Unconditional Parole

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Abstract A simple model is developed to examine the rationale for unconditional (statutory) release of prison inmates before the expiration of their full sentence. Even when types are fixed, so there is not prospect of rehabilitation, some period of statutory release may be optimal. A long period of statutory release does raise the crime rate, though a sufficiently short period may not. The longer are sentences, the longer is the optimal period of statutory release.

Whenever an offender on parole release commits a crime, calls for parole reform are widespread. Some argue for "truth in sentencing" reforms that ensure an offender sentenced to ten years, for example, serves ten years, not five or seven. Recent attention in Canada has focussed on "statutory release," whereby offenders are released on parole at a pre-determined point, (almost) independently of evidence of rehabilitation. I construct a model of statutory release to question the rationale for this kind of early release. I find that statutory release may indeed be optimal in some contexts. Specifically, when sentences are long, criminals discount the future sufficiently that the extra deterrence of prison versus parole is minimal. Meanwhile, parolees on statutory release face a much higher likelihood of arrest and are thus more likely to be deterred from crime. However, when statutory release periods increase, the discounted value of eventual release becomes small, at least at the beginning of parole, and parolees are more likely to return to offending. Provided that they avoid arrest, the incentive to not engage in crime rises as the end of the parole period draws near. Thus the model predicts that statutory release parolees are less likely to re-offend as their time on parole passes.

When sentences are short, increasing the portion of the sentence served on parole reduces the deterrence of punishment. In this case, society may be better of without statutory release. This suggests that far from a trade off, as implicitly assumed by the arguments for "truth-in-sentencing", statutory release and long sentences might be complementary. So long as sentences exceed a certain length, some short period of statutory release is welfare improving. With long sentences, longer periods of statutory release become optimal. This may account for the public acceptance of Section 810 Orders, which are in essence renewable, year-long parole terms, imposed by the court at the end of a sentence.

The model I develop is a variant of Scoones, Kotowitz and Chen (2005), which does not include parole. Though the economics of crime literature has grown rapidly in recent years (see Polinsky and Shavell (2000) for a fairly recent survey), next to none of it that I have found examines parole. Even so, the model ignores the dominant explanations for parole, and these too might be used to justify statutory release. The most common argument for statutory release is that releasing prisoners without restriction or community support greatly increases the chance they will re-offend. There is a good deal of evidence to support this common sense supposition. In the model, reintegration is impossible, and all criminals return to crime at the end of their sentences. This of course leads to the question of why all sentences are not life terms. The model is silent on this point; I simply assume they are not. The implicit justification is either that jailing all the criminals for life is too costly to justify the attendant reduction in harm, or, perhaps more appealingly, that differential deterrence is important, and the crime I consider is not the most grievous in the criminal toolkit.

Another argument for parole I ignore is that people change, and jailing the no longer criminally-minded is unduly costly. This explanation has been examined in a recent paper by Bernhardt, Mongrain and Roberts (2006). In the present model types are fixed and there is no prospect of reform. Even so, information on type would be similarly useful as it can potentially help choose the members of the prison population who would be deterred by the greater scrutiny of a parole officer. Of course, this sort of "informed release" is precisely not

statutory release. For this new-information rationale to extend to *statutory* release, it would have to be that some aspect of offenders' types is better measured on parole than in prison. This may very well be true. Finally, a little noted value of parole is how it allows local law enforcement officials (including parole officers) with the modus operandi, family contacts and likely whereabouts of a town's criminal class[1].

All these issues are absent in the model I develop. The next section gives a very brief overview of the Canadian parole system. Section 2 develops a model of criminal behaviour. Sections 3 and 4 examine the social outcomes with parole and contain the main results of the paper. Section 5 suggests concludes.

1. The Canadian Federal Parole System

The criminal corrections system in Canada is divided between Federally and Provincially sentenced offenders. The distinction depends only on the length of sentence, and not for example the specific crime. All sentences of two years or longer are served in the Federal system; this is the focus of the paper. Sentences shorter than two years are served in the Provincial system, where there is not statutory release[2]. In the Federal system parole is potentially available at several points during the sentence. Day parole refers to parole with the requirement to live in a Community Residential Facility (i.e. a half-way house) and report regularly to a Parole officer. Full parole refers to time spent living independently in the community, again reporting regularly to a parole officer. Non-violent, first (Federal) time offenders may apply for day parole after six months served in a penitentiary; if granted full parole is automatically granted six months later. For other inmates, the first opportunity for full parole arrives with one third of the sentence served. Six months prior to this date, inmates are eligible to apply for day parole. None of these releases are automatic. Finally, at the two-thirds mark, by statute, all inmates must be granted parole, unless a compelling case can be made that this exposes the public to too great a risk. Offenders on statutory release may or may not be required to live at a halfway house.

Most of the decisions regarding offenders on parole are made by the Federal Parole Board. At the time of release, the Board may impose a variety of conditions on offenders, from residency at a halfway house, the extent of mandatory drug testing, programs that must be completed, and the frequency of contact with the parole officer. The parole officer is in charge of the day-to-day monitoring of an offender on parole. The frequency of meeting can vary from twice per week to once every three months. Armed with the conditions imposed by the Board, a parole officer has great powers of investigation, and the right to issue a warrant and have the offender detained. Once detained, an offender may be released by his parole officer, or face another hearing of the Board, which may either release him or revoke his parole. If parole is revoked, the same time line for day and full parole, and statutory release is calculated on the remaining portion of his sentence. Once a sentence is concluded,

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[1] While on parole offenders have very circumscribed rights, and often can lead police to other criminals in their network.
[2] Instead there is time off for good behaviour.

a local police force can apply for a Section 810 Order that imposes parole-like conditions on an ostensibly free citizen.

Because of the earlier opportunities for release based on good performance in prison, the statutory released offenders are typically the most likely to return to crime. In this paper I focus entirely on statutory release. Half-way houses and the activities of parole officers is not modelled formally, but summarised with the assumption that parolees face a significantly higher chance of arrest that do similar individuals not under sentence.

2. The model

There exists a unit population of infinitely lived potentially criminal citizens. Time is discrete. At the start of each period every agent is in one of the following: a free non-criminal, a free criminal, in jail, or on parole. Free citizens and parolees must choose between engaging in criminal activity and working in the legal sector. All criminals are subject to arrest and conviction, leading to a prison term of length J followed by P periods of parole. Each agent is characterised by a type, ψ, distributed according to F, that affects his return from crime. All other parameters are assumed to be identical for every agent. The higher is an agent's ψ-coefficients the greater is his benefit from crime. This coefficient can be thought of as a "moral discount" on the cost of criminal activity. For the time being, assume the ψ-coefficient is fixed for the duration of the agent's life. The per-period payoffs are w in lawful employment, ψc in crime, p on parole, and j in jail. I will assume that $w > p > j$.

2.1 Jail time only

First consider a simpler model without parole, i.e. $P = 0$.

Value functions

Without parole, there are three states in which an agent can be: free not committing crime, free committing crime, and in jail. Each of the three states can be characterised by the value it provides to an agent in that state accounting for the possibility of "changing careers" at the start of each period. The value of lawful employment (working) for agent i is given by

(1)
$$V_i^{NC} = w + \delta V_i$$

where NC stands for "non-criminal", δ is the discount rate, and V is the optimal continuation value, which allows for i's potential to shift into a life of crime.

The value of a crime is given by

(2)
$$V_i^C = \psi_i c + \delta[\lambda V_i^A + (1 - \lambda)V_i]$$

where ψ_i is agent i's type, c is the base value of the immediate return to crime, λ is the probability of arrest when free (i.e. not on parole), and V_i^A is agent i's value at the start of a prison term (i.e. upon apprehension, which I assume occurs after the proceeds of crime have been consumed). Equation (2) says that a criminal gets an immediate return from crime that depends on his ψ-coefficient, with a continuation payoff that depends on whether he is

arrested or not. If he is arrested he moves into prison[3], with a value V_i^A; if not, he has the option of continuing in a life of crime, or moving into legitimate employment.

The value of a prison term of length J follows by release is given by

$$V_i^A = \sum_{t=1}^{J} \delta^{t-1} j = \frac{j(1-\delta^J)}{1-\delta} + \delta^J V_i$$

where j is the per-period payoff of imprisonment. Substituting into (2) and solving for the value of crime yields

(3)
$$V_i^C = \frac{\psi_j c + \delta\lambda j(1-\delta^J)(1-\delta)^{-1}}{1-\delta + \lambda(\delta - \delta^{J+1})}$$

The marginal criminal

The marginal criminal is characterised by the ψ-coefficient that equates the benefit from crime to the value of taking a job in the legitimate sector, $V_i^{NC} = V_i^C$. Since the environment is stationary, this will remain true at the next opportunity to change occupations. That is, all criminals are recidivists. Thus $V_i = V_i^{NC} = V_i^C$. Denoting the type of the marginal criminal by, ψ^*, and substituting from the value functions yields

$$\psi^* = \frac{w}{c} + \frac{(w-j)\lambda(\delta - \delta^{J+1})}{c(1-\delta)}.$$

Since the payoff to crime increases in ψ, the marginal criminal partitions the set of agents into criminals, $\psi_i > \psi^*$, and non-criminals, $\psi_i < \psi^*$. Notice that if there is no chance of arrest, $\lambda = 0$, then $\psi^* c = w$. Furthermore, $\partial\Psi^*/\partial j < 0$ so increasing the period payoff in prison reduced the marginal ψ-coefficient, and increases hence the number of criminals. The reverse is true for changes in w. Also,

$$\frac{\partial \psi^*}{\partial \lambda} = \frac{(w-j)(\delta - \delta^{J+1})}{c(1-\delta)} > 0$$

Provided that $J > 0$ and $w > j$, increasing the arrest rate reduces the number of criminals. Lengthening sentences also reduced the number of criminals:

$$\frac{\partial \psi^*}{\partial J} = \frac{-(w-j)\lambda\delta^{J+1}\ln\delta}{c(1-\delta)} > 0$$

However,

$$\frac{\partial^2 \psi^*}{\partial J^2} = \frac{-(w-j)\lambda\delta^{J+1}(\ln\delta)^2}{c(1-\delta)} < 0$$

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[3] Throughout I assume that conviction is immediate and certain upon arrest.

so the effect is attenuated as sentences lengthen. This makes sense, since potential criminals discount the future, so adding to an already long sentence provides little further disincentive.

2.2 Parole

When part of a sentence to be served on parole, criminals face two distinct decisions on criminal activity owing to the differential arrest rates. Assume that when on parole, criminals face a constant probability $\gamma > \lambda$ of arrest. The higher chance of arrest is due to the extensive investigative powers of parole officers. Since crime is riskier when on parole, it might be that the closer monitoring deters some criminals. Moreover, as I demonstrate below, the length of horizon matters. The longer is the horizon, the more likely is a criminal of a given type likely to violate his parole. The population partitions into $P + 2$ groups. The "λ-deterred", who never commit crimes (non-criminals), and the γ_i – deterred for $t = 1...P + 1$. The latter commit crimes while free but while on parole, are deterred by γ starting in the tth period, with the convention that $t = P + 1$ refers to the never deterred. Depending on the parameters of the model, each of these partitions can range from empty to containing the entire population. Henceforth, the notation will treat these agents by class rather than individual identity, so the i subscripts will be suppressed.

Value functions

As before, the non-criminals (the λ – deterred) live a quiet life earning w each period, yielding the value V^{NC} given by equation 1. For the others, the value of crime is given by (2), where the value of arrest now depends on whether and when the criminal violates parole. In order to work out the value of crime for a given agent, one first requires the value of arrest, i.e. of facing a prison term of J followed by P periods of parole, denoted $V_{D_t}^A$ for an agent who is γ_t – deterred. This in turn depends on the value of parole for such an agent. Let $V_{D_t}^{P_1}$ denote the value at the first period of parole for the γ_t – deterred. This is defined recursively from period P.

Consider a γ_t – deterred agent. Assume for a moment that $1 < t < P$. During the final period of parole, this agent will not commit a crime, but instead will receive the payoff p. In the next period, he will be free and return to crime. The value at the final period is then

$$V_{D_t}^{P_P} = p + \delta V_{D_t}^C$$

If $t < P - 1$, in the penultimate period, this agent's value is

$$V_{D_t}^{P_{P-1}} = p + \delta V_{D_t}^{P_P} = (1 + \delta)p + \delta^2 V_{D_t}^C$$

This recursion continues until period t, at which point

$$V_{D_t}^{P_t} = pF_{D_t} + \delta^{P+1-t}V_{D_t}^C$$

where

$$F_{D_t} = \frac{1 - \delta^{P+1-t}}{1 - \delta}$$

One period prior to this, the value of parole is

$$V_{D_t}^{P_{t-1}} = \psi c + \delta(\gamma W_{D_t}^A + (1-\gamma)V_{D_t}^{P_t}) = \psi c + \delta\gamma W_{D_t}^A + \delta(1-\gamma)(pF_{D_t} + \delta^{P+1-t}V_{D_t}^C).$$

The period prior to this, the value is

$$V_{D_t}^{P_{t-2}} = \psi c + \delta(\gamma W_{D_t}^A + (1-\gamma)(\psi c + \delta\gamma W_{D_t}^A + \delta(1-\gamma)(pF_{D_t} + \delta^{P+1-t}V_{D_t}^C)$$
$$= (1 + \delta(1-\gamma))(\psi c + \delta\gamma W_{D_t}^A) + \delta^2(1-\gamma)^2(pF_{D_t} + \delta^{P+1-t}V_{D_t}^C)$$

Continuing on in this manner leads to

(4)
$$V_{D_t}^{P_1} = Q_{D_t}(\psi c + \delta\gamma W_{D_t}^A) + \delta^{t-1}(1-\gamma)^{t-1}(pF_{D_t} + \delta^{P+1-t}V_{D_t}^C).$$

where

$$Q_{D_t} = \frac{1 - \delta^{t-1}(1-\gamma)^{t-1}}{1 - \delta(1-\gamma)}.$$

This general form nests the all of the value functions for the various levels of $\gamma_t -$ deterrence. For example, when $t = P + 1$, so the agent is never deterred,

$$Q_{D_{P+1}} = \frac{1 - \delta^P(1-\gamma)^P}{1 - \delta(1-\gamma)}$$

and

$$F_{D_{P+1}} = 0.$$

Thus,

$$V_{D_{P+1}}^{P_1} = Q_{D_{P+1}}(\psi c + \delta\gamma W_{D_{P+1}}^A) + \delta^P(1-\gamma)^P V_{D_t}^C$$

Similarly, for the always $\gamma -$ deterred, $Q_{D_1} = 0$, and

$$F_{D_1} = \frac{1 - \delta^P}{1 - \delta},$$

and thus

$$V_{D_t}^{P_1} = (pF_{D_1} + \delta^P V_{D_t}^C).$$

With these value functions, the value at arrest can be found where

$$V_{D_t}^A = K + \delta^J V_{D_t}^{P_1}$$

where

$$K = \frac{j(1-\delta^J)}{1-\delta}.$$

Substituting equation 4, implies

$$(5) \quad V_{D_t}^A = \frac{\delta^J Q_{D_t} \psi c + K + \delta^{J+t-1}(1-\gamma)^{t-1}(pF_{D_t} + \delta^{P+1-t}V_{D_t}^C)}{1-\delta^{J+1}\gamma Q_{D_t}}$$

Remark. Notice that as the length of parole increases, the value of returning to freedom in equation 4 and 5 becomes heavily discounted. This is why longer periods of parole lead to recidivism: the high value of freedom at some distant point is outweighed for some agents by the short-term payoff from crime.

To see this more formally, consider the difference between the value at the start of parole for any two types, t and s, which can be written:

$$V_{D_t}^{P_1} - V_{D_s}^{P_1} = Q_{D_t}(\psi c + \delta\gamma V_{D_t}^A) + \delta^{t-1}(1-\gamma)^{t-1}pF_{D_t} - [Q_{D_s}(\psi c + \delta\gamma V_{D_s}^A) + \delta^{s-1}(1-\gamma)^{s-1}pF_{D_s}]$$

$$+ \delta^P\left((1-\gamma)^{t-1}V_{D_t}^C - (1-\gamma)^{s-1}V_{D_s}^C\right)$$

Likewise, when P grows large, the value of freedom disappears from $V_{D_t}^A$. Thus as P grows large, the difference between these values depends only on the difference between the values for time spent in some form of incarceration. The incentive effect of the end of parole disappears, and the only consideration in the choice to re-offend is the relative payoff between fleeting enjoyment of the fruits of crime and a renewed term in penitentiary and more secure time on parole. Freedom becomes irrelevant.

Finally the value of crime can be found using equation 2:

$$V_{D_t}^C = \frac{(1-\delta^{J+1}Q_{D_t}(\gamma-\lambda))\psi c + \delta\gamma K + \lambda\delta^{J+t}(1-\gamma)^{t-1}pF_{D_t}}{(1-\delta^{J+1}\gamma Q_{D_t})(1-\delta(1-\lambda)) - \delta^{J+P+1}\lambda(1-\gamma)^{t-1}}$$

for $t = \{1,.., P+1\}$.

Thus,

$$V_{D_1}^C = \frac{\psi c + \delta\gamma K + \lambda\delta^{J+1}pF_{D_1}}{(1-\delta(1-\lambda)) - \delta^{J+P+1}\lambda}$$

and

$$V_{D_{P+1}}^C = \frac{(1-\delta^{J+1}Q_{D_{P+1}}(\gamma-\lambda))\psi c + \delta\gamma K}{(1-\delta^{J+1}\gamma Q_{D_t})(1-\delta(1-\lambda)) - \delta^{J+P+1}\lambda(1-\gamma)^P}.$$

These value functions are extremely nonlinear in the parameters of interest, J and P. For this reason, to proceed further, it is convenient to solve the model numerically. Some results from simulations are detailed below.

The marginal criminal

In the equilibrium, higher agent types are more likely to violate parole for longer periods of time. To see why this is so, consider an agent considering violating parole in period P_t. The benefit is the short-term gain from crime; the cost is the greater chance of arrest. Since the value of arrest is higher for higher types, this cost is lower. Further, the gain for cheating is higher for higher types. Together, these conditions imply that if type $\psi*$ will violate in period P_t, so will all other agents of type $\psi > \psi*$.

Thus the equilibrium partition involves disjoint (perhaps empty) intervals of $\psi-$ coefficients, with the lowest interval containing the non-criminals, the next higher the always $\gamma-$ deterred, then those deterred in the second period, and continuing to the never deterred. An example of one such equilibrium is depicted below.

The easiest case to study analytically is that in which the marginal criminal is γ_1- deterred, as the value functions take a particularly simple form. For this case, the type of the marginal criminal solves $V^C_{D_1} = V^{NC}$, where the latter is found as in equation 1. The type of the marginal criminal is then[4]

$$\psi^1 = \frac{w}{c} + \frac{\lambda\delta[w - j - (w - p)\delta^{J+P} - (p - j)\delta^J]}{c(1-\delta)}$$

The type of the marginal criminal increases with an increase in either J or P. However, an additional period in jail increases the marginal type by more than an additional period on parole:

(6)
$$\frac{\partial\psi^1}{\partial J} - \frac{\partial\psi^1}{\partial P} = \frac{\lambda\delta}{c(1-\delta)}\left[(j - p)\delta^J \ln\delta\right] > 0$$

Thus parole has a *deterrence effect* on free citizens, but it is, not surprisingly, less than that of additional periods in prison. This is due to paroles higher payoff. Notice that this difference goes to zero as criminals become more impatient and as the number of periods in jail increases.

3. The Social Costs of Crime

For our rational criminals, crime pays, at least in expectation, even considering the probability of incarceration. For society crime imposes two sorts of costs: the direct harm from crime and the cost of crime prevention. The latter includes the cost of police, courts, prisons and parole. To focus attention on sentence length, I will assume that the probability of arrest and conviction is fixed and policing is costless. The most obvious costs of statutory release is the potential for further criminal activity, and of course the cost of parole officers. The latter is quite clearly offset by the principal benefits of parole, reducing the size of the prison population. In fact, the cost of keeping a person in jail compares favourably with that

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[4] When $p=j$, this reduces to the same condition as in section 2.1

of an Ivy League education, while the direct cost of parole officers and even halfway houses are quite a bit smaller[5].

I am interested in the steady state social costs of statutory release. I assume that the population is sufficiently large and parameters are sufficiently stable that the population shares in each state are constant. In fact, I will distinguish between $J + P + 1$ states, one for each year of prison and parole, and one for freedom. Each of these states imposes costs on society: when free, criminals commit crimes with a low chance of being caught. Despite the higher chance of arrest, some criminals on parole commit crime, and all parolees impose direct costs on taxpayers. Finally, incarceration prevents crime, but is extremely costly, particularly when prison populations rise.

The cost of prison will be ignored in what follows. This is to focus on the question of whether and when statutory release is justified. It clearly limits the scope of the research, a limitation I hope to remove in future work. The key assumption is that very long prison sentences are too costly, so even in the absence of parole, offenders would eventually be released. These costs can include the direct costs of prisons, the cost of allowing criminals time to cement their ties to a "criminal lifestyle" during prison, deterioration of (non-criminal) human capital, externalities imposed on the family of long incarcerated offenders, and perhaps most significantly, the constraint of differential deterrence.

To work out the social costs of a particular choice of P and J one needs to uncover the number of criminals in each of four states: at large (committing crime), in jail (with its attendant cost), on parole and violating (committing crime), and on parole and not committing crime. Because of the Markov arrest probabilities, the system forms a simple ergodic Markov chain[6]. The transition matrix for a γ_2 – deterred parole violator when $J = P = 2$ is illustrated in figure 1.

Figure 1.

	Free	Jail 1	Jail 2	Parole 1	Parole 2
Free	1-λ	λ	0	0	0
Jail 1	0	0	1	0	0
Jail 2	0	0	0	1	0
Parole 1	0	γ	0	0	1-γ
Parole 2	1	0	0	0	0

For the γ_1 – deterred criminals it as if $\gamma = 0$, and the process on parole looks like that in prison. For the never deterred, the final row has $(1-\gamma)$ in the first column, and γ in the second. For non-criminals, all the rows contain one 1 and four zeros.

From these it is a straightforward process to determine the share of time spent in each of the four interesting states, Free, Jail, Parole Not deterred and Parole Deterred. Denote the share

[5] A typical Federal parole officer's caseload varies from 10 to 25 cases.
[6] Every state can be reached from all others in finite time.

of time spent by class $i \in \{NC, D_t\}$ in state $j \in \{F, J, PN, PD\}$ as ω_i^j. For non-criminals all time is spent Free, so $\omega_{NC}^F = 1$ and $\omega_{NC}^J = \omega_{NC}^P = 0$. For the γ_t – deterred

$$\omega_{D_t}^F = \frac{(1 - \gamma S_t)}{L_t}$$

$$\omega_{D_t}^J = \frac{J\lambda}{L_t}$$

$$\omega_{D_t}^{P_N} = \frac{\gamma S_t}{L_t}$$

$$\omega_{D_t}^{P_D} = \frac{\lambda(1-\gamma)^{t-1}(P+1-t)}{L_t}$$

where

$$S_t = \tfrac{1}{\gamma}(1 - (1-\gamma)^{t-1}$$

and

$$L_t = 1 - \gamma S_t + \lambda(J + S_t + (1-\gamma)^{t-1}(P+1-t)$$

It is straightforward to demonstrate that for any criminal, increasing either J or P reduces the time spent in crime. However, $\omega_{D_t}^{P_N}$ also increases in P for the reason remarked upon above: as parole terms lengthen the incentive for not committing crimes induced by a return to freedom is weakened, increasing recidivism during parole.

The *incapacitation effect* of incarceration refers to "keeping criminals off the street". All criminals are incapacitated during in states periods in states "jail" and "parole deterred"; none are in states "free" and "parole not deterred. Given the ordering of types, and the distribution function F, the steady state measure in each state can be simply computed. For example, if there were only positive measures of non-criminals and two values of t, the always deterred and the never deterred, the incapacitation effect would be

$$M^I = [1 - F(\psi^2)]\omega_{D_{P+1}}^J + [F(\psi^2) - F(\psi^1)](\omega_{D_1}^J + \omega_{D_1}^P),$$

where ψ^1 is the critical value for type D_1 and ψ^2 the critical value for type D_{P+1}. In the same circumstances, the number of active criminals would be

$$M^C = [1 - F(\psi^2)](\omega_{D_{P+1}}^F + \omega_{D_{P+1}}^P) + [F(\psi^2) - F(\psi^1)]\omega_{D_1}^F.$$

Increasing the length of sentences, either through J or P, increases both the deterrence and incapacitation effects. The exact trade off between the two policy variables depends on the specific parameters. However, as demonstrated by equation (**), as jail terms become longer, the deterrence effects become more similar, so the principal difference is in incapacitation. The next section begins to explore these issues numerically.

4. Numerical simulations

Though in principal the model could be solved analytically, it is fairly intractable given the high degree of non-linearity in the variables of interest. As a first pass I will describe some

results from a very simple numerical solution. Parole and jail both cost real resources, so the optimal sentence length and its division between jail and parole depends on the cost of each. As noted above, I will assume that the cost of jail is zero and treat the length of term in jail as a parameter. This is to address the question of interest, namely when if ever is statutory release reasonable.

As a first approximation, it seems sensible to assume that the *harm* from crime is linear in the number of criminals. I will also assume that the marginal cost parole is increasing in the numbers in those states, and in particular quadratic. The basic notion is that monitoring parolees costs more at the margin as their numbers increase, but other interpretations are possible. A simple "social welfare function" that incorporates these assumptions is given by

$$W = HM^C - k(\sum_{t=1..P+1}\omega_{D_t}^P)^2$$

where $H < 0$ is the harm from each instance of crime.

The parameters in Table 2 below are assumed to hold in the examples studied. In addition, I have assumed that types are distributed uniformly on $[0,1]$.

Table 2

w	c	p	j	δ	λ	γ	H	k	
0.5	1	0.4	0.2	0.95	0.05	0.2	-10	10	

Many "experiments" are possible given this set of parameters. I have explored the possibilities somewhat, and most of the results are quite as expected. As punishments increase, the measure of criminals falls, and more time is spend in some form of incapacitation. Also, higher types enjoy higher payoffs for all parameter settings. More extensive and systematic results await further research.

The main result

The result of interest is how the optimal length of parole varies with the jail term. To explore this, I computed numerically the value of crime for each possible t for various jail terms J. The results under the parameters in Table 2 are given below:

J	P*
0	0
1	4
2	11
3	12

When $J = 0$, the social cost minimizing level of parole term is also zero: time on parole is insufficient deterrent for crime, despite the higher arrest rate, and spending money on monitoring is inefficient. It is simply better to ignore the laws. (Note that for the parameters above, without punishment the critical type is 0.5, so every second person is a criminal.)

As the term in jail increases statutory release becomes optimal. In fact, as jail terms become harsher, it makes sense to use parole as a low cost way to extend incapacitation. Infinite

parole is not optimal for two reasons: the increasing marginal cost of parole and the fact that increasing parole leads to greater recidivism.[7] To get a sense of the latter effect, consider the type distribution for $J = 2$ for various lengths of parole.

When $P = 3$, for example types are distributed by

Type t	ψ-min	ψ-max
NC	0.0	0.55
D_1	0.56	0.57
D_2	0.58	0.61
D_3	0.62	0.67
D_4	0.68	1

With a relatively light combined punishment, there is little deterrence (recall that without punishment 0.5 of the agents would be criminals). When the length of parole is increased, deterrence increases also. For 8 periods of parole, the type distribution becomes

Type t	ψ-min	ψ-max
NC	0.0	0.57
D_1	-	-
D_2	-	-
D_3	0.57	0.58
D_4	0.59	0.6
D_5	0.61	0.63
D_6	0.64	0.67
D_7	0.68	0.74
D_8	0.75	0.86
D_9	0.87	1

Thus with eight periods of parole there are no types who are always deterred or even deterred in the second period. The total number of criminals has been reduced only modestly with the increased length of parole.

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[7] This result does not come solely from the increasing marginal cost of parole, which I assumed for the purposed of the numerical simulations.

Finally, at the optimal length of parole, $P* = 11$, recidivism has increased further.

Type t	ψ-min	ψ-max
NC	0.0	0.58
D_1	-	-
D_2	-	-
D_3	-	-
D_4	-	-
D_5	0.58	0.59
D_6	0.60	0.61
D_7	0.62	0.63
D_8	0.65	0.66
D_9	0.70	0.71
D_{10}	0.77	0.78
D_{11}	0.79	0.93
D_{12}	.94	1

While few people choose to violate parole in every state, no one remains deterred in the first four periods of parole.

This effect is also visible in the share of total time spent by agents violating parole. When $P = 3$, around 6% of all available time is used by parole violators. For $P = 8$, this share rises to almost 15%. When $P = 11$, over 19% of all time is spent in parole violations.

5. Conclusion

As parole lengths increase, the temptation to return to crime while on parole increases. This means that for short periods of parole there may be no violators. Because of this the incapacitation effect is the same for parole as prison. Furthermore, with long sentences the deterrent effect of parole and prison also become similar. This suggests that relatively short periods of statutory release are likely socially efficient.

The model is still in need of work. In particular, analytic solutions might be available. The most important extension will jointly solve for optimal sentences as the portion spent on parole. In addition the model could be extended to include rehabilitation and regular parole release.

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