Mobile Termination, Network Externalities, and Consumer Expectations

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February 2012

Abstract

We re-examine the literature on mobile termination in the presence of network externalities. Externalities arise when firms discriminate between on- and off-net calls or when subscription demand is elastic. This literature predicts that profit decreases and consumer surplus increases in termination charge. This is puzzling since in reality regulators are pushing termination rates down while being opposed to do so by network operators. This puzzle is resolved when consumers' expectations are assumed passive but required to be fulfilled in equilibrium (as defined by Katz and Shapiro, AER 1985), instead of being responsive to non-equilibrium prices, as assumed until now.

JEL: D43, K23, L51, L96

Keywords: Networks, Rational Expectations, Access Pricing, Interconnection, Regulation, Telecommunications.

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We thank Luís Cabral, Giulio Federico, Doh-Shin Jeon, Bruno Jullien, and Patrick Rey and the seminar audiences at IAE (Barcelona), University of the Basque Country, University of Groningen, University of Vienna, Toulouse School of Economics for helpful comments, ZEW Mannheim, Universitat de Barcelona, European University Institute and Universidad de Évora. Hurkens gratefully acknowledges financial support through grant ECO2009-12695 from the Spanish Ministry of Science and Innovation and acknowledges the support of the Barcelona Graduate School of Economics and of the Government of Catalunya. López gratefully acknowledges the financial support from the Spanish Ministry of Science and Technology under ECO2008-05155 and from Juan de la Cierva Program.

1 Introduction

This paper analyzes the effects of interconnection agreements, and in particular of termination charges, on competition and welfare in the mobile telephony market when consumers form passive (self-fulfilled) expectations about network sizes. It is widely recognized that in these markets network interconnection agreements play an important role. A large literature on termination charges has been developed during the last decade, both theoretical and empirical. Moreover, the debate on termination charges between regulators and operators has been on-going, especially in the European Union. On the other hand, the role of consumer expectations has gone unnoticed. We show in this paper that the role of consumer expectations is in fact very important, as it affects equilibrium outcomes and policy implications drastically.

Interconnection requires mobile operators to provide a wholesale service (called 'call termination'), whereby each network completes a call made to one of its subscribers by a caller from a different network (typically termed as 'off-net' call). In most countries, call termination is provided in exchange for a fee or termination charge to be paid by the originating operator to the terminating operator. Naturally, the termination charge affects the operators' cost of off-net calls and therefore has an impact on retail prices, competition and efficiency. Moreover, the termination charge affects the revenues accruing from providing termination services. This has then an impact on the competition for market share, which again affects retail prices and welfare.

Consumer expectations are relevant whenever there exist network externalities (also called network effects). Two types of network externalities exist. First, tariff-mediated network externalities arise when termination-based price discrimination is applied (i.e., when mobile operators charge different prices for on-net and off-net calls). In this case, consumers must form expectations about the relative size of *each* network, or in other words, about the proportion of the number of on-net and off-net calls that they will place. Second, direct network externalities arise when subscription demand is assumed elastic. In this case consumers must form expectations about the total number of people that will subscribe to *some* network.

In this paper we show that the way the formation of consumer expectations is modeled is crucial for the relationship between termination charges, profit and welfare. We argue that a better understanding of this relationship can help to explain a theoretical result that has puzzled academics and practitioners in the last years. Namely, theoretical results predict that networks would favor low termination charges[1] and that regulators who try to maximize consumer welfare would favor high termination charges. However, in the real world regulators are typically concerned that termination charges are excessive while firms typically oppose reductions in termination charges proposed by the regulators. We show that this puzzle can be solved by considering passive (self-fulfilling) expectations. Our results thus have implications for the regulation of termination charges. In particular, they provide an analytical support for the directives imposed by the European Commission.

Existing literature shows that when firms compete in two-part tariffs, marginal prices (i.e., on-net and off-net prices) will be set equal to perceived marginal cost, so that equilibrium profits accrue from the collection of fixed fees and from the provision of termination services. In this setting, and assuming subscription demand is inelastic, Laffont, Rey and Tirole (1998b) show that the total profit of firms is strictly decreasing in termination charge. Building on this result, Gans and King (2001) show that firms strictly prefer below cost termination charges.[2] The intuition behind this result is the following: when the termination charge is above cost, off-net calls will be more expensive than on-net calls so that consumers will then prefer to belong to the larger network. As a result, lowering the fixed fee will become a more effective competitive tool to increase market share and price competition is thus intensified. Firms prefer instead to soften competition and this can be attained by setting the termination charge below cost, which comes at the expense of reduced total welfare and consumer surplus.[3] This result also holds when the model is extended in various directions. For example, it holds for any number of networks (Calzada and Valletti, 2008), when call externalities are taken into account (Berger, 2005) and when networks are asymmetric (López and Rey, 2009). Also, Hurkens and Jeon (2009) show that the result holds when subscription demand is elastic.

The real world is very different. Regulators around the world, and especially in the European Union, are concerned about too high termination charges and intervene in the markets of termination. For example, at present the European Commission recommends national regulators to lower termination rates to cost by the end of 2012 (EC 2009*a*). Mobile operators have repeatedly opposed the cuts in termination rates imposed by the national regulatory

[1]At least when competition is in two-part tariffs and networks charge different prices for on-net and off-net calls, which is common practice in the industry.

[2]Seminal models of network competition include Armstrong (1998) and Laffont, Rey and Tirole (1998a,b). For a complete review of the literature on access charges see Armstrong (2002), Vogelsang (2003) and Peitz et al. (2004).

[3]Total welfare would be maximized by a termination charge equal to the cost of termination, whereas consumer surplus would be maximized by a termination charge strictly above the cost of termination.

authorities (NRAs) during the last decade. This provides a clear indication that mobile operators expect a reduction in profits when termination charges are decreased.[4] This opposition seems inconsistent with the argument of some operators that excessive termination charges are irrelevant because these will be returned to consumers in the form of lower retail prices for some mobile services, such as hand-set subsidies.[5] Some operators even warned regulators that reducing termination charges would distort competition and hurt consumers because increased subscription fees would reduce mobile penetration.[6,7] Interestingly, the latter argument shows that firms are aware of the fact that network externalities exist and are relevant.

All in all, this suggests that regulatory policy to date has been based on an incomplete understanding of strategic interaction in mobile markets. We intend to improve this understanding by examining the role of consumer expectations. The literature thus far silently assumes that first firms compete in prices, then consumers form expectations about network sizes (and these thus may depend on the prices chosen by firms) and consumers make optimal subscription decisions, given the prices and their expectations. A strong rationality condition is imposed on expectations. Namely, for *all* prices expectations are required to be self-fulfilling. We will refer to such expectations as responsive. Consumers having responsive expectations means that any change of a price, how tiny it may be, by one firm is assumed to lead to an instantaneous *rational* change in expectations of all consumers, such that, given these changed expectations, optimal subscription decisions will lead realized and expected network sizes to coincide. So a unilateral change in price does not lead only to a change in market shares, but it also leads consumers to accurately predict how market shares will change.

In comparison, we propose to relax the assumption of responsive expectations and to replace it by one of fulfilled equilibrium expectations. This concept was first proposed by Katz and Shapiro (1985). Katz and Shapiro (1985) assume that first consumers form expectations about network sizes, then firms compete (in their Cournot model by setting

[4]T-Mobile made this concern explicit in response to the 2006 public consultation procedure in the UK. See Ofcom, 2006, par 7.12.

[5]See Ofcom, 2006, par. 7.7.

[6]Ofcom, 2007, par. 7.8.

[7]Some NRAs did not believe that a reduction in termination charge would lead to an increase in retail price. Others, on the other hand, accepted the argument that above cost termination charges could be used to subsidize marginal consumers to join a network, increase mobile penetration and thereby internalize the network externality. The UK regulator Ofcom calculated the so called externality surcharge to be positive, but very mild and took it into account when determining the termination charge (Ofcom, 2007). The European Commission, however, recommends against applying a surcharge and aims for termination charges equal to cost (EC 2009*b*, par 5.2.4.)

quantities), and finally consumers make optimal subscription or purchasing decisions, given the expectations. These decisions then lead to actual market shares and network sizes. Katz and Shapiro impose that, *in equilibrium*, realized and expected network sizes are the same. We will refer to such expectations throughout the paper as passive (self-fulfilled) expectations. They are passive as they do not respond to out of equilibrium deviations by firms.

Our first set of findings concerns the case where subscription demand is assumed inelastic and where firms may charge different prices for on- and off-net calls. When expectations are assumed passive results about termination charges in mobile network industries are in line with real world observations. Firms typically prefer above cost termination charges and regulators are justified in their efforts to push termination charges down. In particular, and most importantly, we overturn the Gans and King (2001) result. When firms compete in non-linear prices, firms prefer termination charge above cost so that off-net calls are priced at monopoly prices. Fixed fees and on-net prices are not influenced by the termination charge and thus, in this model, there is no waterbed effect.[8] The complete absence of a waterbed effect depends on the assumption of duopoly. We show that in oligopolies with more than two firms a partial waterbed effect exists. In any case, firms prefer termination charges above cost.[9] Total welfare maximizing termination charges are equal to cost, whereas termination charge below cost is optimal if maximizing consumer surplus is the objective.

It turns out that characterizing equilibrium prices by means of first-order conditions is easier when expectations are assumed passive than when expectations are assumed responsive. This allows us to consider several extensions of the baseline model. First, our main result is shown to be robust to the inclusion of call externalities, as in Berger (2005). If the call externality is modest, firms prefer again above cost termination charges.[10] Second, we re-examine Laffont, Rey and Tirole (1998b) when firms compete in linear prices. We find that on-net price is independent of termination charge, and that off-net price is increasing in termination charge. Consequently, profits are maximized by a termination charge above cost.[11] Third, we consider the case of brand loyalty causing asymmetric networks and show

[8]The waterbed effect refers to the fact that the profit that a customer generates on fixed-to-mobile or mobile-to-mobile termination is (partially) competed away on the retail market. The term *waterbed* was first coined by the late Prof. Paul Geroski during the investigations of the impact of fixed-to-mobile termination charges on competition.

[9]Our results are thus in line with the empirical evidence of the existence of a waterbed effect that is not full, provided by Genakos and Valletti (2011).

[10]If the call externality is very strong, however, firms prefer termination charge below cost in order to reduce connectivity breakdown. This is because in this case, even when termination is charged at cost, off-net call prices would be too high, above the monopoly level.

[11]This result depends again on the duopoly assumption. With more than two firms on-net price is de-

that both networks will prefer above cost termination charges. This happens despite the fact that the smaller firm will compete more aggressively for market share when consumers come with termination profit.

We also consider the possibility that subscription demand is elastic. When there are both direct and tariff-mediated network effects, we find that a termination charge above cost reduces participation, consumer surplus and total welfare. From a social point of view it is thus optimal to set termination below cost, as it helps to internalize the network effect. Although *Bill and Keep* (zero termination charges) is not necessarily optimal, it could perform better than cost-based termination charges. On the other hand, firms prefer termination charge above cost, unless the direct network effect is so strong that firms would actually prefer to increase penetration rather than to increase fixed fees. This means that in most European countries — with effective penetration rates already close to 100 per cent — firms prefer above cost termination charges. If there is only a direct network effect, because firms are not allowed to charge different prices for on- and off-net calls, the result is reversed: a termination charge above cost now increases participation, consumer surplus and total welfare.

The related literature that also aims to reconcile theory with real world practice is limited. Very recently, a few attempts have been made in this direction. Armstrong and Wright (2009)[12], Jullien, Rey and Sand-Zantman (2009)[13], and Hoernig, Inderst and Valletti (2010)[14] have in common that they introduce additional realistic features of the telecommunication industry into the Laffont, Rey and Tirole (1998b) framework and then show that for some parameter range joint profits are maximized at termination charges above cost. Moreover, these papers conclude that the need to regulate termination charges is reduced since the socially optimal termination charge would also be above cost. We present in this paper a rather different solution to the puzzle, and also come to a very different conclusion. First, instead of adding one or more realistic features of telecommunication competition to the Laffont, Rey and Tirole (1998b) framework, we confine our attention to the assumption

creasing in termination charge but joint profits are still maximized by termination charge above cost.

[12]Armstrong and Wright (2009) argue that if mobile-to-mobile (MTM) and fixed-to-mobile (FTM) termination charges must be chosen uniformly, as is in fact the case in most European countries, firms will trade off desirable high FTM and desirable low MTM charges and arrive at some intermediate level, which may well be above cost (this is the case if there is some room for mobile market expansion, and income from fixed lines is sufficiently important).

[13]Jullien, Rey and Sand-Zantman (2009) argue that the willingness to pay for subscription is related to the volume of calls. They consider two types of users: light users and heavy users. The former only receive calls and are assumed to have an elastic subscription demand. There is full participation for the latter, who can place calls and obtain a fixed utility from receiving calls.

[14]Hoernig, Inderst and Valletti (2010) consider the existence of calling clubs so that the calling pattern is not uniform but skewed.

on how consumers form expectations. Second, while we also conclude that firms prefer above-cost termination charges, in contrast to the above papers we find that total welfare is maximized with termination charges at or below cost.

The paper proceeds as follows. In the next section we briefly discuss the different assumptions about consumer expectations. Section 3 introduces the general model with passive expectations. Section 4 deals with the models in which all consumers subscribe to one of the networks. We start in subsection 4.1 with the case in which firms use non-linear prices and distinguish between on- and off-net calls. In subsection 4.2 we extend the model to allow for call externalities. Next, in subsection 4.3 we re-examine Laffont, Rey and Tirole (1998b) where firms compete in linear prices and distinguish between on- and off-net calls. The last part of section 4 is dedicated to the case of two asymmetric networks. Section 5 deals with elastic subscription demand, so that the total number of subscribers is endogenous. Firms compete in non-linear prices. We examine both the case of termination-based price discrimination and the case where firms must set the same price for on- and off-net calls. Section 6 concludes. Proofs for sections 4 and 5 are collected in Appendix A and B, respectively.

2 Passive versus Responsive Expectations

Expectations are important in any model with network effects, not just in the case of telecommunication. Examples include two-sided markets such as newspapers or credit cards. Readers care about the number of adds and advertisers care about the number of readers. Merchants care about the number of users of a particular credit card and users care about the number of merchants accepting a particular credit card. Network effects can also occur in financial markets. The riskiness of a bank may very well depend on its size, that is, the number of depositors. Of course, depositors will care about the riskiness and thus about the number of other people who will deposit in a given bank. (See Matutes and Vives, 1996.) Expectations even play a role in markets without network effects. For example, consider a monopolistic upstream supplier of an input to several downstream firms that compete with each other in a final product market. The prices paid for the inputs determine the marginal costs for the downstream firms. If the prices of inputs are set secretly, each downstream firm needs to form expectations about the prices paid by its competitors in order to know how profitable competition will turn out to be and to determine the demand for inputs. In this context passive beliefs seem very reasonable and widely accepted.[15]

Many papers have been written on markets with network effects and some have modeled consumer expectations as passive and some have modeled them as responsive.[16] Very few papers justify or even discuss the assumption about expectations. Katz and Shapiro (1985) do mention the possibility of responsive beliefs in their Appendix, but in their quantity setting framework results are not altered in an important manner.[17] Lee and Mason (2001) point out that the results change dramatically if responsive beliefs are used instead of passive ones in their pricing game. Matutes and Vives (1996) characterize the equilibria under passive beliefs but do point out that with responsive expectations any pair of deposit rates leading to non-negative profits can be sustained as an equilibrium. Griva and Vettas (2011) analyze price competition in a duopoly where products are both horizontally and vertically differentiated and exhibit positive, product-specific network effects. They do so both for the case where prices do not influence consumer expectations (passive) and for the case where firms can influence expectations through prices (responsive). They point out that competition is more intense under the latter assumption.

In order to illustrate the difference between passive and responsive expectations, and to explain why responsive expectations may intensify competition, let us consider in some detail a duopolistic industry with network effects.[18] Each network is located at one end of the Hotelling interval $[0, 1]$ over which consumers are uniformly distributed. Suppose the value of subscribing to network i equals $v_i(\alpha_i)$, where α_i denotes the size of network i. We will assume here that network effects are positive (i.e., $v_i' > 0$) and that networks compete for consumers in flat fees, denoted by F_1 and F_2. Given these fees, market shares are stable at $(\alpha_0, 1-\alpha_0)$ if the consumer at location α_0 is exactly indifferent between the two networks, that is when

$$v_1(\alpha_0) - t\alpha_0 - F_1 = v_2(1-\alpha_0) - t(1-\alpha_0) - F_2,$$

where $t > 0$ denotes the Hotelling transportation cost. In other words, given fees F_1 and F_2 expectations $(\alpha_0, 1-\alpha_0)$ are fulfilled.

[15] See Hart and Tirole (1990), O'Brien and Shaffer (1992), Fontenay and Gans (2005) and Rey and Tirole (2007). McAfee and Schwartz (1994) also consider so called "wary", non-passive, beliefs.

[16] Examples of the first include Katz and Shapiro (1985), Matutes and Vives (1996), Economides (1996), Lee and Mason (2001) and Malueg and Schwartz (2006). Examples of the latter include Crémer, Rey and Tirole (2000), Caillaud and Jullien (2003) and Galeotti and Moraga-González (2009).

[17] Hermalin and Katz (2011) also consider Cournot competition. They argue that "the Cournot model can be viewed as a means of approximating a dynamic process in which consumer expectations with respect to network sizes change slowly over time because consumers observe network sizes and predict that these sizes will remain stable."

[18] This model is very similar to one introduced in Laffont, Rey, and Tirole (1998a,b) which we will also employ in this paper.

Now let us investigate what happens when suddenly firm 1 lowers its price to $F_1 - \Delta$. How will consumers react?

If consumers take into account only the *direct* pecuniary effect of the lower price, the result will be that some consumers will switch to network 1. The consumers who will switch are those at locations $x \in (\alpha_0, \alpha_1)$, where α_1 is defined by

$$v_1(\alpha_0) - t\alpha_1 - (F_1 - \Delta) = v_2(1 - \alpha_0) - t(1 - \alpha_1) - F_2.$$

That is,

$$\alpha_1 = \alpha_0 + \Delta/(2t).$$

After these switches have occurred, network 1 has increased in size. Since network effects are assumed to be positive, the consumer located at α_1 will no longer be indifferent. In fact, all consumers located at $x \in (\alpha_1, \alpha_2)$ will now switch to network 1, where

$$\alpha_2 = \alpha_1 + [v_1(\alpha_1) - v_1(\alpha_0) + v_2(1 - \alpha_0) - v_2(1 - \alpha_1)]/(2t).$$

Note that the right-hand side does not directly depend on Δ, but only *indirectly* through the changed network sizes. Of course, the story continues as network sizes have changed again. Defining recursively

$$\alpha_{k+1} = \alpha_k + [v_1(\alpha_k) - v_1(\alpha_{k-1}) + v_2(1 - \alpha_{k-1}) - v_2(1 - \alpha_k)]/(2t),$$

one observes that in consecutive steps consumers in (α_k, α_{k+1}) will switch to network 1. In the limit, $\lim_{k\to\infty} \alpha_k = \alpha_\infty$ where

$$v_1(\alpha_\infty) - t\alpha_\infty - (F_1 - \Delta) = v_2(1 - \alpha_\infty) - t(1 - \alpha_\infty) - F_2.$$

The limit market shares $(\alpha_\infty, 1 - \alpha_\infty)$ are equal to the responsive expectations given fees $F_1 - \Delta$ and F_2. In contrast, the market shares $(\alpha_1, 1 - \alpha_1)$ correspond to the (now no longer) fulfilled passive expectations $(\alpha_0, 1 - \alpha_0)$.

One thus concludes that lowering the fee is a more competitive tool for gaining market share when expectations are responsive than when expectations are passive: a decrease of the fee by Δ increases market share of network 1 under responsive expectations by $\alpha_\infty - \alpha_0$, while under passive beliefs, market share is only increased by $\alpha_1 - \alpha_0$. Of course, in case of negative network externalities (*e.g.*, congestion effects) competition is more intense under passive beliefs than under responsive beliefs. Beliefs are passive when consumers only take into account the direct pecuniary effect of a price decrease. Beliefs are responsive when consumers also take into account all the indirect, higher order, effects that a decrease of the fee has on network size, positive or negative.

We see that the type of expectations one assumes, together with the type of externalities that prevail, determine the intensity of competition, and therefore the equilibrium outcome. We argue that it is not obvious which type of expectations is more adequate. Passive expectations may be justified when the price that each user pays is unclear to the rest of users because of, for example, private discounts, which are common in the communications industry. In addition, the theoretical two-stage model in which first firms choose prices and then consumers choose which network to join, is a rather static description of a market which in reality is dynamic. It is widely accepted that initial conditions and path dependence are important in markets with network effects. The most appropriate form to model such markets would inherently involve dynamics in which both consumers and firms can change their decisions over time. Analyzing a truly dynamic model is very complex and definitely beyond the scope of the present paper.[19] The static two-stage model is much more tractable, but we should not forget what it is supposed to represent, when we decide which solution concept to apply and how to interpret the equilibrium outcomes.

If we interpret the two-stage model literally, where both firms and consumers make only one decision (once and for all), then the assumption of responsive expectations is quite pleasing. Indeed, the assumption is equivalent to demanding that consumers play a Nash equilibrium of the subscription/purchasing/deposit game, once prices are set, for any prices. An equilibrium with responsive expectations thus corresponds to a subgame perfect equilibrium of the two-stage game. Instead, assuming passive self-fulfilling expectations will just correspond to a Nash equilibrium of this game. The concept of subgame perfect equilibria is usually preferred as it rules out incredible threats. It should be noted though that subgame perfect equilibrium outcomes are not always more appealing than other Nash equilibrium outcomes.[20] This is related to the fact that learning is very slow and evolutionary pressure

[19] Even within a dynamic model with network effects, consumer expectations can be modeled as myopic or as forward looking. Cabral (2011), Driskill (2007) and Laussel and Resende (2007) consider forward looking consumers. Doganoglu (2003) and Mitchell and Skrzypacz (2006) consider myopic consumers. Radner and Sundararajan (2006) allow for a mixture of forward looking and boundedly rational consumers.

[20] A clear example is the ultimatum game, where the first player proposes a division of a dollar and the second player can only accept or reject. The subgame perfect equilibrium predicts that the proposer will propose to take (almost) all and that the responder will accept. There are Nash equilibria in which a more even split is agreed upon, and such outcome can even be evolutionarily stable. (See Gale, Binmore, and Samuelson, 1995.) Experimental evidence clearly shows very little support for the subgame perfect equilibrium outcome.

very weak in subgames that are not reached (very often). If we account for the fact that our two-stage model is really a short-cut description of a complex dynamic market in which both firms and consumers can change their actions, it is even less obvious that the subgame perfect equilibrium is the most adequate solution concept.

Also, if at least firms would be able to commit to a price, the assumption of responsive expectations would again be appealing. Even though calculating responsive expectations involves solving a (possibly complex) fixed-point problem and consumers are probably not sufficiently sophisticated to solve it, an adaptive or evolutive process, as discussed above at length in our Hotelling example, would likely lead the consumers to converge to a solution in the long run. In fact, such an adaptive process would also solve the coordination problem between consumers that could potentially arise in the case there exist multiple fixed points. However, a potential problem with this is that it could take a very long time and, at least in the telecommunication sector, firms may want to change their prices before the consumers have fully adapted. It may be very hard for firms to commit to a price. For example, in the Spanish market, the recent entrant (Yoigo) offers a contract with free on-net calls. Clearly, it would be rational for all consumers in Spain to switch and make free calls for the rest of their lives. This does not and will not happen. Once the market share of the entrant passes a certain threshold, it will certainly withdraw the offer (or go bankrupt). Notice that both responsive and self-fulfilling passive expectations are rational, so the difference between them does not occur for equilibrium prices (when all consumers have correct expectations in either case) but rather when prices are out of equilibrium. And exactly when prices are out of equilibrium, at least one firm has an incentive to deviate, which makes its price commitment not credible.

3 The Model

We consider competition between two full-coverage networks, 1 and 2, indexed by $i \neq j \in \{1, 2\}$. Each has the same cost structure. The marginal cost of a call equals $c = c_O + c_T$, where c_O and c_T denote the costs borne by the originating and terminating network, respectively. To terminate an off-net call, the originating network must pay a reciprocal and non-negative access charge a to the terminating network. The termination mark-up is equal to

$$m \equiv a - c_T.$$

Therefore, the perceived cost of calls is the true cost c for on-net calls, augmented by the termination mark-up for the off-net calls: $c + a - c_T = c + m$.

Networks (i.e., firms) offer differentiated but substitutable services. The two firms compete for a continuum of consumers of unit mass. Each firm i ($i = 1, 2$) charges a fixed fee F_i and may (or may not) discriminate between on-net and off-net calls. Firm i's marginal on-net price is written p_i and off-net price is written $\hat{p}_i$. Consumer's utility from making calls of length q is given by a concave, increasing and bounded utility function $u(q)$. Demand $q(p)$ is defined by $u'(q(p)) = p$. The indirect utility derived from making calls at price p is $v(p) = u(q(p)) - pq(p)$. Note that $v'(p) = -q(p)$. For given prices p and $\hat{p}$, the profit earned on the on-net calls is

$$R(p) = (p - c)q(p),$$

whereas the profit earned on the off-net calls is

$$\hat{R}(\hat{p}) = (\hat{p} - c - m)q(\hat{p}).$$

We assume that $R(p)$ has a unique maximum at $p = p^M$, is increasing when $p < p^M$, and decreasing when $p > p^M$. That is, p^M denotes the monopoly price. We assume that $R(p^M) > f$, where f is the fixed cost per subscriber. This means that the market is viable. The Ramsey price p^R is defined as the lowest break-even price characterized by

$$R(p^R) = f.$$

We make the standard assumption of a balanced calling pattern, which means that the percentage of calls originating on a given network and completed on another given (including the same) network is equal to the fraction of consumers subscribing to the terminating network. Let α_i denote the market share of network i. The profit of network i is therefore equal to:

$$\pi_i \equiv \alpha_i \left(\alpha_i R(p_i) + \alpha_j \widehat{R}(\widehat{p}_i) + F_i - f \right) + \alpha_i \alpha_j m q(\widehat{p}_j). \tag{1}$$

The first term represents the profit made on consumer services (on-net and off-net calls, fixed fee and cost), whereas the second term represents the profit generated by providing termination services.

We assume that the terms of interconnection are negotiated (or regulated) first. Then, for a given access charge a (or equivalently, a given m) the timing of the game is as follows:

1. Consumers form expectations about the number of subscribers of each network i (β_i) with $\beta_1 \geq 0$, $\beta_2 \geq 0$ and $\beta_1 + \beta_2 \leq 1$. We let $\beta_0 = 1 - \beta_1 - \beta_2$ denote the number of consumers that is expected to remain unsubscribed. In the case of full participation $\beta_0 = 0$ and $\beta_1 + \beta_2 = 1$.

2. Firms take these expectations as given and choose simultaneously retail tariffs $T_i = (F_i, p_i, \widehat{p}_i)$ for $i = 1, 2$.

3. Consumers make rational subscription and consumption decisions, given their expectations and given the networks' tariffs.

Therefore, market share α_i is a function of prices and consumer expectations. Self-fulfilling expectations imply that at equilibrium $\beta_i = \alpha_i$.

4 Full Participation

In this section we assume that the networks are differentiated à la Hotelling. Consumers are uniformly located on the segment $[0, 1]$, whereas the two networks are located at the two ends of this segment ($x_1 = 0$ and $x_2 = 1$). A consumer located at x and joining network i obtains a net utility given by

$$w_i - |x - x_i| / (2\sigma),$$

where $\sigma > 0$ measures the degree of substitutability between the two networks, and w_i is the value to a consumer subscribing to network i (as defined below). We assume full participation so that each consumer subscribes to the network that yields the highest net utility. We will focus our attention on the properties of shared market equilibria, where both firms have strictly positive market shares.[21]

4.1 Non-linear pricing and termination-based price discrimination

In this subsection we assume that firms can set a fixed fee, an on-net price and an off-net price, as in Gans and King (2001). We first characterize the prices in a shared market equilibrium. Afterwards we show that such an equilibrium indeed exists and is unique.

[21] Cornered market equilibria, where one firm dominates the whole market, may exist, but are of little relevance in mature markets.

Given the balanced calling pattern assumption and consumer expectations β_1 and β_2, the surplus from subscribing to network i (gross of transportation costs) equals:

$$w_i = \beta_i v(p_i) + \beta_j v(\hat{p}_i) - F_i.$$

Market share of network i is thus given by $\alpha_i = 1/2 + \sigma(w_i - w_j)$, whenever this is between 0 and 1.

Marginal cost pricing. As usual, at equilibrium with strictly positive market shares, network operators find it optimal to set cost-based usage prices. Adjusting F_i so as to maintain net surpluses w_1 and w_2 and thus market shares constant, leads network i to set p_i and $\widehat{p}_i$ so as to maximize

$$\alpha_i \left(\alpha_i R(p_i) + \alpha_j \hat{R}(\widehat{p}_i) + \beta_i v(p_i) + \beta_j v(\hat{p}_i) - w_i - f \right) + \alpha_i \alpha_j m q(\widehat{p}_j).$$

The first-order conditions are

$$(\alpha_i - \beta_i) q(p_i) + \alpha_i (p_i - c) q'(p_i) = 0 \tag{2}$$

and

$$(\alpha_j - \beta_j) q(\widehat{p}_i) + \alpha_j (\widehat{p}_i - c - m) q'(\widehat{p}_i) = 0. \tag{3}$$

At equilibrium, self-fulfilling expectations ($\beta_i = \alpha_i$) yield perceived marginal cost pricing as long as both firms have positive market share: $p_i = c$ and $\widehat{p}_i = c + m$. Note, however, that out of equilibrium firms do not necessarily want to set usage prices equal to marginal cost.

Market shares. If firms set usage prices equal to marginal cost, and if consumers expect market shares β_1 and β_2, the actual market share, α_i, as a function of expectations and fixed fees F_1 and F_2, is given by

$$\alpha_i(\beta_i, F_i, F_j) = \frac{1}{2} + \sigma (F_j - F_i) + 2\sigma \left(\beta_i - \frac{1}{2} \right) (v(c) - v(c+m)). \tag{4}$$

Equilibrium fixed fees. We now characterize the equilibrium fixed fees. Since in a shared market equilibrium network operators find it optimal to set cost-based usage prices, network i's profit can be written as:

$$\pi_i = \alpha_i (\beta_i, F_i, F_j) [F_i - f + \alpha_j (\beta_j, F_j, F_i) R(c+m)], \tag{5}$$

where $R(c+m) = mq(c+m)$ is the profit, per incoming call, from providing termination services. In equilibrium, each firm i is optimizing given the fixed fee of the other network, F_j, and consumer expectations. Using $\partial\alpha_i/\partial F_i = -\sigma$, we have

$$\frac{d\pi_i}{dF_i} = -\sigma\left[F_i - f + \alpha_j\left(\beta_j, F_j, F_i\right)R(c+m)\right] + \alpha_i\left(\beta_i, F_i, F_j\right)\left[1 + \sigma R(c+m)\right].$$

Note that

$$\frac{d^2\pi}{dF_i^2} = -2\sigma(1 + \sigma R(c+m)).$$

This means that a necessary local second-order condition is that $1 + \sigma R(c+m) > 0$, which we will assume to hold.[22] Solving the first-order condition for F_i we obtain

$$F_i = \frac{f + \frac{1}{2\sigma} + (1 + 2\sigma R(c+m))\left[(2\beta_i - 1)\left(v(c) - v(c+m)\right) + F_j\right]}{2\left(1 + \sigma R(c+m)\right)}. \tag{6}$$

Solving the pair of first-order conditions yields:

$$F_i = f + \frac{1}{2\sigma} + \left(\frac{1 + 2\sigma R(c+m)}{3 + 4\sigma R(c+m)}\right)(2\beta_i - 1)\left(v(c) - v(c+m)\right). \tag{7}$$

Substituting the expressions for F_1 and F_2 into Eq. (4) yields

$$\alpha_i = \frac{1}{2} + 2\sigma\left(\frac{1 + 2\sigma R(c+m)}{3 + 4\sigma R(c+m)} - \frac{1}{2}\right)(1 - 2\beta_i)\left(v(c) - v(c+m)\right). \tag{8}$$

Using the fulfilled expectations condition $\alpha_i = \beta_i$, Eq. (8) reduces to a linear equation in α_i with a unique solution: $\alpha_i = 1/2$. Note that the symmetry of the shared market equilibrium is due to the assumption of a symmetric duopoly.[23] There simply does not exist any asymmetric shared market equilibrium. It follows immediately that at the equilibrium

$$F^* = f + \frac{1}{2\sigma}.$$

Previous literature has suggested that lower access charges will result in higher retail prices for mobile subscribers, which is known as the waterbed effect. When consumer expectations are passive, we have that at the equilibrium the fixed fee is equal to the fixed cost f plus the Hotelling mark-up $1/2\sigma$. That is, the waterbed effect is not at work on the fixed component of the three-part tariff.

[22] This condition holds for all $m \geq 0$ and also for $m < 0$ as long as $\sigma < -1/R(c+m)$.

[23] This assumption is relaxed in Section 4.4.

The analysis above has shown that there is a unique candidate for a shared market equilibrium. To establish the existence of such an equilibrium not only requires the local second-order condition mentioned, but also that the described strategies are in fact global maximizers. In particular, one needs to verify that no firm wants to try to corner the market, given the prices chosen by its competitor and given the expectations of consumers. Note that the firm that corners the market by lowering its fixed fee, will also want to adjust the on- and off-net prices. In particular, that firm will want to set the off-net price at zero.[24] This is not costly to the firm (as no off-net calls will be made in a cornered market) but will fool consumers with passive expectations who believe that half of their calls will be off-net. If the indirect utility of making calls at zero price were unbounded, the firm would be able to corner the market and make unbounded profits. Hence, existence of a shared market equilibrium requires an upper bound on the utility obtained from making calls at zero price, as assumed in section 3. This requirement is mild, as consumers can at most make calls 24 hours a day, but is not met by the constant elasticity demand function. The following proposition establishes the conditions for the existence and uniqueness of the shared market equilibrium.[25]

Proposition 1 *Any shared market equilibrium is symmetric and is characterized by* $p_1 = p_2 = c$, $\widehat{p}_1 = \widehat{p}_2 = c + m$ *and* $F_1 = F_2 = f + \frac{1}{2\sigma}$. *A necessary condition for existence is that* $1 + \sigma R(c+m) > 0$. *A sufficient condition is that* σ *is small enough.*

Comparative statics. The symmetric equilibrium profit is

$$\pi_1 = \pi_2 = \frac{1}{4\sigma} + \frac{1}{4}R(c+m).$$

That is, networks gain the full profit from providing termination services (without competing it away through lower fixed fees). The equilibrium profit is increasing in m when $c+m < p^M$ and decreasing when $c + m > p^M$. Equilibrium profits are maximized for the termination mark-up m^* that maximizes the termination profit:

$$\frac{d\pi_i}{dm} \equiv \frac{1}{4}\frac{dR}{dm}(c+m^*) = 0.$$

We thus have

[24] The firm will also increase the on-net price above cost.

[25] For expositional purposes we prove the existence of equilibrium for this case only. Our subsequent results will focus on the characterization of equilibria only.

Corollary 1 *Under non-linear pricing and termination-based price discrimination, shared-market equilibrium profits are maximized with the termination mark-up m^* that maximizes the termination profit, i.e. $m^* = p^M - c > 0$. Total welfare is maximized at $m^W = 0$.*

Laffont, Rey and Tirole (1998b) show that in the case of responsive expectations, the first-order derivative with respect to the equilibrium profit is:

$$\begin{aligned} \frac{d\pi_i}{dm} &= \frac{1}{4}\left[-q(c+m) + mq'(c+m)\right] \\ &= \frac{1}{4}\frac{dR}{dm}(c+m) - \frac{1}{2}q(c+m). \end{aligned}$$

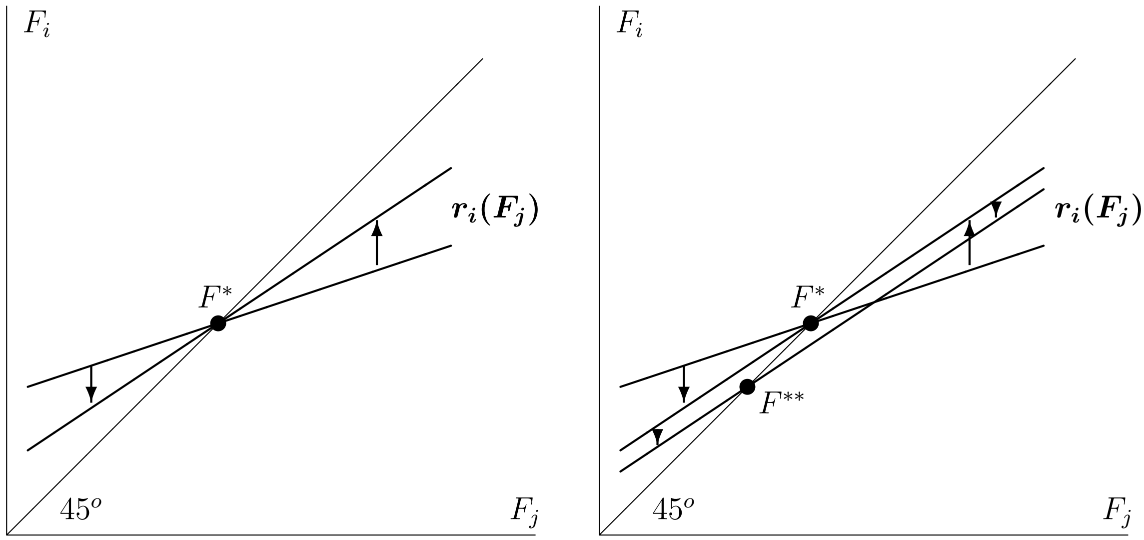
The additional term $-\frac{1}{2}q(c+m)$ is produced by the assumption that consumers change their expectations in response to any variation of prices such that they perfectly foresee realized market shares. As a result,

$$\left.\frac{d\pi_i}{dm}\right|_{m\geq 0} < 0,$$

so that firms prefer below-cost access charges. Gans and King (2001) provides the intuition for this result:[26] When m is positive, off-net calls are more expensive than on-net calls, so that users then wish to belong to the larger network (all else being equal). This makes it easier (or less costly) for firms to gain market share. This leads firms to compete more aggressively for market share (i.e., the reaction functions shift downward) which results in lower fixed fees in equilibrium. Firms prefer thus instead to have a negative termination mark-up.

Sometimes an alternative intuition has been used to explain the negative relation between the termination mark-up and the equilibrium profit. The reasoning goes as follows: When m increases, users become more profitable, in the sense that they bring with them higher termination profits, and this leads firms to compete more aggressively for market share. This is not a completely correct explanation of the impact of m on equilibrium profits in the case of duopoly. Namely, termination profits are only made on calls originated on the rival network. The number of such calls depends on market shares. Firm i terminates $n_i = \alpha_i(1-\alpha_i)$ of such calls. At $\alpha_i = 1/2$, n_i is in fact maximized. This is independent of m. In a symmetric duopoly firms share the market equally in equilibrium. An increase in m will not induce firms to fight more aggressively for consumers because a marginal change in the fixed fee will

[26]Moreover, Gans and King (2001) building on this result show that the optimal termination mark-up from the operators viewpoint is not zero (as Proposition 5 of Laffont, Rey and Tirole, 1998b erroneously concludes) but below cost, so that maximum profits are above (and not bounded above by) the Hotelling level $\frac{1}{4\sigma}$, which is achieved with $m = 0$ (that is, with $a = c_T$).



(a) Passive expectations (b) Responsive expectations

Figure 1: The effect of an increase of m on equilibrium fixed fees in duopoly.

have no impact on n_i. What is true is that, *out of equilibrium*, the level of the termination mark-up influences the optimal fixed fee. For example, if firm i at current prices would have the smaller market share ($\alpha_i < 1/2$ and thus $F_i > F_j$), it would react by lowering its fixed fee when the termination mark-up increases. But at the same time, the rival firm with the higher market share ($\alpha_j > 1/2$) would react by increasing its fixed fee in order to reduce α_j and increase n_j. Hence, the reaction function of each firm rotates around the intersection point with the 45 degree line when m increases, and does not affect the equilibrium fixed fee.

Figure 1 illustrates the above findings. For usage prices fixed at perceived marginal cost, it shows the optimal fixed fee of firm i as a function of the fixed fee of firm j. From Eq. (6) we know this is a linear function with positive slope less than one. The symmetric equilibrium fixed fee is given by the intersection of the reaction function with the 45 degree line. An increase in the termination mark-up leads the smaller (larger) firm to compete more (less) aggressively, rotating the reaction function counterclockwise around the intersection point. (See Figure 1a.) Moreover, in the case of responsive expectations an increase in the termination mark-up shifts the reaction function downward. (See Figure 1b.) This explains why only in this case an increase in the termination mark-up reduces the equilibrium fixed fee (from F^* till F^{**}) and consequently the equilibrium profit.

Oligopolistic competition. The discussion thus far has focussed on duopoly. When expectations are assumed passive, there is no waterbed effect as the equilibrium fixed fee is independent of the termination mark-up. When expectations are responsive, on the other hand, there exists a very strong waterbed effect as the equilibrium fixed fee decreases so fast with m that profits in fact decrease. Nonetheless, if the number of competing networks is larger than two, then there will exist a partial waterbed effect on the fixed component of the three-part tariff even if consumer expectations are passive. To see this, assume that there exist $n \geq 3$ competing networks. The first-order condition is

$$0 = \frac{d\pi_i}{dF_i} = \left[\alpha_i + \frac{d\alpha_i}{dF_i}(F_i - f)\right] + \frac{d\alpha_i}{dF_i}(1 - 2\alpha_i) R(c + m).$$

In a symmetric equilibrium with two networks the second term of the right-hand side of the above equation disappears, thereby m has no impact on the equilibrium fixed fee. Instead, if $n \geq 3$, then at any symmetric equilibrium $\alpha_i = \frac{1}{n} < \frac{1}{2}$, implying that the equilibrium fixed fee will depend on m. Notice that at the symmetric equilibrium

$$F_i = f - \frac{1}{n}\left(\frac{d\alpha_i}{dF_i}\right)^{-1} - \left(1 - \frac{2}{n}\right) R(c + m),$$

thus F_i decreases with m as long as $m < m^*$. The reason is that the number of off-net calls terminated on network i equals $n_i = \alpha_i(1 - \alpha_i)$ which is increasing in α_i at $\alpha_i = 1/n$. As m increases, the profit from terminating calls increases and each firm will compete more fiercely for market share. Yet, the waterbed effect is less than one hundred per cent as the equilibrium profit is still maximized with the termination mark-up that maximizes the termination profit per terminated call:

$$\pi_i = \frac{1}{n}\left[-\frac{1}{n}\left(\frac{d\alpha_i}{dF_i}\right)^{-1} + \frac{1}{n}R(c + m)\right].$$

That is, the equilibrium profit is still maximized with $m^* = p^M - c > 0$. Recall that under passive expectations $\frac{d\alpha_i}{dF_i}$ does not depend on the termination mark-up. Note that total welfare is maximized with termination charges at cost while consumer surplus is maximized with a negative termination mark-up.

Figure 2 explains why there is a partial waterbed effect when expectations are passive and there are at least three firms. It shows the reaction function of firm i against the fixed fee F_j which is assumed to be the same for all firms $j \neq i$. Again, the intersection of this reaction function with the 45 degree line indicates the equilibrium fixed fee. An increase in termination mark-up above 0 leads the reaction function to rotate counterclockwise around the point X, defined as the point where firm i's market share would be 1/2. This is because the firm will fight more fiercely for market share when termination profit per call increases,

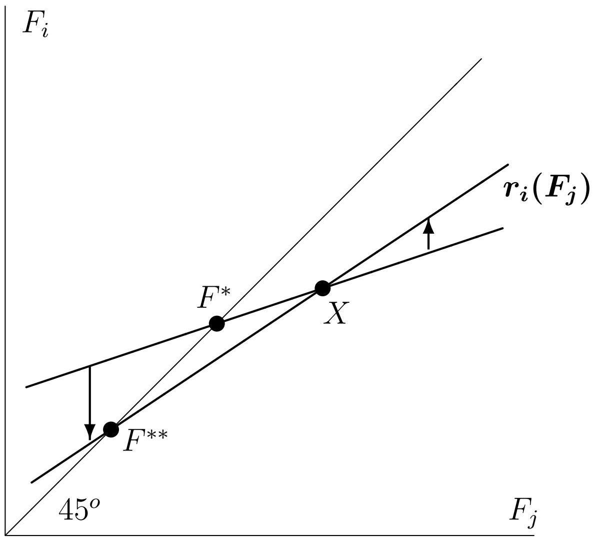


Figure 2: The effect of an increase of m on equilibrium fixed fees in oligopoly (passive expectations).

as long as its market share is less than 1/2. The equilibrium fixed fee thus decreases from F^* till F^{**}.

4.2 Call externalities

In this section we extend the model to consider call externalities, as in Berger (2005). A call externality exists if a consumer derives utility from receiving a call. It seems obvious that call externalities exist, since otherwise nobody would bother to answer a call. How strong such call externalities are is probably an empirical matter. Ofcom (2004) considers that "call externalities probably do not justify any adjustment to call prices. [...] these are likely to be effectively internalized by callers, as a high percentage of calls are from known parties and there are likely to be implicit or explicit agreements to split the origination of calls." On the other hand, Harbord and Pagnozzi (2010) argue that call externalities are strong and that therefore *Bill and Keep* is the appropriate termination charge regime, both from a social and from a private perspective.

We assume that consumers derive utility $\overline{u}(q)$ from receiving calls of volume q, with $\overline{u} = \lambda u$, where $0 < \lambda < 1$ measures the strength of the call externality. If consumers expect market shares β_1 and β_2, then they expect a net surplus

$$w_i = \beta_i \left[v(p_i) + \overline{u}(q(p_i)) \right] + \beta_j [v(\widehat{p}_i) + \overline{u}(q(\widehat{p}_j))] - F_i$$

from subscribing to network i, for $i \neq j \in \{1, 2\}$. The actual market share, α_i, as a function of the consumer expectations and prices, is determined by the indifferent consumer:

$$\begin{aligned}\alpha_i &= \frac{1}{2} + \sigma\beta_i \left[v(p_i) - v(\widehat{p}_j) + \overline{u}(q(p_i)) - \overline{u}(q(\widehat{p}_i))\right] \\ &\quad -\sigma\beta_j \left[v(p_j) - v(\widehat{p}_i) + \overline{u}(q(p_j)) - \overline{u}(q(\widehat{p}_j))\right] + \sigma(F_j - F_i).\end{aligned} \tag{9}$$

Network i's profit is given by Eq. (1). As in Berger (2005) we have

Lemma 1 *In a symmetric equilibrium with $\alpha_1 = \alpha_2 = 1/2$, networks set*

$$p_i = p^* \equiv \frac{c}{1+\lambda} \text{ and } \widehat{p}_i = \widehat{p}^* \equiv \frac{c+m}{1-\lambda}.$$

Therefore, in a symmetric equilibrium $p_i < c$ and $\widehat{p}_i > c + m$, i.e., usage prices do not reflect the perceived cost of calls. Networks find it optimal to internalize the call externality by setting the on-net price below the cost so as to extract the higher consumer surplus through the fixed fee. The off-net price, on the other hand, is set above the cost so as to reduce the utility of rival's customers from receiving calls. If λ tends to 1 (which amounts to say that callers and receivers obtain the same utility from a given call), then the off-net price will tend to $+\infty$, resulting in connectivity breakdown (as shown in Jeon et al. 2004). The first-order condition is

$$\begin{aligned}0 = \frac{d\pi_i}{dF_i} &= -\sigma\left[\alpha_i R(p^*) + \alpha_j \widehat{R}(\widehat{p}^*) + F_i - f\right] + \alpha_i\left[-\sigma R(p^*) + \sigma\widehat{R}(\widehat{p}^*) + 1\right] \\ &\quad +\sigma\left(\alpha_i - \alpha_j\right) m q(\widehat{p}^*),\end{aligned}$$

which defines i's reaction function. Hence, in a symmetric equilibrium ($\alpha_1 = \alpha_2 = 1/2$) the first-order condition is satisfied at $F_i = F^* \equiv f + \frac{1}{2\sigma} - R(p^*)$ or, equivalently,

$$F^* \equiv f + \frac{1}{2\sigma} + \frac{\lambda c}{1+\lambda} q\left(\frac{c}{1+\lambda}\right).$$

The equilibrium profit is thus

$$\pi_1 = \pi_2 = \frac{1}{4\sigma} + \frac{1}{4}\left[R(\widehat{p}^*) - R(p^*)\right].$$

The equilibrium fixed fee is independent of m while equilibrium profits depend on m through the off-net price.

Proposition 2 *Under non-linear pricing, termination-based price discrimination and call* *externalities, symmetric equilibrium profits are maximized with the termination mark-up* m^* *that maximizes the retail profit earned on the off-net calls (gross of termination payments):* $m^* = \arg\max_{m \geq -c_T} R\left(\frac{c+m}{1-\lambda}\right) = \max\{(1-\lambda)\,p^M - c, -c_T\}$*. Hence* $m^* > 0$ *if and only if* $\lambda < \frac{p^M - c}{p^M}$.

In contrast with Jeon et al. (2004) and Berger (2005), the termination mark-up does not affect the fixed fee. The reason is as before: if consumers do not change their expectations with price variations, then $\partial\alpha_i/\partial F_i = -\sigma$, i.e., the fixed fee affects the market share only through the direct pecuniary effect but not via a change in the expectations of consumers. Then, networks maximize shared-market equilibrium profits by setting the termination mark-up m^* that maximizes the retail profit from the off-net calls made by their subscribers. That is, m^* is so that $\widehat{p}^* = \frac{c+m^*}{1-\lambda}$ equals p^M. The equilibrium profits are therefore higher with an above cost access charge than with a below cost access charge when λ is relatively low. When λ is close to 1, there is risk of connectivity breakdown because then $\widehat{p} > p^M$, even if $m = 0$. In this case a below cost access charge brings $\widehat{p}^*$ down towards p^M and increases profits.

The welfare maximizing termination mark-up m^W is such that $\widehat{p}^*$ satisfies the condition $\widehat{p}^* = p = \frac{c}{1+\lambda}$.[27] Therefore, we have

Proposition 3 *In the presence of call externalities, the socially optimal termination mark-up is negative* ($a^W < c_T$) *and given by* $m^W = \max\{-\frac{2\lambda c}{1+\lambda}, -c_T\}$*. Hence* $m^W < 0 < m^*$ *holds when* λ *is relatively low.*

This result is in contrast with Berger (2005), who shows that in the presence of responsive expectations, the best termination charge from the operators' perspective is below cost and smaller than the socially optimal termination charge, that is, $m^* \leq m^W = \max\{-\frac{2\lambda c}{1+\lambda}, -c_T\} < 0$, where the inequality binds when *Bill and Keep* is socially and privately optimal, i.e. when $m^* = m^W = -c_T$. This occurs when externalities are relatively strong. Berger (2005) even argues that regulatory intervention is superfluous in this case since private and social incentives are then perfectly aligned. Our analysis shows that *Bill and Keep* may be optimal from a social point of view, but firms will most likely prefer termination charges above cost. Only if the call externality is extremely high, firms would

[27] As there is full participation and payments are only transfers from one agent to another, what matters is the utility that consumers derive from incoming and outgoing calls, and the cost of these calls. Given that $\overline{\mu} = \lambda\mu$, the socially optimal price maximizes the expression $u(q(p)) + \lambda u(q(p)) - cq(p)$. Hence, this price coincides with the equilibrium on-net price. (Recall that networks set the on-net price to maximize consumer surplus so as to extract it via the fixed fee.)

also prefer *Bill and Keep*. This requires firms setting off-net prices above monopoly price in case termination charge is set at cost. Of course, if the externality is very strong, firms may be tempted to abandon the calling party pays regime and to start charging subscribers for receiving calls.

4.3 Linear pricing and termination-based price discrimination

In this section we analyze competition in linear prices – i.e., networks charge on- and off-net calls but not the fixed fee. Under linear pricing and termination-based price discrimination, and for some given expectations on market shares β_1 and β_2, the variable net surplus offered to network i's customers is

$$w(p_i, \widehat{p}_i) \equiv \beta_i v(p_i) + (1 - \beta_i) v(\widehat{p}_i). \tag{10}$$

Market shares are determined by the indifferent customer:

$$\begin{aligned} \alpha_1 &= \frac{1}{2} + \sigma \left[w(p_1, \widehat{p}_1) - w(p_2, \widehat{p}_2) \right] \\ &= \frac{1}{2} + \sigma \left[\beta_1 \left(v(p_1) - v(\widehat{p}_2) \right) - \beta_2 \left(v(p_2) - v(\widehat{p}_1) \right) \right]. \end{aligned} \tag{11}$$

Differentiating Eq. (1) – where α_i is given by Eq. (11) and $F_i = 0$ – with respect to p_i and $\widehat{p}_i$, we have that at a symmetric equilibrium ($p_1 = p_2 = p$, $\widehat{p}_1 = \widehat{p}_2 = \widehat{p}$, $\alpha_i = \beta_i = 1/2$):

$$[R(p) - f] - \frac{R'(p)}{2\sigma q(p)} = 0, \tag{12}$$

$$[R(p) - f] - \frac{\widehat{R}'(\widehat{p})}{2\sigma q(\widehat{p})} = 0, \tag{13}$$

Let p^D be the equilibrium price in a duopoly model where termination-based price discrimination is not allowed and $m = 0$. From Eq. (12) we have that the equilibrium on-net price p^* equals p^D and is therefore neutral with respect to the access charge. Using Eqs. (12) and (13) we obtain

$$\frac{q(p) + (p - c) q'(p)}{q(p)} = \frac{q(\widehat{p}) + (\widehat{p} - c - m) q'(\widehat{p})}{q(\widehat{p})}. \tag{14}$$

Assuming a constant elasticity demand function[28] ($\eta \equiv -q'(p)(p/q)$), we can rewrite Eq. (14) as follows

$$\frac{\widehat{p}}{p} = \frac{c+m}{c},$$

which coincides with the proportionality rule derived in Laffont, Rey and Tirole (Lemma 1, 1998b). However, here the off-net price is increasing in the termination mark-up: $\frac{d\widehat{p}}{dm} = \frac{p^*}{c} > 0$ (since $\frac{dp^*}{dm} = 0$). It is easily established that $p^R < p^* < p^M$.

In the symmetric equilibrium, i's profit can be written as follows

$$\pi_i = \frac{1}{4}\left[R(p^D) + R(\widehat{p}^*) - 2f\right],$$

where $\widehat{p}^* = p^D\left(\frac{c+m}{c}\right)$. We summarize our results in the next Proposition.

Proposition 4 *Under linear pricing and termination-based price discrimination, in equilibrium the on-net price does not depend on the access charge. Moreover, for a constant elasticity call demand function*

(i) the off-net price increases with the access charge;

(ii) the shared-market equilibrium profits are maximized with the termination mark-up m^* *that maximizes the retail profit earned on the off-net calls* $R(\widehat{p}^*)$*:* $m^* = \left(\frac{p^M}{p^D} - 1\right)c > 0$*;*

(iii) total welfare is maximized by a termination subsidy $m^W < 0$.

In the presence of responsive expectations, and assuming a constant elasticity demand function, Laffont, Rey and Tirole (1998b) obtain that the on-net price decreases with m, and the off-net price increases with m if the degree of substitutability σ is small, though it may decrease with m otherwise. In addition, they obtain that if σ is small, then the access charge that maximizes profits is above the cost ($m > 0$) and higher than the access charge that is socially preferred, which is below the cost ($m < 0$). Nevertheless, they also find that if σ is not that small, then the access charge that maximizes profits may be lower. Indeed, it could be even below cost.[29]

Under passive expectations we obtain different results. Proposition 4 states that the on-net price does not react to the level of the termination mark-up, whereas the off-net price always increases with the termination mark-up (independently of the degree of substitutability between the two networks). Consequently, networks find it profitable to increase

[28]To guarantee existence of equilibrium we need $v(p)$ to be bounded so that we need to cap the demand function by setting $q(p) = \min\{\bar{q}, p^{-\eta}\}$ for some constant $\bar{q}$.

[29]Simulations show that for $\eta = 1.2$, $c_T = 0.5$, $c = 2$, $f = 0$ and $\sigma = 1$, the profit maximizing termination mark-up equals $m^* = -0.10$, whereas the total welfare maximizing termination mark-up is positive.

the access charge above the cost as it exerts upward pressure on the off-net price (towards the monopoly level), which leads to higher profits.

Oligopolistic competition. The neutrality result on the on-net price is specific to the number of competing networks (as the neutrality result on the fixed fee in the case of three-part tariffs analyzed in section 4.1). If the number of competing networks is larger than two, then there will exist a partial waterbed effect on the on-net price even if consumer expectations are passive. As commented earlier, the number of terminated off-net calls increases with a firm's market share as long as it is below 1/2. Since an increase in the termination mark-up increases the profit per terminated call, firms will fight more intensively to gain market share. They do this by lowering the on-net price. Yet, simulations show that the waterbed effect is less than one hundred per cent. In particular, assuming a Logit model with $n \geq 3$ competing networks we obtain that: (i) on-net prices decrease and off-net prices increase with m (in a neighborhood of $m = 0$); (ii) the profit maximizing termination mark-up is positive and increasing in the number of networks; (iii) consumer surplus is maximized with minimum mark-up $m^{CS} = -c_T$ (i.e., with $a = 0$); (iv) total surplus is maximized with a positive termination charge below cost (i.e. $-c_T < m^W < 0$). Table 1 reports optimal termination charges for oligopolies with 2, 3, 4 and 5 firms.[30] More details about prices, profits and welfare can be found in Figure 3.

Termination markup	$n = 2$	$n = 3$	$n = 4$	$n = 5$
m^*	0.414	0.538	0.61	0.66
m^{CS}	-0.50	-0.50	-0.50	-0.50
m^W	-0.293	-0.264	-0.265	-0.274

Table 1: Optimal termination mark-up for firms (m^*), consumer surplus (m^{CS}) and total welfare (m^W). Simulation parameters: $\eta = 2$, $c_T = 0.5$, $c = 1$, $f = 0$, $\mu = 0.25$.

4.4 Asymmetric networks

In this section we analyze the competition between two asymmetric networks, an incumbent and a new entrant. We allow for brand loyalty as in Carter and Wright (1999, 2003) but allow firms to use termination-based price discrimination. The parameter $\gamma > 0$ measures the degree of asymmetry between the networks. The net surplus from subscribing to network

[30] The parameter μ is the degree of product differentiation in a Logit model (see section 5).

1 is

$$w_1 = \gamma/(2\sigma) + \beta_1 v(p_1) + \beta_2 v(\hat{p}_1) - F_1,$$

whereas the net surplus offered to network 2's consumers is

$$w_2 = \beta_2 v(p_2) + \beta_1 v(\hat{p}_2) - F_2.$$

Network i's profit is given by Eq. (1). Thus, as in Section 4.1, in an equilibrium where firms share the market, it is optimal to adopt cost-based usage prices: $p_i = c$ and $\widehat{p_i} = c + m$. The market share of network 1 is thus given by

$$\alpha_1 = 1 - \alpha_2 = \frac{1+\gamma}{2} + \sigma (F_2 - F_1) + 2\sigma \left(\beta_1 - \frac{1}{2}\right)(v(c) - v(c+m)). \tag{15}$$

The first-order condition yields

$$F_i = f + \frac{\alpha_i}{\sigma} + 2\left(\alpha_i - \frac{1}{2}\right) R(c+m). \tag{16}$$

Equilibrium profit of firm i is thus

$$\pi_i = \alpha_i^2 \left(\frac{1}{\sigma} + R(c+m)\right), \tag{17}$$

where α_i is given by Eqs. (15) and (16).

Proposition 5 *In the presence of two asymmetric networks and starting from cost-based termination charges ($m = 0$), in any shared-market equilibrium a small increase in the termination charge:*

(i) raises the fixed fee of the large network and lowers the fixed fee of the small network,

(ii) reduces the difference in market shares between the two networks,

(iii) leads to higher equilibrium profits for both the large and small network,

(iv) reduces total welfare.

López and Rey (2009) analyze competition between two asymmetric networks (an incumbent and a new entrant) in the presence of responsive expectations. They find that in the shared-market equilibrium a below-cost access charge generates higher equilibrium profits (for the large and small network) than any above-cost access charge. Their finding therefore extends the insight of Gans and King to asymmetric networks. In contrast, Proposition 5 states that in the presence of passive expectations, increasing the access charge above the cost, raises the equilibrium profit for both networks (as in the case of symmetric networks studied in Section 4.1) while it reduces consumer and total welfare. In this asymmetric case an increase in termination profit makes the large firm compete less fiercely for market share as this in fact increases the number of calls to be terminated. The small firm will compete more fiercely for market share. This makes equilibrium market shares less asymmetric and reduces market concentration (as measured by the HHI index). This implies that more calls are off-net, which are inefficiently high priced. This explains why firms obtain higher profits while consumer surplus is reduced. Moreover, taking into account that network 1 provides higher value ($\gamma > 0$) to consumers, reducing asymmetry between the firms lowers total and consumer welfare even more.

5 Voluntary Subscription

In this section we do not assume that all consumers will subscribe to one of the two networks. Consumers have the option to stay unsubscribed. Since consumers can only call to subscribers, consumers will care about the total number of people that will subscribe to *some* network. In the case of termination-based price discrimination consumers will care about the number of subscribers to *each* network. The addition of an extra subscriber has a positive benefit for all subscribers. The nature of competition impedes firms to fully internalize this externality. It has been argued by some mobile operators and regulators that the termination charge should include a network externality surcharge so as to facilitate the internalization of the externality. Dessein (2003) and Hurkens and Jeon (2009) show indeed that when subscription demand is elastic, a surcharge may increase penetration and improve total welfare. However, these models assume again responsive expectations. Moreover, these models also predict that firms prefer not to have a surcharge, since profits are higher with termination charges below cost. We will now review this issue under the assumption of passive expectations.

The Hotelling framework is not very well suited to address the issue of elastic subscription demand. Namely, if some consumers in the center of the interval do not subscribe, networks would operate like local monopolists, rather than as competitors.[31] We therefore will use a Logit model in which consumers have random utility.[32]

[31] Armstrong and Wright (2009) consider a Hotelling model with hinterlands to address the possibility of expansion.

[32] See Anderson and de Palma (1992) and Anderson, de Palma and Thisse (1992) for more details about the Logit model.

We consider competition between two networks. Each firm i ($i = 1, 2$) charges a fixed fee F_i and may be allowed (or may not be allowed) to discriminate between on-net and off-net calls. For ease of exposition we will continue to use the notation p_i and $\hat{p}_i$ for on- and off-net call prices of firm i. When termination-based price discrimination is not allowed we impose that $p_i = \hat{p}_i$. Notation and definition of call demand is as before. In particular, given some expectations β_1 and β_2, utility from subscribing to network 1 equals

$$w_1 = \beta_1 v(p_1) + \beta_2 v(\hat{p}_1) - F_1,$$

while subscribing to network 2 yields

$$w_2 = \beta_2 v(p_2) + \beta_1 v(\hat{p}_2) - F_2.$$

Finally, not subscribing at all yields utility w_0.

We now add a random noise term and define $U_1 = w_1 + \mu\varepsilon_1$, $U_2 = w_2 + \mu\varepsilon_2$, and $U_0 = w_0 + \mu\varepsilon_0$. The parameter $\mu > 0$ reflects the degree of product differentiation in a Logit model. A high value of μ implies that most of the value is determined by a random draw so that competition between the firms is rather weak. The noise terms ε_k are random variables of zero mean and unit variance, identically and independently double exponentially distributed. They reflect consumers' preference for one good over another. A consumer will subscribe to network 1 if and only if $U_1 > U_2$ and $U_1 > U_0$; he will subscribe to network 2 if and only if $U_2 > U_1$ and $U_2 > U_0$; he will not subscribe to any network otherwise. The probability of subscribing to network i is denoted by α_i where α_0 represents the probability to remain unsubscribed. The probabilities are given by

$$\alpha_i = \frac{\exp[w_i/\mu]}{\sum_{k=0}^{2} \exp[w_k/\mu]}. \tag{18}$$

Note that for $i = 1, 2$

$$\frac{\partial \alpha_i}{\partial F_i} = -\frac{\alpha_i(1-\alpha_i)}{\mu}, \tag{19}$$

while for $j \in \{0, 1, 2\} \backslash \{i\}$

$$\frac{\partial \alpha_j}{\partial F_i} = \frac{\alpha_i \alpha_j}{\mu}. \tag{20}$$

Consumer surplus in the Logit model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS = \mu \ln \left(\sum_{k=0}^{2} \exp(w_k/\mu) \right) = w_0 - \mu \ln(\alpha_0), \tag{21}$$

where the right-hand side follows from (18). Clearly, consumer surplus is increasing in market penetration $1 - \alpha_0$.

5.1 Equilibrium

We will first establish that also in a setting with voluntary participation firms will set variable price equal to perceived marginal cost. The reason is simply that a firm can offer the same consumer surplus more efficiently by setting variable price closer to perceived marginal cost while adjusting the fixed fee accordingly. This will keep the number of subscribers to each firm constant, but improve the profit of the firm. The reasoning is valid both for the case where firms are not allowed to charge different prices for on- and off-net calls and for the case where this is allowed. Of course, the notion of perceived marginal cost depends on the case under consideration. When firms can price discriminate, perceived marginal cost for on-net calls equals c and perceived marginal cost for off-net calls equals $c + m$. In this case profit is given by Eq. (1).

In the case where price discrimination is not allowed, we denote

$$\tilde{c}_i = \frac{\alpha_i c + \alpha_j (c + m)}{\alpha_i + \alpha_j}$$

as the weighted average marginal cost of calls. Now, i's profit can be rewritten as

$$\pi_i^{UNI} = \alpha_i \left[F_i - f + (\alpha_i + \alpha_j)(p_i - \tilde{c}_i) q(p_i) \right] + \alpha_i \alpha_j m q(p_j).$$

Using these expressions for profit, it is easy to establish the following perceived marginal cost pricing result.

Lemma 2 *(i) When firms can price discriminate between on- and off-net calls, in equilibrium firm i will set $p_i = c$ and $\hat{p}_i = c + m$.*

(ii) When firms cannot price discriminate between on- and off-net calls, in equilibrium firm i will set $p_i = \hat{p}_i = \tilde{c}_i$. In a symmetric equilibrium $\tilde{c}_i = c + m/2$.

Next we will characterize fixed fees in equilibrium. It will be necessary to treat the case of on-net/off-net price discrimination separately from the case where firms set a uniform usage price.

Given the perceived marginal cost pricing result, in the case of termination-based price discrimination, profits can be rewritten as:

$$\pi_i^{PD} = \alpha_i(F_i - f) + \alpha_i\alpha_j mq(c+m).$$

Profits stem from the fixed fee and from termination services. The necessary first-order condition with respect to the fixed fee thus gives

$$0 = \frac{\partial \pi_i^{PD}}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i}(F_i - f) + \alpha_i + [\alpha_i \frac{\partial \alpha_j}{\partial F_i} + \alpha_j \frac{\partial \alpha_i}{\partial F_i}]mq(c+m).$$

Substituting (19) and (20) and re-arranging yields

$$F_i = f + \frac{\mu}{1-\alpha_i} - mq(c+m)\frac{\alpha_j(1-2\alpha_i)}{1-\alpha_i}.$$

Looking for a symmetric solution with $\alpha_i = \alpha_j = \alpha$, we find the following relation between equilibrium fixed fee and equilibrium number of subscribers per firm:

$$F^{PD} = f + \frac{\mu}{1-\alpha} - mq(c+m)\frac{\alpha(1-2\alpha)}{1-\alpha}. \tag{22}$$

We will denote the right-hand side of equation (22) by $F_{PD}^{FOC}(\alpha, m)$ and we will refer to this curve as the equilibrium curve (when termination-based price discrimination is allowed).

When termination-based price discrimination is not allowed, profits can also be rewritten as

$$\pi_i^{UNI} = \alpha_i\left[F_i - f + (\alpha_i + \alpha_j)(p_i - c)q(p_i)\right] + \alpha_i\alpha_j m[q(p_j) - q(p_i)].$$

Keeping both call prices fixed at $c + m/2$, this expression further simplifies to:

$$\pi_i^{UNI} = \alpha_i\left[F_i - f + (\alpha_i + \alpha_j)\frac{m}{2}q(c+m/2)\right].$$

Profits stem from the fixed fee and from the fact that all calls are charged at $c+m/2$, while termination payments cancel out against termination revenues. The necessary first-order condition with respect to the fixed fee thus gives

$$0 = \frac{\partial \pi_i^{UNI}}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i}\left[F_i - f + (\alpha_i + \alpha_j)\frac{m}{2}q(c+m/2)\right] + \alpha_i\left[1 + (\frac{\partial \alpha_i}{\partial F_i} + \frac{\partial \alpha_j}{\partial F_i})\frac{m}{2}q(c+m/2)\right].$$

Substituting (19) and (20) and re-arranging yields

$$F_i = f + \frac{\mu}{1-\alpha_i} + \frac{m}{2} q(c + m/2) \left[-2\alpha_i - \alpha_j + \frac{\alpha_i \alpha_j}{1-\alpha_i} \right].$$

Looking for a symmetric solution with $\alpha_i = \alpha_j = \alpha$, we find the following relation between equilibrium fixed fee and equilibrium number of subscribers per firm:

$$F^{UNI} = f + \frac{\mu}{1-\alpha} + \frac{m}{2} q(c + m/2) \left[\frac{4\alpha^2 - 3\alpha}{1-\alpha} \right]. \tag{23}$$

We will denote the right-hand side of equation (23) by $F_{UNI}^{FOC}(\alpha, m)$ and we will refer to this curve as the equilibrium curve (when termination-based price discrimination is not allowed).

From (18) we know that expectations being fulfilled in the case of a symmetric solution $(F, p, \hat{p})$ requires that the number of subscribers per firm (denoted by α), must satisfy

$$\alpha = \frac{\exp[(\alpha v(p) + \alpha v(\hat{p}) - F)/\mu]}{2\exp[(\alpha v(p) + \alpha v(\hat{p}) - F)/\mu] + \exp[w_0/\mu]}.$$

This can be rewritten as

$$F = \alpha v(p) + \alpha v(\hat{p}) - w_0 - \mu \log\left(\frac{\alpha}{1-2\alpha} \right). \tag{24}$$

We will denote the right-hand side of equation (24) by $F^{RE}(\alpha, m)$ and we will refer to this curve as the rational expectations curve.

A symmetric equilibrium with fulfilled expectations with (respectively, without) termination-based price discrimination is thus found by solving the system of equations (22) (respectively, (23)) and (24). It is easily verified that this system of equations always admits a solution. Namely, for any given and fixed m, the (continuous) equilibrium curve is bounded on the interval $[0, 1/2]$ while the rational expectation curve approaches $+\infty$ as $\alpha \downarrow 0$ while it approaches $-\infty$ as $\alpha \uparrow 1/2$. The following lemma gives a sufficient condition for the uniqueness of such a solution. We will denote this solution (indexed by m) as $(F_{PD}(m), \alpha_{PD}(m))$, in the case of termination-based price discrimination, and as $(F_{UNI}(m), \alpha_{UNI}(m))$, in the case of no termination-based price discrimination. Since we will be particularly interested in how profits and welfare behave in a neighborhood of $m = 0$, we introduce $(F^*, \alpha^*) = (F(0), \alpha(0))$. Note that for $m = 0$, it does not matter whether termination-based price discrimination is allowed or not, since firms do not find it optimal to discriminate. Hence, in this case we may omit the index referring to the exact case at hand. However, when doing comparative statics with respect to termination mark-up m around 0, one should distinguish between the case of termination-based price discrimination and uniform call prices.

Lemma 3 *(i) For $|m|$ small enough and $\mu > v(c)/4$, the system of equations [(22) and (24)] has a unique solution.*

(ii) For $|m|$ small enough and $\mu > v(c)/4$, the system of equations [(23) and (24)] has a unique solution.

5.2 Comparative statics

We now investigate how the equilibrium behaves in a neighborhood of $m = 0$. We first establish that an increase in the termination mark-up above 0 reduces equilibrium fixed fees, reduces the number of subscribers if on-net/off-net price discrimination is allowed and increases the number of subscribers if on-net/off-net price discrimination is not allowed.

Proposition 6 *(i) In the case of termination-based price discrimination, a marginal increase in the termination mark-up above* 0 *lowers overall subscription and lowers equilibrium fixed fees.*

(ii) In the case of no termination-based price discrimination, a marginal increase in the termination mark-up above 0 *increases overall subscription and lowers equilibrium fixed fees.*

From Lemma 2 we know that an increase in the termination fee raises usage price[33], since termination costs are passed onto consumers. This also means that consumers bring with them termination profits. Competition for customers becomes fiercer and this leads firms to charge lower fixed fees in equilibrium. This means that there is a waterbed effect at play. However, the strength of the waterbed effect depends on whether on-net/off-net price discrimination is allowed. We have noted before, in the case of inelastic subscription (and duopoly), that with price discrimination there is no waterbed effect (the fixed fee is independent of the termination charge) while in the case of uniform call prices there is a profit neutrality result and thus a one hundred per cent waterbed effect. It turns out that with elastic subscription the waterbed effect is also stronger when no price discrimination is allowed. In fact, Proposition 6 states that the waterbed effect in case of no price discrimination is so strong that the number of subscribers increases, while with price discrimination the waterbed effect is weak and the number of subscribers decreases.

[33] When termination-based price discrimination is allowed, only off-net usage price increases.

It is not obvious how profits are affected by an increase in termination mark-up. Namely, an increase in the termination charge improves termination profits. We will now analyze the total effect on equilibrium profits (and on welfare). In equilibrium, profit equals

$$\pi(m, F, \alpha) = \alpha(F - f) + \alpha^2 m q(\hat{p}).$$

Note that this expression is valid independently of whether on-net/off-net price discrimination is allowed.[34] We first analyze how profits change along the rational expectations curve $F^{RE}(\alpha, 0)$ (when termination charge is fixed at $a = c_T$) as market penetration is varied. Note that profits, in this case, are just equal to $\Pi = \alpha(F^{RE}(\alpha, 0) - f)$ so that

$$\frac{\partial \pi}{\partial \alpha} = F^{RE}(\alpha, 0) - f + \alpha \frac{\partial F^{RE}}{\partial \alpha}.$$

Using that at $m = 0$, $F^{RE}(\alpha, 0) = F^{FOC}(\alpha, 0) = \mu/(1-\alpha) + f$, one obtains

$$\frac{\partial \pi}{\partial \alpha} = \alpha \frac{2v(c)(1-\alpha)(1-2\alpha) - \mu}{(1-\alpha)(1-2\alpha)}.$$

The sign is ambiguous since it is negative for mature markets (when $\alpha \approx 1/2$) while it is positive for $\alpha \approx 0$ and $\mu < 2v(c)$. If the sign is negative, it means that colluding networks would prefer to increase fixed fees and lower market penetration. This is the case if competition is effective in terms of boosting penetration and lowering prices. We will refer to this case as one of effective competition. If the sign is positive, it means the opposite, that is, firms would prefer to increase penetration and reduce fixed fees. This would be the case in which externalities are important and are not well internalized under competition. We will refer to this case as one of strong network externalities.

Next, we consider how the profit changes as the termination charge is changed, keeping market penetration constant. An increase in m increases termination profits, but decreases the fixed fee. At $m = 0$ these effects exactly cancel out.

$$\left.\frac{\partial \pi}{\partial m}\right|_{m=0} = \alpha \frac{\partial F^{RE}}{\partial m} + \alpha^2 q(c) = 0,$$

where the second equality follows from Eq. (24) and $v'(c) = -q(c)$. Putting the two effects together shows that

[34] We omit indices to the case at hand when it does not matter. Recall that in the case of no on-net/off-net price discrimination $\hat{p} = p = c + m/2$.

$$
\begin{aligned}
\left.\frac{d\pi}{dm}\right|_{m=0} &= \frac{\partial \pi}{\partial m} + \frac{\partial \pi}{\partial F}\frac{dF}{dm} + \frac{\partial \pi}{\partial \alpha}\frac{\partial \alpha}{\partial m} \\
&= \alpha^2 q(c) + \alpha\left(\frac{\partial F}{\partial m} + \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial m}\right) + (F-f)\frac{\partial \alpha}{\partial m} \\
&= \alpha^2 q(c) + \alpha\left(-\alpha q(c) + \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial m}\right) + (F-f)\frac{\partial \alpha}{\partial m} \\
&= \frac{\partial \alpha}{\partial m}\left(F - f + \alpha\frac{\partial F}{\partial \alpha}\right).
\end{aligned}
$$

Note that the expression between brackets in the last line is just the derivative with respect to α of profits along the rational expectations curve.

Suppose first the case that profits increase along the rational expectations curve (low μ and low market penetration). If on-net/off-net price discrimination is allowed, then an increase of the termination charge lowers profits, since $\partial\alpha/\partial m < 0$ from Proposition 6(i). When price discrimination is not allowed, then an increase of the termination charge increases profits, since $\partial\alpha/\partial m > 0$ from Proposition 6(ii).

Suppose next the case that profits decrease along the rational expectations curve. If on-net/off-net price discrimination is allowed, then an increase in the termination charge increases the profits. When price discrimination is not allowed, then an increase of the termination charge lowers profits.

The second case happens exactly when colluding firms would set a higher fixed fee than what they would choose under competition, and when market penetration would be lower under monopoly than under duopoly. That is, the second case is the one of effective competition. This situation is the more likely scenario, especially for European countries where penetration rates are close to 100 per cent.

We now turn our attention to the effects of termination charges on consumer and total surplus. Note that total surplus is just the sum of consumer surplus and industry profit:

$$TS = CS + 2\pi.$$

From (21) we know that $dCS/dm = (\partial\alpha/\partial m)(2\mu/(1-2\alpha))$. Hence

$$
\begin{aligned}
\left.\frac{dTS}{dm}\right|_{m=0} &= \left(\frac{\partial\alpha}{\partial m}\right)\left(\frac{2\mu}{1-2\alpha}+2\left(F-f+\alpha\frac{\partial F}{\partial\alpha}\right)\right)\\
&= 2\left(\frac{\partial\alpha}{\partial m}\right)\left(2\alpha v(c)+\frac{\mu}{1-\alpha}\right).
\end{aligned}
$$

Since the second factor is positive, total surplus increases whenever market penetration (or consumer surplus) increases.

Proposition 7 *(i) Suppose on-net/off-net price discrimination is allowed. In order to maximize either consumer surplus or total surplus, the termination charge has to be set strictly below the cost of termination. Firms' profits are maximized by a termination charge above the cost of termination if and only if* $\mu > 2v(c)(1-\alpha^*)(1-2\alpha^*)$ *(that is, if and only if competition is effective).*

(ii) Suppose on-net/off-net price discrimination is not allowed. In order to maximize either consumer surplus or total surplus, the termination charge has to be set strictly above the cost of termination. Firms' profits are maximized by a termination charge below the cost of termination if and only if $\mu > 2v(c)(1-\alpha^*)(1-2\alpha^*)$ *(that is, if and only if competition is effective).*

This result is in stark contrast with Dessein (2003) and Hurkens and Jeon (2009) who implicitly use responsive expectations. Dessein (2003) does not allow for termination-based price discrimination while Hurkens and Jeon (2009) do. Both papers find that firms always prefer termination charge below the cost of termination. Moreover, they both find that only in the (plausible) case of effective competition, consumer surplus and total welfare are maximized when termination charge is above cost. Instead, we find that an externality surcharge only improves penetration when termination-based price discrimination is not allowed.

One might expect that firms, for marketing purposes, will never charge on-net prices above off-net prices, even if that from a theoretical point of view is optimal when termination charges are below cost. Under this assumption, termination charges below cost will result in no on-net/off-net price discrimination while termination charges above cost will result in on-net/off-net price discrimination. The socially optimal termination charge would then be exactly equal to cost while, in the case of effective competition, firms would see their profits increase both when termination charge is reduced below cost and when termination charge is increased above cost. This could potentially explain both that regulators are right when they propose to set termination charges at cost, and that operators complained when termination charges were reduced (but were still far above cost). Moreover, it is consistent with the recent campaign by smaller operators to go all the way to Bill and Keep, that is, zero termination charges.

6 Conclusion

This article has studied how consumer expectations affect retail competition when network externalities exist. As in Katz and Shapiro (1985) and related literature on network externalities, we assume that first consumers form expectations about network sizes, then firms compete, and last consumers make rational subscription and consumption decisions based on their expectations and the chosen prices. Expectations must be fulfilled in equilibrium. Instead, in the literature on termination charges and tariff-mediated network externalities (starting from Laffont, Rey and Tirole, 1998b), rational expectations are imposed interim: any change of a price by one firm leads to a *rational* change in consumer expectations for which subscription decisions are such that realized and expected network sizes coincide. We have shown that the way consumers form expectations and how these react to price variations have important implications in terms of the impact of termination charges on retail competition. We have shown that if expectations are modeled as in Katz and Shapiro (1985), results regarding the impact of mobile termination rates on retail competition are in fact in line with real world observations. In particular, we overturn the Gans and King (2001) result: in our model when firms compete in fixed fees and charge different prices for on- and off-net calls, they prefer a termination charge above cost so that off-net calls are priced at monopoly levels (socially optimal termination charges are equal to cost). Moreover, fixed fees and on-net prices are neutral with respect to the termination charge and thus, in the case of full participation and two networks, there is no waterbed effect at all. If the number of competing networks is larger than two, then there exists a partial waterbed effect on the subscription fee. Our theoretical results are thus in line with the empirical evidence of the existence of a waterbed effect that is not full, provided by Genakos and Valletti (2011).

Our results provide formal support to the relatively commonly-held view in the decision practice on mobile markets that firms benefit from high termination rates. Given the current debate on the optimal level of mobile termination rates, our results have direct policy implications. Mobile network operators (MNOs) have opposed cuts in termination charges over the past decade, and they keep doing so. This is of course a clear sign that mobile operators fear to see their profits reduced. The arguments these operators employ to defend their opposition against lowering termination charges sometimes make reference to the existence of a waterbed effect. They warn regulators that cutting termination rates may lead to higher prices and that may hurt consumers. Regulators have not been very empathetic to this argument and sometimes even denied the existence of a waterbed effect. For example, the Australian Competition and Consumer Commission (ACCC, 2007), wrote "The Commission considers that these trends of lower average retail prices [...] demonstrate that the converse of the 'waterbed' effect has been in operation." The New Zealand Commerce Commission (NZCC, 2006) initially discarded the existence of a waterbed effect, but after the intervention by Prof. Hausman, acting on behalf of one of the MNOs, acknowledged the possible existence of a waterbed effect. However, the NZCC noted that to the extent that there is a waterbed effect, whereby retail mobile prices are adjusted in some way in response to regulation, it considered it likely that mobile prices will decline under regulation but at a slower rate than without. The UK regulator (Ofcom 2004) accepts the existence of a waterbed effect, but does not believe it is full because the retail market is not yet fully competitive. On the other hand, Ofcom (2004, 2007) and some other NRAs did accept the suggestion that an externality surcharge to promote subscription was appropriate. Our model shows that this conclusion is not warranted and that the Recommendation of the European Commission to not allow for such a mark-up is correct (unless termination-based price discrimination would be prohibited).

The present paper has considered a number of theoretical models (linear and non-linear pricing, duopoly and oligopoly, symmetric and asymmetric firms, elastic and inelastic subscription demand, and call externalities) and shows that a waterbed effect often does exist but that it is always less than full, so that consumer welfare is improved when termination charges are reduced toward or even below cost. A further important lesson from our paper is that more competition in the telecommunication market may not be effective if it is not accompanied by continued adequate regulation of the monopolistic bottlenecks. In fact, regulation may be even more important in these cases since the number of off-net calls decreases with the HHI index.

We have assumed that the expectations of consumers do not change with price variations (off the equilibrium path) and that expectations are fulfilled in equilibrium. We believe this to be a plausible assumption in the context of telecommunication markets. Notwithstanding, we believe it will be important to consider a truly dynamic model where consumers face switching costs and where expectations are formed endogenously. A key question would be whether the results in such a dynamic model resemble the ones obtained in the static model with passive or with responsive expectations. We hope that this article will stimulate further research extending the analysis in this direction. Although the current paper has already considered a wide range of models, a number of issues that deserve further investigation have remained unaddressed. For example, how do passive expectations affect equilibrium outcomes when (i) different types of consumers are allowed for?; (ii) the called party also pays? (as is the case in Canada, Hong Kong, Singapore and the US); (iii) when both fixed and mobile operators compete with each other?

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Appendix A

Proof of Proposition 1:

We have already established that there is a unique candidate for a shared market equilibrium. We need to show that for σ small enough, this candidate solution is indeed an equilibrium. We fix the strategy for firm 2 as $p_2 = c$, $\hat{p}_2 = c + m$ and $F_2 = f + 1/(2\sigma)$. Moreover, we fix consumer expectations at $\beta_1 = \beta_2 = 1/2$. We need to calculate the optimal response of firm 1. Recall from our discussion of the perceived marginal cost pricing that firm 1, in order to maximize its profit, can adjust its fixed fee F_1 so as to keep market share constant at α_1. The on- and off-net price have to satisfy the first-order conditions (2) and (3), respectively. Denote these prices by $p_1(\alpha_1)$ and $\hat{p}_1(\alpha_1)$. One derives immediately that for $0 < \alpha_1 < 1$

$$R'(p_1(\alpha_1)) = \frac{q(p_1(\alpha_1))}{2\alpha_1} > 0 \tag{25}$$

and

$$\widehat{R}'(\hat{p}_1(\alpha_1)) = \frac{q(\hat{p}_1(\alpha_1))}{2(1-\alpha_1)} > 0. \tag{26}$$

Hence, $p_1(\alpha_1) < p^M$ and $\hat{p}_1(\alpha_1) < p^M$.

Let $F_1(\alpha_1)$ denote the fixed fee that yields firm 1 indeed a market share of α_1. That is,

$$F_1(\alpha_1) = f + \frac{1-\alpha_1}{\sigma} + \frac{1}{2}[v(p_1(\alpha_1)) + v(\hat{p}_1(\alpha_1)) - v(c) - v(c+m)].$$

Finding the optimal response for firm 1 boils down to finding the optimal market share. The profit of firm 1, as a function of chosen market share, is

$$\Pi_1(\alpha_1) = \alpha_1\left(\alpha_1 R(p_1(\alpha_1)) + (1-\alpha_1)\widehat{R}(\hat{p}_1(\alpha_1)) - f + F_1(\alpha_1)\right) + \alpha_1(1-\alpha_1)mq(c+m).$$

Note that $\Pi_1(0) = 0$ and that, since $F_1(1) < f + v(0)$, for σ small enough

$$\Pi_1(1) < R(p^M) + v(0) < 1/(4\sigma) + mq(c+m)/4 = \Pi_1(1/2).$$

Because the profit function is continuous, there exist $\underline{\alpha}$ and $\bar{\alpha}$ with $0 < \underline{\alpha} < \bar{\alpha} < 1$ so that the profit function will be maximized on the interval $[\underline{\alpha}, \bar{\alpha}]$.

Because of the envelope theorem, the partial derivatives with respect to on-net and off-net price are equal to zero, so that

$$\frac{d\Pi_1}{d\alpha_1} = 2\alpha_1 R(p_1(\alpha_1)) + (1-2\alpha_1)\widehat{R}(\hat{p}_1(\alpha_1)) - f + F_1(\alpha_1) - \frac{\alpha_1}{\sigma} + (1-2\alpha_1)mq(c+m).$$

Note that at $\alpha_1 = 1/2$ the first order derivative indeed equals zero since $p_1(1/2) = c$, $\hat{p}_1(1/2) = c + m$, and $F(1/2) = f + 1/(2\sigma)$. Using expressions (25) and (26) we can write the second-order derivative as

$$\frac{d^2\Pi_1}{d\alpha_1^2} = 2R(p_1(\alpha_1)) + \frac{q(p_1(\alpha_1))p_1'(\alpha_1)}{2} - 2\widehat{R}(\hat{p}_1(\alpha_1)) - \frac{\alpha_1 q(\hat{p}_1(\alpha_1))\hat{p}_1'(\alpha_1)}{2(1-\alpha_1)} - 2(\frac{1}{\sigma} + mq(c+m)).$$

Clearly, for small enough σ this is strictly negative as the first 4 terms of this expression are bounded on the interval $[\underline{\alpha}, \bar{\alpha}]$.

Proof of Lemma 1:

For given rival strategies, maximizing π_i with respect to p_i, while adapting F_i so as to keep market shares constant, yields

$$\alpha_i \left[\alpha_i \left(q(p_i) + (p_i - c)\, q'(p_i) \right) + \frac{dF_i}{dp_i} \right] = 0. \tag{27}$$

For a constant α_i, differentiating Eq. (9) with respect to p_i yields

$$\sigma \left[\beta_i \left(q(p_i) - \overline{u}'(q(p_i))q'(p_i) \right) + \frac{dF_i}{dp_i} \right] = 0. \tag{28}$$

In equilibrium, expectations are fulfilled ($\beta_i = \alpha_i$), then from Eqs. (27) and (28) we have that $c - p_i = \overline{u}'(q(p_i))$. Since $\overline{u}(q) = \lambda u(q)$ and $u'(q) = p$, it follows that

$$p_i = p^* \equiv \frac{c}{1+\lambda}. \tag{29}$$

Similarly, for given rival strategies, the first-order derivative of i's profit with respect to $\widehat{p_i}$, while adapting F_i so as to maintain market shares constant, yields

$$\alpha_i \left[\alpha_j \left(q(\widehat{p}_i) + (\widehat{p}_i - c) q'(\widehat{p}_i) \right) - \alpha_j m q'(\widehat{p}_i) + \frac{dF_i}{d\widehat{p}_i} \right] = 0. \tag{30}$$

By differentiating α_i with respect to $\widehat{p}_i$ we obtain

$$-\sigma \beta_i \overline{u}'(q(\widehat{p}_i)) q'(\widehat{p}_i) - \sigma \beta_j q(\widehat{p}_i) - \sigma \frac{dF_i}{d\widehat{p}_i} = 0. \tag{31}$$

Comparing Eqs. (30) and (31), we have that $\beta_i \overline{u}'(q(\widehat{p}_i)) q'(\widehat{p}_i) = \alpha_j (\widehat{p}_i - c - m) q'(\widehat{p}_i)$. Using $\overline{u}'(q) = \lambda \widehat{p}_i$, we obtain $\beta_i \lambda \widehat{p}_i = \alpha_j (\widehat{p}_i - c - m)$, where $\beta_i = \alpha_i$. Hence

$$\widehat{p}_i = \widehat{p}^*(\alpha_i) \equiv \frac{(1 - \alpha_i)(c + m)}{1 - \alpha_i (1 + \lambda)}. \tag{32}$$

Proof of Proposition 5:

Profits in equilibrium are given by (17). Totally differentiating with respect to m gives

$$\frac{d\pi_i}{dm} = 2\alpha_i \frac{d\alpha_i}{dm} \left(\frac{1}{\sigma} + m q(c+m) \right) + \alpha_i^2 \left(q(c+m) + m q'(c+m) \right).$$

Evaluating this derivative at $m = 0$ yields

$$\left. \frac{d\pi_i}{dm} \right|_{m=0} = \frac{2\alpha_i}{\sigma} \left. \frac{d\alpha_i}{dm} \right|_{m=0} + \alpha_i^2 q(c). \tag{33}$$

Totally differentiating (15) and (16), using $\alpha_2 = 1 - \alpha_1$, gives

$$\frac{d\alpha_i}{dm} = \sigma \left(\frac{dF_j}{dm} - \frac{dF_i}{dm} \right) + 2\sigma \left(\beta_i - \frac{1}{2} \right) q(c+m)$$

and

$$\frac{dF_i}{dm} = \frac{1}{\sigma} \frac{d\alpha_i}{dm} + 2 \frac{d\alpha_i}{dm} m q(c+m) + 2 \left(\alpha_i - \frac{1}{2} \right) \left(q(c+m) + m q'(c+m) \right).$$

Evaluating this derivative at $m = 0$ yields

$$\left. \frac{dF_i}{dm} \right|_{m=0} = \frac{1}{\sigma} \left. \frac{d\alpha_i}{dm} \right|_{m=0} + 2 \left(\alpha_i - \frac{1}{2} \right) q(c).$$

Thus, we have that

$$\left. \frac{d\alpha_i}{dm} \right|_{m=0} = \frac{2\sigma}{3} \left[2 \left(\frac{1}{2} - \alpha_i \right) + \left(\beta_i - \frac{1}{2} \right) \right] q(c).$$

Self-fulfilling expectations imply that at equilibrium $\beta_i = \alpha_i$, thus

$$\left.\frac{d\alpha_i}{dm}\right|_{m=0} = -\frac{2\sigma}{3}\left(\alpha_i - \frac{1}{2}\right) q(c) \quad i = 1, 2, \tag{34}$$

and

$$\left.\frac{dF_i}{dm}\right|_{m=0} = \frac{4}{3}\left(\alpha_i - \frac{1}{2}\right) q(c).$$

That is, starting from cost-based access charges, a slight increase in m, raises F_1 (lowers α_1) and lowers F_2 (raises α_2), which in turn reduces the asymmetry between the networks. Substituting Eq. (34) into Eq. (33) we get that

$$\left.\frac{d\pi_i}{dm}\right|_{m=0} = \frac{2}{3}\alpha_i\left(1 - \frac{\alpha_i}{2}\right) q(c) > 0 \quad i = 1, 2.$$

Finally, total surplus equals

$$\begin{aligned} TS(m) \;\; = \;\; & \alpha_1[\gamma/(2\sigma) + \alpha_1 v(c) + (1-\alpha_1)(u(q(c+m)) - cq(c+m))] \\ & +(1-\alpha_1)[(1-\alpha_1)v(c) + \alpha_1(u(q(c+m)) - cq(c+m))] \\ & -\frac{1}{2\sigma}[\frac{\alpha_1}{2}\alpha_1 + \frac{1-\alpha_1}{2}(1-\alpha_1)]. \end{aligned}$$

Using that at $m = 0$, $\alpha_1 = (\gamma + 3)/6$, one easily verifies that

$$\left.\frac{dTS}{dm}\right|_{m=0} = -q(c)\frac{\gamma^2}{27} < 0.$$

Appendix B

Proof of Lemma 2:

(i) Suppose that $(p_i, \hat{p}_i) \neq (c, c+m)$. We claim that firm i can improve its profit by changing its tariff from $(p_i, \hat{p}_i, F_i)$ to $(c, c+m, \tilde{F}_i)$ where $\tilde{F}_i$ is defined by

$$\beta_i v(p_i) + \beta_j v(\hat{p}_i) - F_i = \beta_i v(c) + \beta_j v(c+m) - \tilde{F}_i.$$

Such a change leaves the expected utility for subscribing to any of the networks unaltered, and will thus lead to the same subscription decisions. The difference in profit for firm i is thus equal to

$$\alpha_i[\tilde{F}_i - F_i - \alpha_i(p_i - c)q(p_i) - \alpha_j(\hat{p}_i - (c+m))q(\hat{p}_i)] =$$
$$\alpha_i(\alpha_i[v(c) - v(p_i) + v'(p_i)(p_i - c)] + \alpha_j[v(\hat{c}) - v(\hat{p}_i) + v'(\hat{p}_i)(\hat{p}_i - (c+m))]) > 0$$

where the equality follows from self-fulfilled expectations ($\beta_k = \alpha_k$) whereas the inequality follows from the fact that $v(\cdot)$ is a strictly convex and decreasing function. The deviation is thus profitable.

(ii) Suppose that $p_i \neq \tilde{c}_i$. We claim that firm i can improve its profit by changing its tariff from (p_i, F_i) to $(\tilde{c}_i, \tilde{F}_i)$ where $\tilde{F}_i$ is defined by

$$(\beta_i + \beta_j)v(p_i) - F_i = (\beta_i + \beta_j)v(\tilde{c}_i) - \tilde{F}_i.$$

Such a change leaves the utility for subscribing to any of the networks unaltered, and will thus lead to the same subscription decisions. Given self-fulfilled expectations ($\beta_k = \alpha_k$) the difference in profit for firm i is thus equal to

$$\alpha_i[\tilde{F}_i - F_i - (\alpha_i + \alpha_j)(p_i - \tilde{c}_i)q(p_i)] = \alpha_i(\alpha_i + \alpha_j)[v(\tilde{c}_i) - v(p_i) + v'(p_i)(p_i - \tilde{c}_i)] > 0$$

where the inequality follows from the fact that $v(\cdot)$ is a strictly convex and decreasing function. The deviation is thus profitable.

Proof of Lemma 3:

Let $m = 0$ and $\alpha \in (0, 1/2)$. Let $K \in \{PD, UNI\}$. Then

$$\frac{\partial F_K^{FOC}}{\partial \alpha} = \frac{\mu}{(1-\alpha)^2} > 0,$$

while

$$\frac{\partial F^{RE}}{\partial \alpha} = 2v(c) - \frac{\mu}{\alpha(1-2\alpha)} < 0$$

whenever $\mu > v(c)/4$. So, for $m = 0$, the equilibrium curve intersects the rational expectations curve from below. By continuity, the same holds for $|m|$ small enough. Hence, there is exactly one solution.

Proof of Proposition 6:

Note that

$$\left.\frac{dF^{RE}}{dm}\right|_{m=0} = -\alpha^* q(c)$$

while

$$\left.\frac{dF^{FOC}_{PD}}{dm}\right|_{m=0} = -\alpha^* q(c)\frac{1-2\alpha^*}{1-\alpha^*}$$

and

$$\left.\frac{dF^{FOC}_{UNI}}{dm}\right|_{m=0} = -\alpha^* q(c)\frac{3-4\alpha^*}{2(1-\alpha^*)}.$$

In the case of on-net/off-net price discrimination, an increase in m lowers the rational expectations curve by more than the equilibrium curve, since $0 < (1-2\alpha^*)/(1-\alpha^*) < 1$. The intersection point thus shifts to the south-west, lowering subscription rate and fixed fee.

In the case of no on-net/off-net price discrimination, an increase in m lowers the rational expectations curve by less than the equilibrium curve, since $1 < (3-4\alpha^*)/(2(1-\alpha^*))$. The intersection point thus shifts to the south-east, lowering fixed fee and increasing subscription rate.

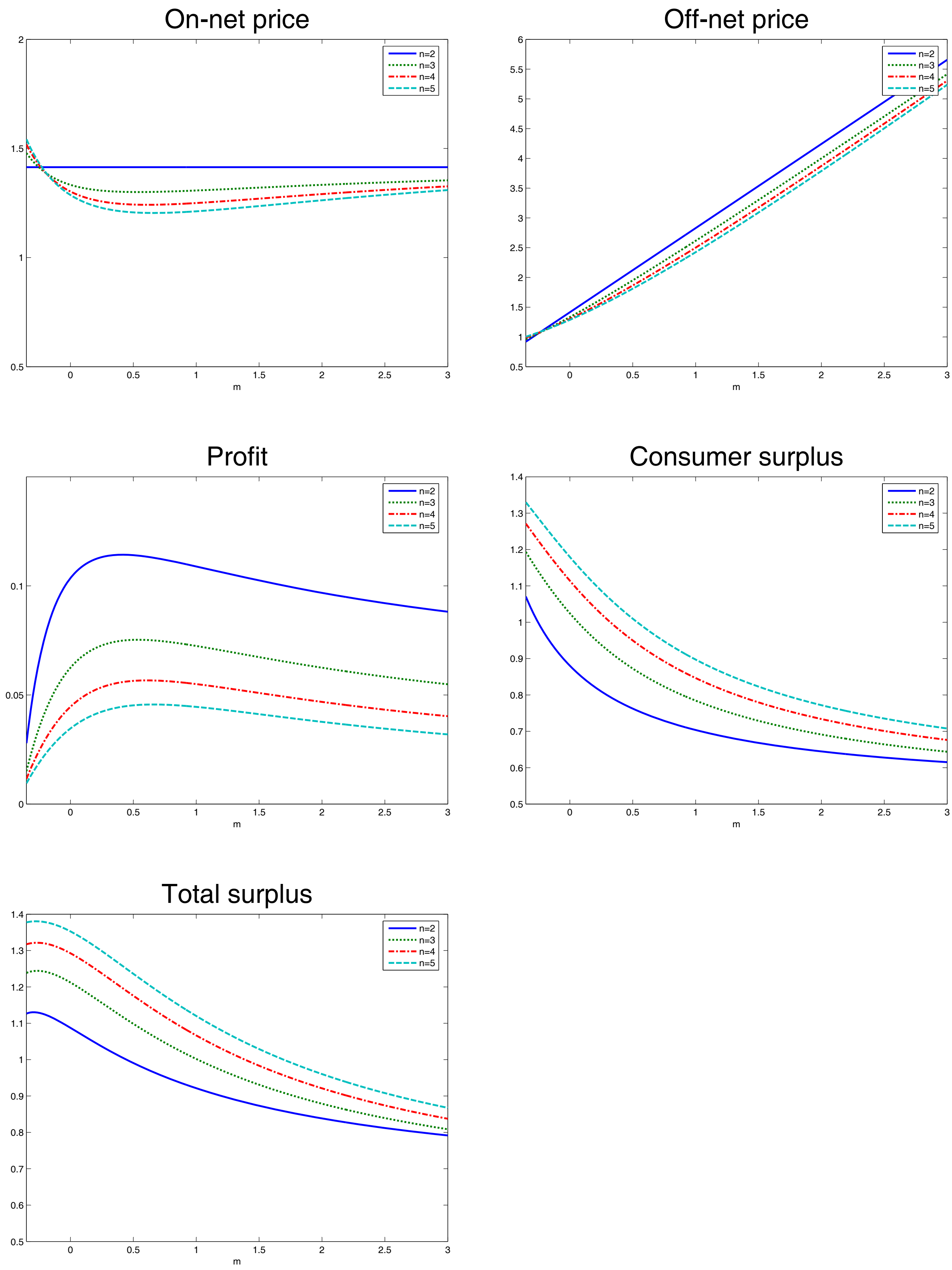

Figure 3: Effect of termination mark-up m on on-net price, off-net price, profit, consumer surplus and total welfare in the case of linear pricing.