/[1 - \bar{t}\ln(\Delta/\bar{t})]] = 0$ the condition is always violated.

It remains to consider whether $p_A = p_B = 0$ constitutes an equilibrium. This is clearly not the case, since these prices yield zero profits to both firms. With a minimal deviation upward a firm could attract the nearest consumers with the highest transportation cost parameters and make a positive profit. This completes the proof of part *i)* in Lemma 2.

We now turn to the proof of part *ii)*. Assume now that $\underline{t} > 0$ and $k \leq \sqrt{e}$. Since firms are symmetric assume without loss of generality that $p_A \leq p_B$. In the following we will distinguish between the cases: $\underline{x} \leq 1$ (or, equivalently $p_B \leq p_A + \underline{t}$) and $\underline{x} > 1$ (or, equivalently $p_B > p_A + \underline{t}$). Consider first the interval $p_A \leq p_B \leq p_A + \underline{t}$. Firms's profits are then $\Pi_A^{\varnothing|\varnothing}\Big|_{p_A \leq p_B} = \int_0^{\overline{x}}\int_{\underline{t}}^{\bar{t}} f(t)p_A dtdx + \int_{\overline{x}}^{\underline{x}}\int_{\underline{t}}^{t^c} f(t)p_A dtdx$ and $\Pi_B^{\varnothing|\varnothing}\Big|_{p_A \leq p_B} = \int_{\overline{x}}^{\underline{x}}\int_{t^c}^{\bar{t}} f(t)p_B dtdx + \int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}} f(t)p_B dtdx$. Maximization yields the reaction function $p_i(p_j) = \left[p_j + (\bar{t} - \underline{t})/\ln\left(\bar{t}/\underline{t}\right)\right]/2$ with $i, j = \{A, B\}$ and $i \neq j$. These reaction functions give prices $p_A^* = p_B^* = p^* = H(\underline{t}, \bar{t})$. The corresponding profits are $\Pi_i^{\varnothing|\varnothing} = \left(\bar{t} - \underline{t}\right)/\left[2\ln(\bar{t}/\underline{t})\right]$ for $i = \{A, B\}$. Note that the equilibrium prices satisfy the condition $p_A \leq p_B \leq p_A + \underline{t}$. We can conclude then that if firm A sets the price $p_A = p^*$ by choosing the same price firm B gets the highest profit on the interval $p^* \leq p_B \leq p^* + \underline{t}$: $\Pi_B^{\varnothing|\varnothing}(p^*, p^*)$.

We also need to show that given $p_A = p^*$ firm B does not have an incentive to deviate by choosing a price on the interval $p^* + \underline{t} < p_B \leq p^* + \bar{t}$, where $p_B < p_A + \bar{t}$ guarantees that firm B makes positive profits. We denote firm B's price on the interval $p^* + \underline{t} < p_B < p^* + \bar{t}$ as $p_B = p^* + d$, with $\underline{t} < d < \bar{t}$ and firm B's profit as $\Pi_B^{\varnothing|\varnothing}(p^*, p^* + d)$. Comparing firm B's profits we get $\Pi_B^{\varnothing|\varnothing}(p^*, p^* + d) - \Pi_B^{\varnothing|\varnothing}(p^*, p^*) = \left[d\left[\bar{t} - d + d\ln(d/\bar{t})\right]/(\bar{t} - \underline{t}) + \left[\underline{t} - d + d\ln(d/\bar{t})\right]/\ln(\bar{t}/\underline{t})\right]/2$. Taking the derivative of the RHS of the latter equality w.r.t. d we get $\partial(\Pi_B^{\varnothing|\varnothing}(p^*, p^* + d) - \Pi_B^{\varnothing|\varnothing}(p^*, p^*))/\partial d = 2A(d)$, where $A(d) := \left[\bar{t} - d + 2d\ln(d/\bar{t})\right]/(\bar{t} - \underline{t}) + \ln(d/\bar{t})/\ln(\bar{t}/\underline{t})$. Taking the derivative of $A(d)$ w.r.t. d yields $\partial A(d)/\partial d = \left[\bar{t} - \underline{t} + d\ln(\bar{t}/\underline{t}) + 2d\ln(\bar{t}/\underline{t})\ln(d/\bar{t})\right] / \left[d\left(\bar{t} - \underline{t}\right)\ln(\bar{t}/\underline{t})\right]$. Note that the denominator of $\partial A(d)/\partial d$ is always positive and we denote the numerator as $B(d)$. Taking the derivative of the numerator we get $\partial B(d)/\partial d = \left[2\ln(d/\bar{t}) + 3\right]\ln(\bar{t}/\underline{t})$, which increases in d. Moreover, provided that $\bar{t}/\underline{t} < e^{\frac{3}{2}}$ it holds that $\partial B(d)/\partial d|_{d=\underline{t}} > 0$. Hence, under the condition $\bar{t}/\underline{t} < e^{\frac{3}{2}}$ it is true that $\partial B(d)/\partial d > 0$ for $\underline{t} < d < \bar{t}$. This in turn implies that $\partial A(d)/\partial d$ increases on $\underline{t} < d < \bar{t}$. Note that provided $\bar{t}/\underline{t} < e^{\frac{1}{2}}$ it holds that $B(\underline{t}) > 0$ since the term $d\ln(\bar{t}/\underline{t}) + 2d\ln(\bar{t}/\underline{t})\ln(d/\bar{t})$ is then positive. Hence, for $\underline{t} < d < \bar{t}$ both the numerator and denominator of $\partial A(d)/\partial d$ are positive and we have that $\partial A(d)/\partial d > 0$, which implies that $A(d)$ is an increasing function. Note also that $A(\bar{t}) = 0$, hence, $A(d) < 0$ for $\underline{t} < d < \bar{t}$ and the difference $\Pi_B^{\varnothing|\varnothing}(p^*, p^* + d) - \Pi_B^{\varnothing|\varnothing}(p^*, p^*)$ decreases on the interval $\underline{t} < d < \bar{t}$. Moreover, if $d = \underline{t}$, then $\Pi_B^{\varnothing|\varnothing}(p^*, p^* + d) - \Pi_B^{\varnothing|\varnothing}(p^*, p^*) = (-1/2)(\underline{t})^2\ln(\bar{t}/\underline{t})/(\bar{t} - \underline{t}) < 0$. It follows then that for $\underline{t} < d < \bar{t}$ the difference $\Pi_B^{\varnothing|\varnothing}(p^*, p^* + d) - \Pi_B^{\varnothing|\varnothing}(p^*, p^*)$ is negative. Note finally that $\bar{t}/\underline{t} < e^{\frac{1}{2}}$ is a stricter condition than $\bar{t}/\underline{t} < e^{\frac{3}{2}}$. This completes the proof of part *ii)* in Lemma 2. *Q.E.D.*

Proof of Proposition 1. We first find equilibria in every information scenario and then compare the profits of firms in the different information scenarios.

Claim 1. Let $\underline{t} = 0$. Assume that $I_i = X$ with $i = \{A, B\}$. In equilibrium firm i sets the price $p_i^*(x) = 2\bar{t}\,|1 - 2x|/3$ on its own turf and $p_i^*(x) = \bar{t}\,|1 - 2x|/3$ on the competitor's turf. Firm i serves all consumers with $t \geq \bar{t}/3$ on its own turf and all consumers with $t < \bar{t}/3$ on the competitor's turf and realizes the profit $\Pi_i^{X|X} = 5A(\underline{t}, \bar{t})/18$.

Proof of Claim 1. Since firms are symmetric, we will only consider the region $x \leq 1/2$. A consumer in this region chooses firm A if $t \geq t^c$. Both firms treat the consumer transportation cost parameters as a random variable and maximize their expected profits for a given value of x: $E\left[\Pi_A^{X|X}|x \leq 1/2\right] = p_A \Pr\{t \geq t^c\} = f(t)p_A\left[\bar{t} - (p_A - p_B)/(1 - 2x)\right]$ and $E\left[\Pi_B^{X|X}|x \leq 1/2\right] = p_B \Pr\{t < t^c\} = f(t)p_B(p_A - p_B)/(1 - 2x)$. Solving the corresponding maximization problems yields equilibrium prices $p_A^*(x) = 2\bar{t}(1-2x)/3$ and $p_B^*(x) = \bar{t}(1-2x)/3$ for $x \leq 1/2$, which give $\underline{t}^c = \bar{t}/3$. To compute firm A's equilibrium profit we sum up the revenues across the demand regions: $\Pi_A^{X|X} = \int_0^{1/2}\int_{\underline{t}^c}^{\bar{t}}\left[2\bar{t}(1 - 2x)/3\right]f(t)dtdx + \int_0^{1/2}\int_0^{\bar{t}_c}\left[\bar{t}(2x - 1)/3\right]f(t)dtdx = 5A(\underline{t}, \bar{t})/18$. Since firms are symmetric, $\Pi_B^{X|X} = \Pi_A^{X|X}$. This completes the proof of Claim 1.

Claim 2. Let $\underline{t} > 0$ and $k < \sqrt{e}$. Assume that $I_i = X$ with $i = \{A, B\}$. In equilibrium firm i sets the price $p_i^*(x) = \bar{t}\,|1-2x|/2$ on its own turf and the price $p_i^*(x) = 0$ on the competitor's turf. Every firm serves all consumers on its own turf and realizes the profit $\Pi_i^{X|X} = \bar{t}/8$.

Proof of Claim 2. Since firms are symmetric, we will only consider $x \leq 1/2$. A consumer in this region chooses firm A if $t \geq t^c$. Both firms treat consumer transportation costs as a random variable and maximize their expected profits for a given value of x: $E\left[\Pi_A^{X|X}|x \leq 1/2\right] = p_A \Pr\{t \geq t^c\} = f(t)p_A\left[\bar{t} - (p_A - p_B)/(1-2x)\right]$ and $E\left[\Pi_B^{X|X}|x \leq 1/2\right] = p_B \Pr\{t < t^c\} = f(t)p_B\left[(p_A - p_B)/(1-2x) - \underline{t}\right]$. Solving the corresponding maximization problems yields equilibrium prices $p_A^*(x) = \bar{t}(1-2x)/2$ and $p_B^*(x) = 0$ for $x \leq 1/2$, which give $\underline{t}^c = \bar{t}/2$ such that $\bar{t}/2 < \underline{t}$. Firms' equilibrium profits are: $\Pi_A^{X|X} = \int\limits_0^{1/2}\int\limits_{\underline{t}}^{\bar{t}}\left[\bar{t}(1-2x)/2\right]f(t)dtdx = \bar{t}/8$. Since firms are symmetric, $\Pi_B^{X|X} = \Pi_A^{X|X}$. This completes the proof of Claim 2.

Claim 3. Let $\underline{t} = 0$ or $\underline{t} > 0$ and $k < \sqrt{e}$. Assume that $I_i = T$ with $i = \{A, B\}$. In equilibrium firm i sets the price $p_i^*(t) = t$, serves all consumers on its own turf and realizes the profit $\Pi_i^{T|T} = A(\underline{t}, \bar{t})/2$.

Proof of Claim 3. Both firms treat consumer brand preference as a random variable and maximize their expected profits: $E\left[\Pi_A^{T|T}|t\right] = f(t)p_A \Pr\{x < x^c\} = f(t)p_A\left[1/2 + (p_B - p_A)/2t\right]$ and $E\left[\Pi_B^{T|T}|t\right] = f(t)p_B \Pr\{x \geq x^c\} = f(t)p_B\left[1/2 - (p_B - p_A)/2t\right]$. Maximization of the expected profits with respect to the corresponding prices yields $p_A^*(t) = p_B^*(t) = t$, which give $x^c = 1/2$. Firm A realizes the profit $\Pi_A^{T|T} = \int\limits_0^{x^c}\int\limits_{\underline{t}}^{\bar{t}} tf(t)dtdx = A(\underline{t}, \bar{t})/2$. It holds that $\Pi_A^{T|T} = \Pi_B^{T|T}$. This completes the proof of Claim 3.

Claim 4. Let $\underline{t} = 0$ or $\underline{t} > 0$ and $k < \sqrt{e}$. Assume that $I_i = X \cup T$ with $i = \{A, B\}$. In equilibrium firm i sets the price $p_i^*(x,t) = t\,|1-2x|$ on its own turf and $p_i^*(x,t) = 0$ on the competitor's turf, serves all consumers on its own turf and realizes the profit $\Pi_i^{XT|XT} = A(\underline{t}, \bar{t})/4$.

Proof of Claim 4. Since firms are symmetric, we will only consider firms' pricing decisions in the region $x \in [0, 1/2]$. Firm A has a cost advantage in this region, hence its best-response to any price of firm B is to render consumers indifferent by setting $p_A(p_B) = p_B + t(1-2x)$. Firm B's best-response is to undercut firm A's price by setting $p_B(p_A) = p_A - t(1-2x) - \varepsilon$ whenever it is feasible (i.e., $p_A - t(1-2x) > 0$). Otherwise, firm B sets $p_B = 0$. As undercutting is not possible in the equilibrium, we get $p_B^*(x,t) = 0$ and $p_A^*(x,t) = t(1-2x)$. Firm A's profit is $\Pi_A^{XT|XT} = \int_0^{1/2}\int_{\underline{t}}^{\bar{t}} t(1-2x)f(t)dtdx = A(\underline{t},\bar{t})/4$. Due to the symmetry. it holds that $\Pi_A^{XT|XT} = \Pi_B^{XT|XT}$. This completes the proof of Claim 4.

With $\underline{t}=0$ the comparison of firms' profits in the different information scenarios is straightforward and yields $\Pi_i^{XT|XT} < \Pi_i^{X|X} < \Pi_i^{T|T}$. Consider now $\underline{t}>0$ and $k<\sqrt{e}$. It is straightforward that $\Pi_i^{X|X} < \Pi_i^{XT|XT}$. By substituting in k we get $\Pi_i^{\varnothing|\varnothing} - \Pi_i^{XT|XT} = 4(k-1)/(k+1) - \ln k$. The RHS of the latter equality increases in k on the interval $1<k<\sqrt{e}$ and approaches zero if $k \to 1$, hence, $\Pi_i^{XT|XT} < \Pi_i^{\varnothing|\varnothing}$. Since $\Pi_i^{T|T} = A(\underline{t},\bar{t})/2$ and $\Pi_i^{\varnothing|\varnothing} = H(\underline{t},\bar{t})/2$ and it holds that $A(\underline{t},\bar{t}) > H(\underline{t},\bar{t})$, it follows that $\Pi_i^{\varnothing|\varnothing} < \Pi_i^{T|T}$. These comparisons yield the ranking $\Pi_i^{X|X} < \Pi_i^{XT|XT} < \Pi_i^{\varnothing|\varnothing} < \Pi_i^{T|T}$. *Q.E.D.*

Proof of Proposition 2. Consider first the case $\underline{t}=0$. We use the demand regions and prices as stated in the proof of Proposition 1 to find the consumer surplus. Since in the information scenarios $I_A = I_B = T \cup X$ and $I_A = I_B = T$ every firm serves only its own turf, we use the formula $\int_0^{1/2}\int_0^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_0^{\bar{t}} U_B(x,t)f(t)dtdx$ to compute $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$ and $CS^{T|T} = v - 5A(\underline{t},\bar{t})/4$. We also get $CS^{X|X} = \int_0^{1/2}\int_{\underline{t}^c}^{\bar{t}} U_A(x,t)f(t)dtdx + \int_{1/2}^{1}\int_0^{\bar{t}^c} U_A(x,t)f(t)dtdx + \int_0^{1/2}\int_0^{\underline{t}^c} U_B(x,t)f(t)dtdx + \int_{1/2}^{1}\int_{\bar{t}^c}^{\bar{t}} U_B(x,t)f(t)dtdx = v - 31A(\underline{t},\bar{t})/36$. The comparison is straightforward and yields the ranking $CS^{T|T} < CS^{X|X} < CS^{XT|XT}$. Social welfare follows immediately from adding up profits and consumer surplus as $W^{I_A|I_B} = CS^{I_A|I_B} + \Pi_A^{I_A|I_B} + \Pi_B^{I_A|I_B}$, from where we get $W^{XT|XT} = W^{T|T} = v - A(\underline{t},\bar{t})/4$ and $W^{X|X} = v - 11A(\underline{t},\bar{t})/36$. The comparison is straightforward and yields the ranking $W^{X|X} < W^{XT|XT} = W^{T|T}$.

Consider now $\underline{t}>0$ and $k<\sqrt{e}$. Note that in all the symmetric information scenarios firms share the market equally, hence, social welfare is always same and is given by $SW^{XT|XT} = SW^{T|T} = SW^{X|X} = SW^{\varnothing|\varnothing} = v - 2\int_0^{1/2}\int_{\underline{t}}^{\bar{t}} txf(t)dtdx = v - A(\underline{t},\bar{t})/4$. We can use the formula $CS^{I_A|I_B} = W^{I_A|I_B} - \Pi_A^{I_A|I_B} - \Pi_B^{I_A|I_B}$ to derive consumer surplus as $CS^{T|T} = v - 5A(\underline{t},\bar{t})/4$, $CS^{\varnothing|\varnothing} = v - H(\underline{t},\bar{t}) - A(\underline{t},\bar{t})/4$, $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$ and $CS^{X|X} = v - (3\bar{t}+\underline{t})/8$. Since social welfare is same in all the symmetric information scenarios, the ranking of consumer surplus follows directly from the ranking of the profits as $CS^{T|T} < CS^{\varnothing|\varnothing} < CS^{XT|XT} < CS^{X|X}$. *Q.E.D.*

Proof of Proposition 3. We first derive equilibria in all the relevant information scenarios to be able to compare firms' individual (joint) profits to see whether they have incentives to share information without (with) transfers.

Claim 1. Let $\underline{t} = 0$ *or* $\underline{t} > 0$ *and* $k < \sqrt{e}$*. Consider the information scenario* $I_A = T \cup X$ and $I_B = T$*. In equilibrium firm A sets the price* $p_A^*(x,t) = \max\{t/2 + t(1-2x), 0\}$ *and serves all consumers with* $x \leq 3/4$*. Firm B sets the price* $p_B^*(t) = t/2$*. Firms realize profits* $\Pi_A^{XT|T} = 9A/16$ *and* $\Pi_B^{XT|T} = A/8$.

Proof of Claim 1. Since firm A has full information, it can undercut firm B as long as it can set a non-negative price, which yields firm A's equilibrium strategy $p_A(p_B) = \max\{p_B + t(1-2x), 0\}$. Undercutting is possible whenever $t(2x-1) \leq p_B(t)$. Firm B treats x as a random variable and maximizes its expected profit given firm A's equilibrium strategy $E\left[\Pi_B^{XT|T}|t\right] = f(t) p_B \Pr\{t(2x-1) > p_B\} = p_B(1-p_B/t)/\left[2(\bar{t}-\underline{t})\right]$. Solving the maximization problem for p_B yields $p_B^*(t) = t/2$, which gives $p_A^* = \max\{t/2 + t(1-2x), 0\}$, such that $t/2 + t(1-2x)$ is non-negative whenever $x \leq x^c = 3/4$. Firms A and B realize profits $\Pi_A^{XT|T} = \int_0^{x^c} \int_{\underline{t}}^{\bar{t}} f(t)\left(t/2 + t(1-2x)\right) dtdx = 9A(\underline{t},\bar{t})/16$ and $\Pi_B^{XT|T} = \int_{x^c}^{1} \int_{\underline{t}}^{\bar{t}} f(t)(t/2) dtdx = A(\underline{t},\bar{t})/8$, respectively. This completes the proof of Claim 1.

Claim 2. Let $\underline{t} = 0$ *or* $\underline{t} > 0$ *and* $k < \sqrt{e}$*. Consider the information scenario* $I_A = T \cup X$ and $I_B = X$*. In equilibrium firm A sets the prices* $p_A^*(x,t) = t(1-2x)$ *for any* $x \leq 1/2$ *and* $p_A^*(x,t) = (2x-1)\max\{0, \bar{t}/2 - t\}$ *for any* $x > 1/2$*. Firm B sets the price* $p_B^*(x) = \max\{\bar{t}(x-1/2), 0\}$ *and serves all consumers with* $x > 1/2$ *and* $t > t^m$*, where* $t^m = \max\{\bar{t}/2, \underline{t}\}$*. Firms realize profits* $\Pi_A^{XT|X} = A(\underline{t},\bar{t})/4 + \left[t^m(\bar{t}-t^m)/(\bar{t}-\underline{t}) - \underline{t}\right]/8$ *and* $\Pi_B^{XT|X} = \left[\bar{t}(\bar{t}-t^m)\right]/\left[8(\bar{t}-\underline{t})\right]$.

Proof of Claim 2. Having full information on consumer preferences firm A can undercut firm B as long as it can set a non-negative price, which is the case if $t(2x-1) \leq p_B(x)$ holds. Firm B treats t as a random variable and maximizes its expected profits given A's equilibrium strategy separately in the regions $x \leq 1/2$ and $x > 1/2$. In the region $x \leq 1/2$ firm A can undercut any price set by firm B, hence, $p_B^*(x) = 0$ for $x \leq 1/2$. Firm B's expected profit in the region $x > 1/2$ is $E\left[\Pi_B^{XT|X}|x > 1/2\right] = p_B \Pr\{t(2x-1) > p_B | x > 1/2\} = p_B\left[\bar{t} - p_B/(2x-1)\right]/(\bar{t}-\underline{t})$. Maximization of this profit yields $p_B^*(x) = \bar{t}\,(x-1/2)$. Hence, in the region $x > 1/2$ firm A can undercut firm B only if $t \leq \bar{t}^c = \bar{t}/2$. Note that $\bar{t}^c > \underline{t}$ if $\underline{t} = 0$ and $\bar{t}^c < \underline{t}$ if $\bar{t}/\underline{t} < \sqrt{e}$. Firms' equilibrium profits are $\Pi_A^{XT|X} = \int_0^{1/2}\int_{\underline{t}}^{\bar{t}} [t(1-2x)]\, f(t)dtdx + \int_{1/2}^{1}\int_{\underline{t}}^{t^m} \left[(2x-1)(\bar{t}/2-t)\right] f(t)dtdx = A(\underline{t},\bar{t})/4 + \left[t^m(\bar{t}-t^m)/(\bar{t}-\underline{t}) - \underline{t}\right]/8$ and $\Pi_B^{XT|X} = \int_{1/2}^{1}\int_{t^m}^{\bar{t}} \left[\bar{t}(x-1/2)\right] f(t)dtdx = \left[\bar{t}(\bar{t}-t^m)\right]/\left[8(\bar{t}-\underline{t})\right]$. This completes the proof of Claim 2.

Claim 3. Assume that $I_A = T \cup X$ *and* $I_B = \varnothing$. *If* $\underline{t} = 0$, *then in equilibrium firms A and B set the prices* $p_A^*(x,t) = \max\{p_B^* + t(1-2x), 0\}$ *and* $p_B^* \approx 0.28\bar{t}$ *and realize profits* $\Pi_A^{XT|\varnothing} \approx 0.32\bar{t}$ *and* $\Pi_B^{XT|\varnothing} \approx 0.05\bar{t}$, *respectively. If* $\underline{t} > 0$ *and* $k < \sqrt{e}$, *then in equilibrium firms set the prices* $p_A^*(x,t) = \max\{H(\underline{t},\bar{t})/2 + t(1-2x), 0\}$ *and* $p_B^* = H(\underline{t},\bar{t})/2$ *and realize profits* $\Pi_A^{XT|\varnothing} = 5H(\underline{t},\bar{t})/16 + A(\underline{t},\bar{t})/4$ *and* $\Pi_B^{XT|\varnothing} = H(\underline{t},\bar{t})/8$, *respectively.*

Proof of Claim 3. Consider first $\underline{t} > 0$ and $k < \sqrt{e}$. Firm A maximizes its profit given the price set by firm B. Firm A's optimal strategy is $p_A(p_B) = \max\{0, t(1-2x) + p_B\}$, which gives $\bar{t}^c = p_B/(2x-1)$ and $\bar{x} = 1/2 + p_B/(2\bar{t})$ and $\underline{x} = 1/2 + p_B/(2\underline{t})$. Depending on the relation of $\underline{x}$ and 1 two cases are possible in the equilibrium: $\underline{x}(p_B^*) < 1$ if $p_B^* < \underline{t}$ and $\underline{x}(p_B^*) > 1$ if $\underline{t} < p_B^* < \bar{t}$. We show first that $\underline{t} < p_B^* < \bar{t}$ cannot characterize firm B's equilibrium price. Assume that $\underline{t} < p_B^* < \bar{t}$. Firm B sets p_B to maximize its profit $\Pi_B^{XT|\varnothing} = \int_{\bar{x}}^{1}\int_{\bar{t}^c}^{\bar{t}} f(t)p_B dtdx$ given firm A's optimal strategy. The optimal price p_B solves the equation $p_B\left[2\ln\left(p_B/\bar{t}\right) - 1\right] + \bar{t} = 0$. There is no analytical solution to this problem, the value $p_B = 0.28\bar{t}$ is however a good numerical approximation, which fulfills the second order condition. Note that $0.28\bar{t} < \underline{t}$, hence, $\underline{t} < p_B^* < \bar{t}$ is not possible in equilibrium. We show next that in equilibrium $p_B^* < \underline{t}$. Assume this is the case. Firm B sets p_B to maximize its profit $\Pi_B^{XT|\varnothing} = \int_{\bar{x}}^{\underline{x}}\int_{\bar{t}^c}^{\bar{t}} f(t)p_B dtdx + \int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}} f(t)p_B dtdx = \left[p_B(\bar{t} - \underline{t} - p_B\ln(\bar{t}/\underline{t}))\right]/\left[2(\bar{t}-\underline{t})\right]$, which yields $p_B^* = H(\underline{t},\bar{t})/2$. Under the constraint $k < \sqrt{e}$ it holds that $H(\underline{t},\bar{t})/2 < \underline{t}$, hence, $p_B^* = H(\underline{t},\bar{t})/2$ is indeed the equilibrium price. It is straightforward to compute firms' profits as $\Pi_A^{XT|\varnothing} = 5H(\underline{t},\bar{t})/16 + A(\underline{t},\bar{t})/4$ and $\Pi_B^{XT|\varnothing} = H(\underline{t},\bar{t})/8$. Consider finally $\underline{t} = 0$, in which case $\Pi_B^{XT|\varnothing} = \int_{\bar{x}}^{1}\int_{\bar{t}^c}^{\bar{t}} f(t)p_B dtdx$ and $p_B^* \approx 0.28\bar{t}$ yielding $\Pi_A^{XT|\varnothing} \approx 0.32\bar{t}$ and $\Pi_B^{XT|\varnothing} \approx 0.05\bar{t}$. This completes the proof of Claim 3.

Claim 4. Assume that $I_A = T$ and $I_B = \varnothing$. *If* $\underline{t} = 0$, *then in equilibrium firms A and B set prices* $p_A^*(t) = (t + p_B^*)/2$ *and* $p_B^* \approx 0.85\bar{t}$ *and realize profits* $\Pi_A^{T|\varnothing} \approx 0.47\bar{t}$ and $\Pi_B^{T|\varnothing} \approx 0.23\bar{t}$, *respectively. If* $\underline{t} > 0$ *and* $k < \sqrt{e}$, *then in equilibrium firms set prices* $p_A^*(t) = (t + p_B^*)/2$ *and* $p_B^* = 3H(\underline{t},\bar{t})/2$ *and realize profits* $\Pi_A^{T|\varnothing} = 21H(\underline{t},\bar{t})/32 + A(\underline{t},\bar{t})/8$ *and* $\Pi_B^{T|\varnothing} = 9H(\underline{t},\bar{t})/16$, *respectively.*

Proof of Claim 4. Assume first that $\underline{t} > 0$ and $k < \sqrt{e}$. Firm A takes p_B as given and maximizes its expected profit $E\left[\Pi_A^{T|\varnothing} |t\right] = f(t)p_A \Pr\{x \leq x^c\} = f(t)p_A(1/2 + (p_B - p_A)/2t)$, which yields firm A's equilibrium strategy as $p_A(p_B) = (t + p_B)/2$. We also get $t^c = p_B/(4x-1)$ and $\overline{x} = 1/4 + p_B/(4\overline{t})$ and $\underline{x} = 1/4 + p_B/(4\underline{t})$. Depending on the relation between $\underline{x}$ and 1 two cases are possible in equilibrium: $\underline{x}(p_B^*) \geq 1$ if $3\underline{t} \leq p_B^* < 3\overline{t}$ and $\underline{x}(p_B^*) < 1$ if $p_B^* < 3\underline{t}$. We show first that the first case does not emerge. Assume that $3\underline{t} < p_B^* < 3\overline{t}$. In equilibrium firm B chooses its price to maximize the profit $\Pi_B^{T|\varnothing} = \int\limits_{\overline{x}}^{1}\int\limits_{t^c}^{\overline{t}} f(t)p_B dtdx$. The optimal price p_B solves the equation $p_B[1+\ln(9)] - 3\overline{t} - 2p_B \ln\left(p_B/\overline{t}\right) = 0$. There is no analytical solution to this problem, the value $p_B = 0.85\overline{t}$ is however a good numerical approximation which fulfills the second order condition. Note that $0.85\overline{t} < 3\underline{t}$, hence, $3\underline{t} \leq p_B^* < 3\overline{t}$ cannot hold in equilibrium. Assume further that p_B^* satisfies $p_B^* < 3\underline{t}$. Firm B maximizes its profit $\Pi_B^{T|\varnothing} = \int\limits_{\overline{x}}^{\underline{x}}\int\limits_{t^c}^{\overline{t}} f(t)p_B dtdx + \int\limits_{\underline{x}}^{1}\int\limits_{\underline{t}}^{\overline{t}} f(t)p_B dtdx$, which yields $p_B^* = 3H(\underline{t},\overline{t})/2$. Under the constraint $k < \sqrt{e}$ it holds that $3H(\underline{t},\overline{t})/2 < 3\underline{t}$, hence, $p_B^* = 3H(\underline{t},\overline{t})/2$ is indeed the equilibrium price. It is straightforward to compute firms' profits as $\Pi_A^{T|\varnothing} = 21H(\underline{t},\overline{t})/32 + A(\underline{t},\overline{t})/8$ and $\Pi_B^{T|\varnothing} = 9H(\underline{t},\overline{t})/16$. Consider finally $\underline{t} = 0$, in which case $\Pi_B^{T|\varnothing} = \int\limits_{\overline{x}}^{1}\int\limits_{\overline{t}^c}^{\overline{t}} f(t)p_B dtdx$ and $p_B^* \approx 0.85\overline{t}$ yielding $\Pi_A^{T|\varnothing} \approx 0.47\overline{t}$ and $\Pi_B^{T|\varnothing} \approx 0.23\overline{t}$. This completes the proof of Claim 4.

Claim 5. Assume that $I_A = X$ and $I_B = \varnothing$. *If* $\underline{t} = 0$, *in equilibrium firms A and B set prices* $p_A^*(x) = (\max[\overline{t}(1-2x), 0] + p_B^*)/2$ *and* $p_B^* \approx 0.465\overline{t}$ *and realize profits* $\Pi_A^{X|\varnothing} \approx 0.28\overline{t}$ *and* $\Pi_B^{X|\varnothing} \approx 0.12\overline{t}$, *respectively. If* $\underline{t} > 0$ and $k < \sqrt{e}$, *then in equilibrium firms set prices* $p_A^*(x) = (\overline{t}(1-2x) + p_B^*)/2$ *if* $x \leq 1/2$, $p_A^*(x) = (\underline{t}(1-2x) + p_B^*)/2$ *if* $x > 1/2$ *and* $p_B^* = (\overline{t}-\underline{t})/\ln((2\overline{t}-\underline{t})/\underline{t})$. *Firms realize profits* $\Pi_A^{X|\varnothing} = \left[(\overline{t}-\underline{t})/\ln((2\overline{t}-\underline{t})/\underline{t})\right](1/8 + 1/\left[4(2\overline{t}-\underline{t})\right]) - \left[(\overline{t}-\underline{t})/\ln((2\overline{t}-\underline{t})/\underline{t})\right]^2/\left[8(2\overline{t}-\underline{t})\right] + \overline{t}/\left[8(2\overline{t}-\underline{t})\right]$ *and* $\Pi_B^{X|\varnothing} = (\overline{t}-\underline{t})/\left[4\ln((2\overline{t}-\underline{t})/\underline{t})\right]$.

Proof of Claim 5. Consider first $\underline{t} > 0$ and $k < \sqrt{e}$. Firm A takes p_B as given and maximizes its expected profits $E\left[\Pi_A^{X|\varnothing} |x \leq 1/2\right] = \Pr\{t \geq t^c\} p_A = \left[\overline{t} - (p_A - p_B)/(1-2x)\right] f(t)p_A$ and $E\left[\Pi_A^{X|\varnothing} |x > 1/2\right] = \Pr\{t < t^c\} p_A = \left[(p_B - p_A)/(2x-1) - \underline{t}\right] f(t)p_A$, which yields firm A's equilibrium strategy as $p_A(p_B) = \left[\overline{t}(1-2x) + p_B\right]/2$ if $x < 1/2$ and $p_A(p_B) = \left[\underline{t}(1-2x) + p_B\right]/2$ if $x > 1/2$. From firm A's equilibrium strategy we derive $\underline{t}^c(x, p_B) = \overline{t}/2 - p_B/[2(1-2x)]$ and $\overline{t}^c(x, p_B) = \underline{t}/2 + p_B/[2(2x-1)]$. Solving $\overline{t}^c(x, p_B) = \overline{t}$ and $\overline{t}^c(x, p_B) = \underline{t}$ we get $\overline{x} =$

$1/2+p_B/\left[2(2\bar{t}-\underline{t})\right]$ and $\underline{x}=1/2+p_B/\bar{t}$. Note that $\underline{t}^c(0,p_B)=(\bar{t}-p_B)/2$ such that $(\bar{t}-p_B)/2<\underline{t}$ for any $p_B\geq 0$ and $k<\sqrt{e}$. Two cases are possible in equilibrium depending on the relation between $\underline{x}(p_B^*)$ and 1: $\underline{x}(p_B^*)\geq 1$ if $\underline{t}\leq p_B^*<2\bar{t}-\underline{t}$ and $\underline{x}(p_B^*)<1$ if $p_B^*<\underline{t}$. We show that the first case does not emerge. Assume that $\underline{t}\leq p_B^*<2\bar{t}-\underline{t}$. Then firm B maximizes its profit $\Pi_B^{X|\varnothing}=\int_{\overline{x}}^{1}\int_{\bar{t}^c}^{\bar{t}}f(t)p_B dtdx$. The optimal price p_B^* solves the equation $(2\bar{t}-\underline{t})-p_B+2p_B\ln(p_B/(2\bar{t}-\underline{t}))=0$. There is no analytical solution to this problem, the value $p_B=0.28(2\bar{t}-\underline{t})$ is however a good numerical approximation which fulfills the second order condition. Note that $0.28(2\bar{t}-\underline{t})<\underline{t}$ if $k<\sqrt{e}$, hence, $\underline{t}\leq p_B^*<2\bar{t}-\underline{t}$ cannot hold in equilibrium. Assume now that $p_B^*<\underline{t}$. In equilibrium firm B maximizes its profit $\Pi_B^{X|\varnothing}=\int_{\overline{x}}^{\underline{x}}\int_{\bar{t}^c}^{\bar{t}}f(t)p_B dtdx+\int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}}f(t)p_B dtdx$, which yields $p_B^*=(\bar{t}-\underline{t})/\ln((2\bar{t}-\underline{t})/\underline{t})$. Under the constraint $k<\sqrt{e}$ it holds that $(\bar{t}-\underline{t})/\ln((2\bar{t}-\underline{t})/\underline{t})<\underline{t}$, hence, $p_B^*=(\bar{t}-\underline{t})/\ln((2\bar{t}-\underline{t})/\underline{t})$ is indeed the equilibrium price. Firms realize profits $\Pi_B^{X|\varnothing}=(\bar{t}-\underline{t})/\left[4\ln((2\bar{t}-\underline{t})/\underline{t})\right]$ and $\Pi_A^{X|\varnothing}=\left[(\bar{t}-\underline{t})/\ln((2\bar{t}-\underline{t})/\underline{t})\right](1/8+1/\left[4(2\bar{t}-\underline{t})\right])-\left[(\bar{t}-\underline{t})/\ln((2\bar{t}-\underline{t})/\underline{t})\right]^2/\left[8(2\bar{t}-\underline{t})\right]+\bar{t}/\left[8(2\bar{t}-\underline{t})\right]$. Consider finally $\underline{t}=0$. Note that $\underline{t}^c(0,p_B^*)=(\bar{t}-p_B^*)/2>0$ if $p_B^*<\bar{t}$ and $\bar{t}^c(1,p_B^*)=p_B^*/2<\bar{t}$ if $p_B^*<2\bar{t}$. Assume that $p_B^*<\bar{t}$. Firm B maximizes its profit $\Pi_B^{X|\varnothing}=\int_0^{\widehat{x}}\int_0^{t^c}f(t)p_B dtdx+\int_{\widetilde{x}}^{1}\int_{\bar{t}^c}^{\bar{t}}f(t)p_B dtdx$, which yields $p_B^*\approx 0.465\bar{t}<\bar{t}$, hence this price constitutes the equilibrium. Firms A and B realize profits $\Pi_A^{X|\varnothing}\approx 0.28\bar{t}$ and $\Pi_B^{X|\varnothing}\approx 0.12\bar{t}$, respectively. This completes the proof of Claim 5.

The comparison of firms' profits leads to the results stated in Proposition 3. *Q.E.D.*

Proof of Proposition 4. Consider first $\underline{t}=0$. Consumer surplus in the information scenario with $I_A=X\cup T$ and $I_B=X$ is $CS^{XT|X}=\int_0^{1/2}\int_0^{\bar{t}}U_A(x,t)f(t)dtdx+\int_{1/2}^{1}\int_0^{\bar{t}/2}U_A(x,t)f(t)dtdx+\int_{1/2}^{1}\int_{\bar{t}/2}^{\bar{t}}U_B(x,t)f(t)dtdx=v-3\bar{t}/8$. As was shown in the proof of Proposition 2, $CS^{XT|XT}=v-3\bar{t}/8$, hence, $CS^{XT|X}=CS^{XT|XT}$. Social welfare follows immediately from adding up firms' profits and consumer surplus such that $W^{XT|X}\approx v-0.155\bar{t}<W^{XT|XT}=v-0.125\bar{t}$.

Consider finally $\underline{t}>0$ and $k<\sqrt{e}$. Consumer surplus in the information scenario with $I_A=X\cup T$ and $I_B=\varnothing$ is $CS^{XT|\varnothing}=\int_0^{\overline{x}}\int_{\underline{t}}^{\bar{t}}U_A(x,t)f(t)dtdx+\int_{\overline{x}}^{\underline{x}}\int_{\underline{t}}^{t^c}U_A(x,t)f(t)dtdx+\int_{\overline{x}}^{\underline{x}}\int_{t^c}^{\bar{t}}U_B(x,t)f(t)dtdx+\int_{\underline{x}}^{1}\int_{\underline{t}}^{\bar{t}}U_B(x,t)f(t)dtdx=v-\left[A(\underline{t},\bar{t})+H(\underline{t},\bar{t})\right]/2$ and social welfare is $SW^{XT|\varnothing}=v-(4A(\underline{t},\bar{t})+H(\underline{t},\bar{t}))/16$. Consumer surplus in the information scenario $I_A=X\cup T$ and $I_B=T$ is

$CS^{XT|T} = \int\limits_0^{x^c}\int\limits_{\underline{t}}^{\bar{t}} U_A(x,t)f(t)dtdx + \int\limits_{x^c}^{1}\int\limits_{\underline{t}}^{\bar{t}} U_B(x,t)f(t)dtdx = v - A(\underline{t},\bar{t})$ and social welfare is $SW^{XT|T} = v - 5A(\underline{t},\bar{t})/16$. Straightforward comparison yields that $CS^{XT|\varnothing} > CS^{XT|T}$ and $SW^{XT|\varnothing} > SW^{XT|T}$.

In the information scenario $I_A = X \cup T$ and $I_B = X$ consumers enjoy $CS^{XT|X} = \int\limits_0^{1/2}\int\limits_{\underline{t}}^{\bar{t}} U_A(x,t)f(t)dtdx + \int\limits_{1/2}^{1}\int\limits_{\underline{t}}^{\bar{t}} U_B(x,t)f(t)dtdx = v - (3\bar{t}+2\underline{t})/8$. We showed in the proof of Proposition 2 that $CS^{XT|XT} = v - 3A(\underline{t},\bar{t})/4$, hence, $CS^{XT|X} > CS^{XT|XT}$. Since in the information scenarios with $I_A = X \cup T$, $I_B = T$ and $I_A = I_B = X \cup T$ every firm serves its own turf, it follows that $SW^{XT|X} = SW^{XT|XT}$. *Q.E.D.*

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