Dynamics of Networks If Everyone Strives for Structural Holes[1]

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Dynamics of Networks If Everyone Strives for Structural Holes

Abstract

When entrepreneurs enter "structural holes" in information networks, they can make a profit by exploiting the access and control benefits that these provide. That is Ronald Burt's 13-year-old argument. Ever since, evidence for the suggested advantages from the occupancy of strategic network positions has steadily accumulated. What has not been shown, however, is whether those who strive for such structural advantages can actually obtain and maintain them. We ask what would happen if people followed Burt's advice and indeed entered structural holes by manipulating their local networks in pursuit of access and control benefits. Using an explicit model of network entrepreneurship, we characterize the networks that are obtained after beneficial links have been added and costly ones removed, and we assess the returns that network entrepreneurs earn on their social capital investments in these networks. An important class of stable networks turns out to be "balanced complete bipartite networks." In these networks, benefits are evenly distributed, so no one has a structural advantage. This result sharply contrasts with Burt's typical example of what a network may look like after entrepreneurial activity, in which the majority of access and control benefits accrue to only one individual.

Introduction

Ideas about how positions in social networks are of use to people occupying them are abundant in sociology. The structure of one's social environment has been shown to matter in a number of ways. Balanced triads generate less cognitive inconsistency (Heider 1946; Krackhardt 1999). Job search through weak ties is more successful (Granovetter 1995 [1974]). Non-excludable trading parties have larger profit margins (Cook and Emerson 1978; Willer 1999). Dense structures and closure in networks facilitate trust (Coleman 1988; Raub and Weesie 1990; Buskens 2002; Burt 2005a). And, ties between otherwise unconnected groups benefit the broker (Burt 1992). The last example will be the focus of this paper.

Those results constitute an important body of sociological regularities, but from an agency perspective, they are only half the story. They enable us to understand consequences of certain network positions, but not how actors manage to reach these positions. If some networks are more beneficial than others, actors can be expected to modify the less beneficial ones to their advantage (Flap 2003, pp. 12–13). By the mere non-atomic nature of networks, such actions will be interdependent and partly collective, and thus trigger non-trivial network evolution. Sociologists have only recently begun to explore this agency component of social network analysis in more detail (e.g., Doreian and Stokman 1997). In this paper, we will further develop theoretical insight into which networks can be expected to emerge when actors purposively choose their relationships. In particular, we develop a model in which actors strive for structural holes.

When Ronald Burt (1992) launched his idea of structural holes, he went beyond structuralism. He did not assume that actors would simply reap the fruits from structural advantages that they happen to have over others. He suggested the possibility that entrepreneurs, just as they can strategically put financial and human resources to work, exploit social resources and turn them into a profit: “You enter the structural hole between two players to broker the relationship between them” (Burt 1992, p. 34). Burt even went so far as to argue that social capital *implies* prior strategic networking: “I will treat motivation and opportunity as one and the same...a network rich in entrepreneurial opportunity surrounds a player motivated to be entrepreneurial. At the other extreme, a player innocent of entrepreneurial motive lives in a network devoid of entrepreneurial opportunity” (1992, p. 36). Burt’s “structural entrepreneur personality index” (2005a, p. 34) is an attempt to quantify this inclination to exploit social resources.

Nevertheless, the agency component in Burt’s argument was never as fully developed as the structure component. Burt proposed a precise measure of structural advantage, the “constraint” formula (1992, p. 54), but the network dynamics he sketches in his book are instructions on how to unilaterally reduce one’s score on the constraint measure; he thereby neglects the interdependence between actors and possible cascades of subsequent network adaptations by other actors. Burt does not study what happens if multiple actors try to improve their network positions. Moreover, the book – and the subsequent literature – lacks a description of stable networks in which no entrepreneur can make any further improvement, i.e., networks from which no further social capital can be extracted. Burt deals with some of these issues informally in his forthcoming book (2005a, ch. 5). Although his considerations are very insightful, they lack a strict theoretical deduction. For example, he speculates on the emergence of stable networks if network benefits are extended beyond brokerage to include closure, if these benefits and also the costs of ties are heterogeneous among actors, and if network structure also changes randomly. We think that such speculation is unwarranted.

We will limit ourselves to formalizing network dynamics with a single type of benefit, with homogeneous actors, and without random change. This, as we will see, is already quite challenging. Clearly, these are strong, simplifying assumptions, since strategic networking will be more salient in some settings than in others, and in any particular setting not everybody will be equally interested in occupying strong entrepreneurial positions. Still, our model provides an insightful benchmark that can straightforwardly be modified to accommodate more complex assumptions.

In Burt's typical example of a network after entrepreneurial activity, the majority of benefits are held by a single individual. But it is not obvious why others would not follow this first entrepreneur's example, and what stable network structure would eventually result. Moreover, some economic studies of network dynamics in information and communication settings have identified the "star," in which a single entrepreneur receives all of the profit, as the unique stable network (Jackson and Wolinsky 1996; Bala and Goyal 2000; Goyal and Vega-Redondo 2004). In this paper, we show that the "star" is not stable if everyone tries to minimize their network constraint. By pursuing the following aims, we reach this conclusion.

Our first aim is to model the network entrepreneurship that Burt suggests and to characterize stable networks. This enables us to study the distribution of profits in these stable networks and the possibility that actors can profit from strategic networking. This requires an appropriate choice of a model of network formation, including a stability concept indicating in which networks no actor can further improve his network position.

We model Burtian network dynamics after the actor-based approaches of Myerson (1991), Jackson and Wolinsky (1996), Snijders (1996, 2001, 2005), and Gould (2002). A secondary aim is to propose a stability concept for general use in actor-based models of network formation.

We first review the structural hole argument and the dynamic networks literature. Then we introduce a model of network entrepreneurship, explain our stability concept, and identify two types of stable networks. Using simulation, we show that one of these two types – balanced complete bipartite networks – evolves with a much higher likelihood than the other.

Structural Holes

Structural holes are "disconnections or nonequivalencies between players" and hence "entrepreneurial opportunities for information access, timing, referrals and control" (Burt 1992, p. 2). There exists a structural hole between two players if there is a potential for beneficial information flow between them. The word "disconnections" in the above definition refers to the absence of a tie or path through which the information can flow.

A network rich in structural holes thus contains many exploitable brokering opportunities: "The structural hole is an opportunity to broker the flow of information between people and to control the form of projects that bring together people from opposite sides of the hole" (1997, p. 340). The network entrepreneur recognizes these opportunities and places himself in the hole by initiating ties with both players. Just as the investment banker and the human resource manager generate returns from financial and human capital, so does the network entrepreneur seek rents in information structure.

"When you take the opportunity to be the *tertius*, you are an entrepreneur in the literal sense of the word – a person who generates profit from being between others" (1992, p. 34). Occupying the hole and being essential to the information flow between the two, the entrepreneur can charge a brokering fee.

Burt introduced a formula for quantifying the benefits from spanning structural holes, the "constraint" measure. Entrepreneurial opportunities are considered constrained if there exists a feasible alternative road along which the information you are intending to broker can travel: "Contact *j* constrains your entrepreneurial opportunities to the extent that: (a) you've made a large investment of time and energy to reach *j*, and (b) *j* is surrounded by few structural holes with which you could negotiate to get a favorable return on the investment" (1992, p. 54).[2] The constraint measure c_i captures the extent to which this is the case for each contact *j* of actor *i*:

$$c_i \equiv \sum_{j \neq i} \left(p_{ij} + \sum_{k \neq i, k \neq j} p_{ik} p_{kj} \right)^2 \qquad (1)$$

where p_{ij} is the proportion of time *i* has invested in contact *j*. Burt assumes that an actor distributes his time equally over his contacts: If *i* is connected to *j*, $p_{ij} = 1/d_i$, where d_i is actor *i*'s degree. If *i* and *j* are not connected, $p_{ij} = 0$. c_i lies between 0 and 9/8 (a proof for c_i having this range can be obtained from the authors) and is 0 for isolates. This implies that isolates have the lowest constraint. It seems more plausible, though, that it is better to be connected in some way than not to be connected at all, and we think Burt did not

[2] In recent work (e.g., Burt 2005a), Burt multiplies his index with 100 to compare integer values of the index. We will use the original formulation in this paper. Of course, both formalizations are equivalent.

intend isolates to be least constrained. Therefore, we additionally assume that $c_i = 2$ (greater than 9/8) for isolates.

FIGURE 1 ABOUT HERE

The higher the score on this measure c_i, the more structural opportunities are constrained, and as a result, the lower the network benefits. Consider as an example figure 1. In the network on the left, actor *A* is essential for all information flow. His constraint score is $c_A = 4\ (¼ + 0)^2 = ¼$. In the middle network, *B* and *C* can also communicate directly rather than through *A*. This constrains the relations between *A* and *B* and *A* and *C*. *A*'s constraint score is now $c_A = 2\ (¼ + 0)^2 + 2\ (¼ + ¼\ ½)^2 = 13/32$. In fact, a network without one of the two "redundant" ties with *B* and *C* would be better for *A* (on redundancy, see Burt 1992, p. 51). In the network on the right, *A*'s constraint is lower, namely $c_A = 3\ (⅓ + 0)^2 = ⅓ < 13/32$. Therefore, actor *A* is willing to give up his relation with *C* in the middle network of figure 1 and move to the network on the right.

The constraint formula has been found negatively related to a wide range of objective indicators of success (see Burt 2000, 2002, and 2005a for extensive reviews). Producer profit margins are larger for firms in buyer-supplier networks (Burt et al. 2002; Talmud 1994; Yasuda 1996). Jobs are more desirable (Bian 1994; Leenders and Gabbay 1999; Lin 1999; Lin et al. 2001). Salaries are higher (Burt 1997, 1998; Podolny and Baron 1997; Burt, Hogarth, and Michaud 2000; Mehra et al. 2001; Mizruchi and Sterns 2001). And negative correlations have been found with positive performance evaluations, peer reputations, promotions, and good ideas (Gabbay 1997; Burt 2001, 2004). Given that this evidence indicates a (negative) association between the constraint formula and network benefits, we use constraint as an (reverse) indicator for the utility actors can extract from a network.

Dynamic Networks

What the evidence above does *not* tell us is whether investments in brokerage relations pay off. Burt has suggested, but not shown, that those who remove redundant ties and add non-redundant ones will eventually obtain the returns on these investments. Current network manipulations that lead to an improved network position in the short run may trigger subsequent changes by others that could ultimately make oneself worse off. This temporal interdependence in decision making is not trivial, and its examination requires an explicit model. Recently, some models have been proposed to examine such dynamics. These models can be categorized into two approaches, both of which we review. Earlier reviews of models of network dynamics can be found in, e.g., Weesie and Flap (1990), Doreian and Stokman (1997), Stokman and Doreian (2001), Dutta and Jackson (2003), Breiger et al. (2003), and Jackson (2003). In the first approach, network dynamics are a process in which structure changes as a result of probabilistic processes, while in the second approach, networks evolve as a consequence of tie changes made by purposive actors.[3] Our model is of the latter type.

In the former approach, tie formation is an event, such as an introduction to one's friend's friend or the sharing of information. Ties represent the long-lasting effects of

[3] "E-state structuralism" (Fararo and Skvoretz 1986; Skvoretz et al. 1996) constitutes a hybrid of these two approaches. Networks change as a result of actors' behavior, but this behavior is not network manipulation itself.

these events and will never disappear; from now on you will know your friend's friend or you will be informed, respectively. The probability of the event depends on the current network structure. Two people are more likely to be introduced to each other the more they have both been introduced to other people beforehand, and they may more readily share information if they have done so previously. Watts (1999) and Barabasi and Albert (1999) propose scenarios in which ties are added to an empty network. Watts states that "small worlds" constitute the end result of a process that takes some middle position between two extreme scenarios – one where new ties are established between two individuals only through the introduction by a shared friend, and one where new friendships are made at random. Barabasi and Albert argue that small worlds are created following a "power law": Nodes are more likely to connect with nodes that already have many connections. One example they offer is the establishment of new links between websites. Carley's (1991) constructualist model of group formation is also based on a probabilistic notion of social interaction, even though individuals' preferential attachment is implicit. Individuals share information. Those who have more knowledge in common are assumed to be more likely to interact. Since interaction leads to even more shared knowledge, and so on, the equilibrium in her model is separate cliques that have fully common knowledge. Robins et al. (2005) give an overview of this line of network dynamics research and provide a rather advanced study in this category themselves. Employing well-known sociological network principles in their model of network formation, such as the tendency toward triadic closure, they find that small-world networks might emerge, but a wide range of other types of structures are also possible.

The second approach to network formation is one where actors try to optimize or improve their network position, assuming they obtain a certain utility from occupying that position. Clearly, a model of this type better fits the network formation process that we have set out to explicate than a probabilistic model. Burt's network entrepreneurs are precisely such optimizing actors, with brokerage benefits as network-derived utility.

A classic study that takes this approach is Cartwright and Harary (1956). They show that Heider's (1946) balancing principles for friendship formation would always lead to two entirely segregated groups. Lacking a specification of the dynamics through which these equilibria would be obtained, their study was elaborated by others who proposed various models of network formation (cf. Doreian and Mrvar 1996; Hummon and Doreian 2003; Wang and Thorngate 2003). In each of these models, an actor is randomly selected to add or delete a tie that involves him. Macy et al. (2004) model Heiderian balancing processes as a dynamic network process in which opinions of actors and ties between actors change simultaneously. They find that segregation is highest with an intermediate number of opinion dimensions.

Another classic example of actors' consciously trying to improve their network position is Schelling's model of residential segregation (1969, 1971). Actors move if they can find a vacant place whose location they prefer over that of their present residence. Bonacich (2004) analyzes the dynamics of exchange networks with a Schelling-like checkerboard model. What makes this model less suitable for our purposes is that the structure of the social environment is predefined (i.e., on a checkerboard) and that actors cannot choose relations freely, but can only search for empty spots on the checkerboard.

The models that allow for free choice of ties and various forms of calculative behavior of actors are similar to the model we will propose subsequently; we will review them in a bit more detail. There exist two subclasses of models of this kind. First are equilibrium models in which actors simultaneously propose ties to other actors. They can select any combination of actors they wish. Actors derive utility from their positions in the resulting network. Stable networks are those that are induced by Nash equilibria, combinations of strategies that are each a best response to the strategies of the other actors. Gould (2002) proposed such a model to explain the emergence of status hierarchies. His model is special in the sense that actors are allowed to propose a *level* of attachment to all other actors. Utility depends on this level of attachment, the status of the actors that one connects to, and the extent to which attachment is reciprocated. A prominent instance of this type of model in economics is Bala and Goyal (2000), in which actors can unilaterally link to others and derive utility from the number of other actors they are directly or indirectly linked to.

In both Gould's model and that of Bala and Goyal, any initiative toward other actors has direct consequences in terms of the utility these other actors derive from their choices. However, we will assume that ties are formed only if both actors want to connect. This assumption is based on the idea that relevant information can be exchanged only if both actors agree to come together and talk. No one can force a tie upon anyone else. Myerson (1991, p. 448) proposed a model that uses this assumption. All actors simultaneously propose ties. Ties that are mutually proposed are formed, and actors derive utility from the resulting network. This type of model, which we will call the Gould-Myerson (GM) model, is attractive but has one important drawback: It shortcuts the network dynamics. Instead of representing network evolution as a continuous process in which one actor can react to changes by other actors elsewhere in the network, it assumes that all actors make their decisions simultaneously and that these decisions are binding. There are often numerous equilibria, and the lack of a specification of the network evolution process then leaves one unable to identify the network that is most likely to evolve. From an evolutionary viewpoint, many of these equilibria can hardly be considered stable networks. We will show later that we can identify a subset of these equilibria that have more appealing stability properties. These stability properties can be explained by describing adaptive processes through which actors solve coordination problems and reach specific equilibrium networks. The second subclass of actor-oriented models of network formation specifies such network evolution processes as well.

In sociology, Snijders (1996, 2001) develops methods for the statistical analysis of longitudinal network data. These models allow one to test hypotheses on the specific utility function that actors optimize when making changes to the network. Snijders's statistical model has been applied to, for example, the emergence and stability of friendship networks (Zeggelink et al. 1996; Van der Bunt et al. 1999; Whitmeyer 2002). The model assumes that actors are randomly selected one by one and given the opportunity to remove or add one tie. Actors then investigate whether some change increases their utility. If so, they make that change; if not, they leave the network as it is. Originally, the model assumed that tie formation was unilateral, and thus each actor could force a tie upon some other actor. These ties were directed; an example would be a phone call. In the latest editions, one can also specify a model in which ties are undirected and formed only if both actors want that tie.[4] This particular scenario is implied by Jackson and Wolinsky's (1996) concept of "pairwise stability." A network is pairwise stable if no pair of actors wishes to connect and no single actor wishes to remove a tie. Although sequential tie formation under mutual consent reaches stasis only once a pairwise stable network has formed, Jackson and Wolinsky do not explicitly mention this model of network formation in their original article. In this type of model, which we will call the Snijders-Jackson-Wolinsky (SJW) model, actors are myopic; they do not consider what other actors will change after they make a change themselves. In the GM model, in contrast, the actors consider all other actors' possible actions in a setting in which everyone chooses all desired ties simultaneously and only once. We will show later on that the SJW and GM models can be unified despite these differences.

A Model of Network Entrepreneurship

Burt's (1992) network entrepreneurs are actors who change structure in pursuit of greater network-derived benefits. A model of network entrepreneurship should therefore be of the second type. Moreover, we can use the explicit formula that Burt used to quantify an actor's lack of network benefits, the aforementioned constraint measure. Through alterations of their network positions, actors maximize utility, which should be decreasing with the score of that individual on the constraint measure. The literature provides two candidate models for such network entrepreneurship, the GM model and the SJW model. As we will show now, these models can be unified.

[4] To make statistical estimation possible, Snijders adds a random utility component to the utility function of actors. Note that including noise in actor-oriented models of network formation moves them one, probably realistic, step toward the purely probabilistic models.

We first introduce the necessary notation. Let $n \geq 2$ be the number of actors, $N = \{1,2,...,n\}$ the set of actors, and X the n times n adjacency matrix with x_{ij} indicating the tie strength of the connection from i to j. $d_i = \sum_j x_{ij}$ is actor i's degree.

In the GM model, each actor proposes whom he wants to be connected to. $s_i \in \{0,1\}^{(n-1)}$ is a pure strategy of actor i. The utility function $u_i(s)$ assigns a numerical value to each strategy profile s. Gould considers Nash equilibrium as stability concept.

Definition 1. A strategy profile s^* is a *Nash* equilibrium if $u_i(s^*) \geq u_i(s_i, s^*_{-i})$ for all i and s_i, where s^*_{-i} is the set of all strategy vectors in s^* excluding the one of i.

Moreover, Burt's ties are non-reflexive, so $x_{ii} = 0$ for all i, they are undirected, so $x_{ij} = x_{ji}$ for all i and j, and they are unvalued, so $x_{ij} \in \{0, 1\}$. Let $g^N = \{ij \mid i < j \in N\}$ denote the complete network of all non-reflexive, undirected, and unvalued connections ij. Let $g + ij$ denote network g with the tie ij added to it and $g - ij$ denotes network g with tie ij removed. We say that the strategy profile s *induces* the network $g \subseteq g^N$ if $ij \in g \Leftrightarrow s_{ij} = s_{ji} = 1$; i.e., only ties that are proposed by both actors are part of the network. The network constraint formula depends only on the network formed and not on the ties proposed. Therefore, we assume that proposing ties is costless. This implies that the utility function $u_i(s)$ is the same for combinations of strategies that induce the same network. Formally, the utility function has the property that $u_i(s') = u_i(s'')$ if s' and s'' induce the same network g. With some abuse of notation, we can also write $u_i(g)$ as the utility of a certain network g, given that it does not matter precisely what strategies induce this network.

Specifying the model for our purposes, actor *i*'s utility is a decreasing function of the constraint measure c_i as indicated above, e.g. $u_i(g) = - c_i(g)$.[5]

Now, we define as a first stability concept the Nash network:

Definition 2. A network g^* is a *Nash* network if some s^* inducing g^* is a Nash equilibrium.

Nash network is a rather weak stability concept for undirected networks, since many networks that can hardly be considered stable are included. The problem is that if one actor does not propose a tie to another, the second actor has no incentive to propose a tie to the first, because a tie is formed only if it is proposed by both actors at the same time. No one can increase his utility through any proposal if no one else is proposing ties, which makes "nobody proposing any tie" a Nash equilibrium. The network induced by this set of strategies is the empty network, and this is a Nash network. Given our utility function, every *pair* of actors wants to initiate the first tie in the empty network, because isolates have the lowest possible utility. Thus, many Nash equilibria are due to trivial coordination problems.

In the SJW model, such coordination problems do not exist. This model asks what an actor would change given the current status of the network if he were offered that possibility. An actor is allowed to delete a tie or add a tie with permission from the new

[5] We make the strong assumption here that reducing network constraint is the only utility argument for all actors and that the utility derived from a lack of constraint is the same for all actors. We thereby neglect other utility arguments related to, e.g., closure (Burt 2005a, ch. 3) and indirect brokerage (Burt 2005b). Neither do we investigate the effects of heterogeneity among actors (see Burt 2005a, ch. 1.4), and in our model, the value of brokerage is not only an effect but a goal as well (cf. Burt 2005a, pp. 240–41). We will consider these issues in more detail in the discussion section.

contact; i.e., it does not make this other actor worse off. Stability is reached if no actor can profitably delete a tie or add an acceptable tie. Jackson and Wolinsky (1996) call this pairwise stability:

Definition 3. A network g is *pairwise stable* if both

(i) $\forall ij \in g,\ \ u_i(g) \geq u_i(g-ij) \wedge u_j(g) \geq u_j(g-ij)$ and

(ii) $\forall ij \notin g,\ \ \left(u_i(g+ij) > u_i(g) \Rightarrow u_j(g+ij) < u_j(g)\right) \wedge \left(u_j(g+ij) > u_j(g) \Rightarrow u_i(g+ij) < u_i(g)\right)$.

Condition (i) states that no actor wants to sever a tie, and condition (ii) states that no pair of actors wishes to add a tie. On the one hand, pairwise stability has the advantage that pairs of actors can add ties if they both want to have a tie, which seems a clear indication that a network is unstable, since networks can be Nash networks despite this possibility. On the other hand, the disadvantage of pairwise stability is that it does not consider the simultaneous removal of multiple ties. If an actor would profit from removing two ties, this network could never be a Nash network in the GM game, because not proposing these two ties would have been a better strategy. Networks can be pairwise stable despite profitable deviations that involve multiple ties, even though such deviations arguably make a network less stable. Thus, Nash networks in the GM model and pairwise stable networks in the SJW model both contain a subset of networks that could be considered unstable.

That makes the intersection of these two sets of networks a candidate for an appropriate set of stable networks. Goyal and Joshi (2004) introduced the concept of pairwise Nash in the GM model, which corresponds to the intersection of Nash networks and pairwise stable networks (see also Cálvo-Armengol 2004). Gilles and Sarangi (2004) call the same stability concept *strong pairwise stability* using the SJW model. In words, pairwise Nash networks are networks in which (i) no actor wants to delete a subset of his ties and (ii) no pair of actors wants to add a tie between them. But clearly, there is an asymmetry here: An actor can delete all his ties but add only one tie – if that tie does not make the other actor worse off.

Even more problematic is the assumption in that actors do not simultaneously add and delete ties. For example, pairwise Nash implies that an actor does not contemplate improving his network position by replacing one contact with another. This, however, seems a rather straightforward change in a network. To resolve the problems of asymmetry and non-simultaneity in the deletion and addition of ties, we introduce our own stability concept, "unilateral stability." In our definition of this concept we use the concept of "unilateral obtainability":

Definition 4. A network $g' \subseteq g^N$ is *unilaterally obtainable* from g by i through $S \subseteq N \setminus \{i\}$ if

(i) $\forall jk \in g' \setminus (g' \cap g), (j = i \vee k = i) \wedge \{j,k\} \subseteq S + i$

(ii) $\forall jk \in g \setminus (g' \cap g), (j = i \vee k = i)$

One network is unilaterally obtainable from another by a proposing actor i and through a subgroup S, if each tie that is added or deleted involves actor i and if each tie that is added involves also a member of S. That is, a network is unilaterally obtainable from another network if we can pick one actor from the initial network, give him a new set of ties and then we obtain the new network. Now, we can define our core stability concept.

Definition 5. A network $g \subseteq g^N$ is *unilaterally stable* if $\forall i$, $\forall S \subseteq N \setminus \{i\}$, $\forall g' \subseteq g^N$ unilaterally obtainable from g by i through S, $u_i(g') > u_i(g) \Rightarrow u_j(g') < u_j(g)$ for some $j \in S$.

In words, a network is called unilaterally stable if no actor *i* can change the ties that he is involved in himself such that two conditions are fulfilled: (i) *i* is strictly better off; (2) none of the actors in *S* to whom actor *i* proposes a new tie is worse off than in the original network. Unilateral stability is defined here in terms of the SJW model. It can also be formulated as a refinement of Nash equilibrium in the GM model, in which case we refer to "initiative proof" Nash equilibria (see AUTHOR 2005). Hence, the concept of unilateral stability can be used in both models. From the definitions, it is clear that all unilaterally stable networks are pairwise Nash, and all pairwise Nash networks are pairwise stable.

Networks of Structural Entrepreneurs

We now identify networks to which no entrepreneur can profitably make any further change. Our first result states that two actors will always connect if they have no shared contacts.

Theorem 1. Adding a tie without creating closed triads is always beneficial for both actors involved in the new tie.

Proof. For purposes of legibility, we moved all proofs to the Appendix.

Theorem 1 establishes the unconditional benefits from brokerage. If an actor adds a tie without creating a closed triad, then this actor will be on the shortest path between the new contact and all the contacts the actor already had. And vice versa, the new partner comes to mediate the information the focal actor receives and passes this along to his old contacts. Scores on the constraint measures of both actors drop. The added value of an additional tie decreases as more ties are added because an actor has to distribute his time among more neighbors and can thus broker less information per pair of neighbors, but this marginal utility never becomes zero.

As we will see below, the reverse of theorem 1 is not true. Sometimes actors want to add ties that cause closed triads, and networks with triads can even be pairwise stable. We did not encounter any unilaterally stable network with closed triads, but we could not prove that they do not exist.

Corollary 1. The shortest path between any pair of actors in a pairwise stable network has length less than or equal to 2.

The shortest path between two actors can be of length 2 or less only if both actors are directly connected or can reach each other through a broker. If neither condition obtains for some pair of actors, then these actors can add a tie without creating a closed triad, which is profitable by theorem 1. Note that since pairwise stability is a weaker stability concept than unilateral stability, corollary 1 also holds for unilaterally stable networks.

Corollary 2. A network of disconnected parts cannot be pairwise stable.

A network of disconnected parts contains many brokerage opportunities. Every entrepreneur wants to add a tie to someone in another part because that tie will never create a closed triad. Such networks can therefore not be pairwise stable (and, consequently, not unilaterally stable, either). The following set of definitions describes a family of networks that includes important stable networks.

Definition 6.

- An *m-partite* network is a network in which the actors can be divided into m groups such that there are no ties within these groups.
- A *complete m-partite* network is an m-partite network in which all the possible ties between the actors in the m groups exist (see Wasserman and Faust 1994, p. 120).
- A *balanced m-partite network* is an m-partite network such that the difference between the number of actors in the largest group and the number of actors in the smallest group is at most 1.
- If $m = 2$, m-partite networks are called bipartite networks.
- If $m > 2$, m-partite networks are called multipartite networks.

Now we can formulate the following corollary of theorem 1.

Corollary 3. Pairwise stable networks that are bipartite networks are necessarily complete bipartite networks (otherwise some actors are at a distance greater than 2 and one can add ties without creating closed triads).

Let us take a closer look at complete bipartite networks. The network on the left in figure 1 is an example of a complete bipartite network. Actors B, C, D, and E form a first group, actor A a second. There are no within-group ties: B, C, D, and E are not directly connected to one another. All between-group ties are present: A is tied to B, C, D, and E. We will call complete bipartite networks in which one group contains only one actor – such as the example network – "stars."

Definition 7. A *star* is a complete bipartite network in which one of the groups consists of only one actor**.**

Corollary 3 says that in order for a bipartite network to be stable, it will have to be complete. Otherwise there would be a brokerage opportunity. As the reader can verify, eliminating any tie in the example network makes both actors involved in that tie worse off: Actor A would have constraint ⅓ instead of ¼, and the other actor would become an isolate. The next result states that the example network we just described is *not* pairwise Nash, despite its completeness.

Theorem 2. A complete bipartite network of size *n* is pairwise Nash, unless it is a star with $n > 4$.

Theorem 2 identifies an important class of pairwise Nash networks, and thus also of pairwise stable networks. As soon as both groups in a bipartite network consist of at least two actors and it is complete, this network is pairwise stable. The example network is a star with $n = 5$, and by theorem 2 it is not pairwise stable. The reason is that any two peripheral actors may wish to connect: B and C lower their constraint from 1 to 61/64 if they connect, even though this tie is redundant. This is a property of Burt's constraint formula. In some cases, when the broker has many ties and can therefore spend little time passing information, it is better to establish a direct connection. This is what happens in the example network. B and C connect because A spends too much time with D and E. The middle network in figure 1 evolves. Given the argument for instability of larger stars, it is surprising that in other unbalanced complete bipartite networks actors in the larger groups never have an incentive to connect. If one broker can be too busy, then why cannot two?

Clearly, in no bipartite network does an actor want to remove one or more ties, since the network is void of closed triads. This illustrates the weakness of the concept of Nash network: *All* bipartite networks, incomplete and complete, are Nash networks.

Complete bipartite networks have another nice property, namely that they are efficient in the Pareto sense.

Definition 8. A network is *Pareto-efficient* if there is no other network such that no actor is worse off and at least one actor is better off.

If our actors could cooperate and enforce agreements – which we have assumed they cannot – then they would not be able to leave a Pareto-efficient network. There would always be some actor vetoing a transition. The following theorem tells us that the class of stable networks identified in theorem 2 is Pareto-efficient.

Theorem 3. Complete bipartite networks are Pareto-efficient.

It may be the case that complete bipartite networks are the only efficient networks, but we were not able to prove this conjecture. We did verify that this conjecture holds for networks up to size 8.

We now specify a necessary and sufficient condition under which complete bipartite networks are not only pairwise stable but also unilaterally stable.

Theorem 4. A complete bipartite network is unilaterally stable if and only if it is balanced.

Theorems 3 and 4 together identify a class of networks that are efficient and stable under the strictest stability condition. In a complete bipartite network with a minimal difference in group size (i.e., groups are of equal size for an even number of actors and differ by only one actor in the case of an odd number of actors), no actor can profit from deleting and adding permitted ties or be made better off in any network, whether unilaterally obtainable or non-obtainable, without another actor being made worse off. Figure 2 shows the 10-actor balanced complete bipartite network. We were able to identify a second class of pairwise stable networks.

Theorem 5. All complete multipartite networks are pairwise stable if the groups are of equal size and contain more than one actor.

Figure 2 shows the Octahedron, which meets the theorem 5 requirements. There are three pairs of unconnected actors, each of which constitutes a group. Note that this network is neither unilaterally stable nor pairwise Nash. If he deletes two ties, an actor's constraint drops from 9/16 to 1/2. The network is nevertheless pairwise stable because if an actor is allowed to delete only a single tie, he prefers to keep it. His constraint then increases to 43/72. Because the deletion of two ties makes an actor better off, this network is not a Nash equilibrium of this game using the GM-model. Therefore, this is also an indication that pairwise stability might be a bit too weak a stability concept. Moreover, the Octahedron is extremely inefficient. All actors would fare better in the six-actor balanced complete bipartite network, which gives each actor constraint ⅓. More generally, this class of complete multipartite networks consists of inefficient pairwise stable networks and thus constitutes a set of "social traps."

FIGURE 2 ABOUT HERE

Simulation

We have identified two classes of networks to which structural entrepreneurship gives rise. First, in balanced complete bipartite networks, each pair is connected either directly or at distance 2 via $\frac{1}{2}n$ possible brokers. They contain no redundant ties, but no one has entrepreneurial advantages, since everyone is brokering the same number of only indirectly connected pairs. Also, if Burt's constraint formula correctly quantifies brokerage benefits, they are Pareto-efficient. No alternative network makes one actor better off without making another worse off. Second, in the multipartite networks of theorem 5, all pairs are brokered, and in addition, the majority of pairs are directly connected. These networks contain many redundant ties and are consequently Pareto-inefficient. Every actor would fare better in the balanced complete bipartite network of the same size. Yet these multipartite networks are pairwise stable. No entrepreneur can profitably add or delete a single tie.

On the basis of our analysis, we could expect either type of network to arise in a world in which entrepreneurs pursue access and control benefits by changing ties one by one. Simulating such a world enables us to investigate which of the two types of networks is more likely to emerge. Such simulations also help us identify potential pairwise stable structures that the two classes of networks mentioned above do not cover.

The simulation we built executes the following steps:

1. Start from some network.
2. Randomly select one actor who is allowed to delete a tie or add a tie and can count on the consent of the actor to whom he wants to add a tie.

3. The actor changes the tie that reduces his constraint most in the subsequent network (if more ties lead to the same reduction in constraint, he chooses randomly between the options; if he cannot make a change that strictly reduces his constraint, he does not change anything).
4. Repeat steps 2 and 3 until no actor can profitably delete or, with consent, add a tie.

Networks that are formed after these steps have been executed are necessarily pairwise stable; otherwise they could not have passed step 4. We first ran such simulations starting with each of the 13,597 non-isomorphic networks of sizes 2 through 8. This allowed us to identify all pairwise stable networks for these network sizes, since whenever the initial network was pairwise stable, the simulation ended instantly. For each of these networks, we checked whether they were unilaterally stable as well.

Table 1 shows the number of stable networks by network size and stability concept. Figure 2 displays some of the interesting examples of stable networks. For n = 2, we have the connected pair as the only pairwise stable network. For n = 3, the three-line is the only pairwise stable network. For n = 4, the "Box" is the only pairwise stable network. For n = 5, there are two pairwise stable networks, the Pentagon and the five-actor 2,3-complete bipartite network, which are both unilaterally stable. For n = 6, there are four pairwise stable networks: the 2,4-complete bipartite and the 3,3-complete bipartite as well as the "Bag" (see figure 2) and the Octahedron. The Octahedron is not pairwise Nash, and only the 3,3-complete bipartite network is unilaterally stable. For n = 7, there are three pairwise stable networks, including the 2,5-complete bipartite and the 3,4-complete bipartite. The latter is also unilaterally stable. For n = 8, there are seven pairwise stable networks: the 2,6-complete bipartite, the 3,5-complete bipartite, the 4,4-complete bipartite, the "Twisted Cube" (see figure 2), the complete 4-partite network, and two other networks. The three densest networks, including the balanced complete 4-partite network, are not pairwise Nash. The twisted cube is the second unilaterally stable network for $n = 8$, in addition to the balanced complete bipartite network. Note that the twisted cube is a regular structure in which everyone has three ties and occupies a regularly equivalent (Wasserman and Faust 1994, pp. 473–74) position with all the others.

In addition, we checked which of the more than 12 million non-isomorphic networks of sizes 9 and 10 fulfilled a specific stability condition. In this way, we found 9 pairwise stable networks for $n = 9$ and 14 pairwise stable networks for $n = 10$. The unilaterally stable structures for $n = 10$ are the balanced complete bipartite network and another regular network in which every actor has four ties. Finally, for sizes 11 through 16, we checked whether the networks that resulted from the simulations were unilaterally stable as well. The results are also summarized in table 1. For these network sizes, it is not guaranteed that every pairwise stable network has been found, because not all starting networks were considered.[6] In fact, we know from the analytical results that some networks that we did not find in the simulations are nevertheless pairwise stable. This is why for some network sizes, the lower bound of the number of pairwise stable networks (table 1) exceeds the number of networks found in the simulations (table 2). Still, these results show that the number of pairwise stable networks per network size is very small

[6] Checking all structures for $n = 10$ took about 5 days with our software and computers, which implies that for $n = 11$ it would take about 500 days.

and increases only slowly with network size. There is no network size for which we identified more than two unilaterally stable networks.

TABLE 1 ABOUT HERE

To further investigate the likelihood of emergence of certain pairwise stable networks through an adaptive process, we drew a subsample of networks stratified on density for network size *n* = 9 through 25. We used all networks for densities with a small number of different networks but drew a random sample of connected networks for other densities. We never let a simulation start from a disconnected network, but since disconnected networks cannot be pairwise stable by corollary 2 and since the minimal density of pairwise stable networks seems to be around 0.4, not starting from the sparsest networks is not problematic. We decrease the number of networks per network density for larger network sizes, in order to have comparable numbers of networks per network size.[7] In sum, in our construction of the set of initial networks, we attempt to minimize bias toward networks of a particular density while keeping the set feasibly small (for a complete overview of the sampling, see AUTHOR 2005). For each network, we let the simulation converge to pairwise stability twice. Convergence always occurs, and it does so reasonably fast. For *n* = 25, the maximum number of iterations to reach a pairwise stable network is just above 200.

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[7] Numbers of sampled networks per density are decreased at *n* = 13 and *n* = 17. As a result, there is a drop in sampled networks at *n* = 13 and *n* = 17 and an increase thereafter due to the increasing number of different densities if the network is larger.

Examining the entire range of *n* from size 2 through 25 in table 2, we can make several important observations. The number of pairwise stable networks increases as *n* increases, although not entirely monotonically. Clearly, the number of pairwise stable networks is very small compared with the number of existing networks. The simulation shows that the balanced complete bipartite network is by far the most likely to emerge from the simulation. The less equal the group sizes of a complete bipartite network are, the less likely it emerges in the simulation. Table 2 shows the proportions of simulations from which the most equal and the second most equal complete bipartite networks emerge as pairwise stable networks. These two networks cover about 85% of the resulting networks, except for some cases in which *n* is small (all percentages are taken out of the networks of a particular size). If *n* is odd, the balanced complete bipartite network alone even accounts for 75–80% of the resulting pairwise stable networks. There are no other pairwise stable networks that obtain in a large proportion of simulations except for the Pentagon (19%) and the third pairwise stable network for $n = 7$ (34%).[8] For $n > 8$, no other network occurs in more than 8% of the simulations, and for $n > 16$, in no more than 3%. Strikingly, the other unilaterally stable networks that are not bipartite do not emerge in larger percentages than other pairwise stable networks. The twisted cube, for example, occurs in only 1% of the simulations for $n = 8$. Complete multipartite networks are obtained in only a negligible number of cases.

TABLE 2 ABOUT HERE

[8] This network has two groups of three actors who are all tied to each other as in the 3,3-complete bipartite network, except that two actors (one in each group) are tied not to each other but both to the seventh actor.

The density of pairwise stable networks varies between 0.4 and 0.9. Another question is whether we can predict the structure of the outcome network from the properties of the ingoing network. Because the dynamic process toward pairwise stable networks does not vary much in terms of the outcome networks, we restrict ourselves here to the relationship between density of the ingoing network and the resulting network. Not surprisingly, denser starting networks lead to denser pairwise stable networks. Pairwise stable multipartite networks will thus be somewhat more likely to emerge if we start from very dense networks. Furthermore, one can see from table 2 that the correlation between the two densities increases with network size.

It should be noted that the correlations as well as the proportions presented in table 2 are contingent upon the sample of networks that is used. However, the findings are so strong and the prevalence of the balanced complete bipartite networks is so obvious that we do not really worry about this. One thing we did was weight networks for $n < 9$ by the inverse of the size of the isomorphism group that they represented.[9] The statistics then correspond with a sample of random networks. This left the substantive result – that the balanced complete bipartite network is the strongest attractor – unaffected.

[9] Sizes of isomorphism groups were determined using Nauty 2.2 (see McKay 1990).

Discussion

We have attempted to explicitly model Burt's network entrepreneurship: Optimize relationships in terms of brokerage opportunities, initiate relationships with others who are otherwise unconnected, and resolve relationships if they are redundant in terms of access and control benefits. More specifically, we have assumed that everyone tries to minimize his "network constraint," Burt's measure for brokerage. We then answered the question "What networks will evolve?"

In short, the answer is balanced complete bipartite networks. These networks consist of two groups of similar size with all intergroup ties and no intragroup ties present. Such networks meet the strongest stability criterion, and most simulations ended in such networks. The balanced complete bipartite network strongly contrasts with the networks commonly depicted in the literature as outcomes of entrepreneurial activity. Burt's typical example of what a network looks like after entrepreneurial activity has taken place is that of one actor brokering two dense, otherwise separated groups. Moreover, some economic models of network dynamics in information and communication settings identify the star as the stable network.

The difference between these networks is considerable not only in terms of structure, but also in the distribution of benefits among the entrepreneurs. Burt's single-broker structure and the star are both winner-take-all networks. Balanced complete bipartite networks, by contrast, are egalitarian. They benefit each entrepreneur equally.

Balanced complete bipartite networks have other interesting properties. No one really is a broker. Even though each entrepreneur attempts to occupy a brokering position, in these equilibria, two-step information flow between any two persons travels through at least ½ (n – 1) third parties. Thus, betweenness centrality (Freeman 1979; Wasserman and Faust 1994: 189-191) is not particularly high for any single actor. Moreover, although everybody has a low network constraint, no one has a substantial comparative advantage over someone else. Hence the conclusion: *If everyone wants to be in the center, there is no center*.

The other stable networks that we identified, namely, the symmetric multipartite networks, are also egalitarian. Every actor is equally well off. In addition, these networks constitute social traps because they are inefficient in terms of their network constraint. Especially the even-sized networks that are divided in n/2 groups of size 2 have numerous redundant relationships despite their pairwise stability.

Considering that Burt is often concerned with who has the lowest network constraint compared with the others, rather than how low someone's constraint is in absolute terms, one might wonder what happens to our result if actors try to minimize not their absolute network constraint but instead the relative network constraint compared with those of other actors in the network. Therefore, we analyzed and simulated a model in which utility decreases with relative rather than absolute constraint. The relative constraint of an actor is his absolute constraint divided by the sum of the absolute constraints of the other actors. Results were very similar. Also under this utility function, the balanced complete bipartite networks are the dominant networks, in the sense that they are pairwise stable and emerge in the majority of simulations.

Another property of balanced complete bipartite networks is that for $n > 4$ they are not stars. Economists have also recently modeled network dynamics as a process in which actors maximize information-based utility. Jackson and Wolinsky (1996), Bala and

Goyal (2000), and Goyal and Vega-Redondo (2004), using three distinct utility functions, all find the star to be the dominant equilibrium network. Goyal and Vega-Redondo even contend that their utility function is a measurement for the richness of structural holes in someone's network. Our model does not use the constraint measure for this, as proposed by Burt (2005a). A theoretical reason for the difference between the two models is that control benefits are not subject to decay over longer paths in Goyal and Vega-Redondo's model. This implies that brokerage of indirectly received information is as valuable as brokerage of directly received information. In contrast, Burt's constraint measure implies that brokerage of information one receives indirectly is worthless, and only brokering directly received information creates value. Although both assumptions are quite extreme, we choose to insist strictly on using the constraint measure for three reasons. Actors who take indirect brokerage benefits into account must have information on the structure of the entire network. Their model is thus scope-limited to settings in which such information is readily available. The actors in our model need to know only which of their contacts are in contact with one another and which not. Second, the constraint measure has empirically been shown to explain success (see the evidence discussed on page 6). And third, Burt (2005b) demonstrates in a recent paper that the returns on indirect brokerage are in some contexts not visible at all, and if they are found, they are considerably smaller than returns on direct brokerage. The economists do not have a body of empirical confirmations to back up their utility functions.

Still, that different formalizations of theoretically the same concept yield such different results begs the question, How robust are our results for changes in assumptions on the utility function? Our intuition behind this is that two properties of the network constraint are crucial for our finding. First, network ties are in principle cheap, so if they are well chosen, one wants as many ties as possible. Second, closed triads are bad, such that you almost never want to create a tie if it closes a triad. This intuition is to some extent confirmed by Robins et al. (2005), who find in a stochastic network evolution context that networks converge to complete bipartite networks with a low probability of closed triads and relatively small costs for having ties in general. In addition, if we would add indirect constraint as is done by Burt (2005b) – i.e., an actor's indirect constraint is the average of the constraints of his neighbors – we would still find the same networks to be stable because indirect constraint is optimized, given that each individual constraint is optimized. However, this conclusion does not hold if we take into account redundancy over two steps; i.e., there is limited new information in multiple contacts if these are linked to many of the same third parties (see Reagans et al. 2004 for a generalization of constraint in this direction). Complete bipartite networks are full of such redundancies over two steps and, therefore, unlikely to be optimal if such redundancies are taken into account.

An important next question is how we can test our results, given that the dominant stable network type we found is not the prototypical social network. We know that friendship networks tend to be small-world networks – various clusters connected through a few intercluster ties (Watts and Strogatz 1998). And we know something about the causes. People prefer to associate with people who are like themselves and therefore tend to know their contacts' contacts as well (Homans 1950; Newcombe 1961; Lott and Lott 1965; Byrne 1971; Cohen 1977; Verbrugge 1977; Kandel 1978). Segregated networks have been shown to be extremely stable (Carley 1991; Mark 1998; Macy et al.

2003). However, most of these are settings in which people typically derive additional utility rather than disutility from closed triads. Burtian network dynamics are therefore not applicable to these settings. Testing the model in such contexts would not be appropriate. The settings to which Burt's network entrepreneurship (and thus also our results) applies are competitive settings where non-redundant, first-hand information is important. In such settings, balanced complete bipartite networks should be observable.

In conclusion, we investigated the implication of Burt's theory of structural holes if everyone were a network entrepreneur. In addition, we provided an application of a methodology and a stability concept that can be used to study actor-driven network evolution. The model, however, is too stylized to predict actual network structures. It assumes all actors to be strategic and structural holes to be the only thing that matters. Most empirical networks, including those observed in settings in which structural holes have been shown to be beneficial, do not resemble complete bipartite networks.

Therefore, we finish with some remarks on possible extensions of our analysis. A seemingly obvious one would be adding explicit costs for maintaining ties because this is common in the literature. This would be particularly interesting if we assumed heterogeneity between actors in costs of "bridging" ties. Some actors might be "natural" entrepreneurs, while others do not have the inclination or courage to step up to strangers and build bridging ties, or they just do not observe these brokerage opportunities. Another way to include heterogeneity among actors into the model might be to assume that structural holes are not the only things that matter. In many settings, other competing incentives will be present, such as balance in friendship networks. As Burt (2005a, ch. 5) notes in the last chapter of his recent book, stability might emerge in networks even with many brokerage opportunities still open, since a considerable number of actors are not interested in brokerage or are not able to observe these structural holes. Finally, considering the two types of social capital that Burt (2005a, ch. 3) distinguishes, it may be fruitful to use a utility function that is a hybrid of a brokerage-based utility function and a closure-based utility function. One could then make the relative importance of closure a parameter and study the consequences for network stability. This would change our results, because complete bipartite networks do not include any closed triads. Clearly, actors who care little about structural holes but a lot about friendship and trust will end up in networks full of redundant ties and unexploited brokerage opportunities.

Appendix

Proof of theorem 1. We rewrite the constraint of actor *i* as:

$$c_i \equiv \frac{1}{{d_i}^2} \sum_j \left[1 + \sum_q \frac{1}{d_q} \right]^2$$

where d_i is the number of actors *i* is linked to, *j* is the index for neighbors of *i*, and *q* is the index for neighbors of *i* that are also connected to *j*. This can be done because $p_{ij} = 1/d_i$ for all neighbors *j* of *i*. Suppose that two actors *i* and *r* can add a tie without creating a closed triad. Neither before nor after tie addition are there any actors *q* who are connected to both *i* and *l*. Let c_i* denote the network constraint of *i* after the initiation of the new tie, and let *j* continue to stand for the index of neighbors in the old network. Then,

$$c_i^* \equiv \frac{1}{(d_i + 1)^2} \left[1 + \sum_j \left(1 + \sum_q \frac{1}{d_q} \right)^2 \right]$$

Using straightforward calculations, this implies that

$$c_i^* - c_i = \frac{1}{(d_i+1)^2}\left[1+\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2\right] - \frac{1}{d_i^2}\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2 = \frac{1}{(d_i+1)^2} - \frac{(d_i+1)^2-d_i^2}{d_i^2(d_i+1)^2}\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2$$

$$\le \frac{1}{(d_i+1)^2} - \frac{(d_i+1)^2-d_i^2}{d_i(d_i+1)^2} = \frac{-d_i-1}{d_i(d_i+1)^2} = -\frac{1}{d_i(d_i+1)} < 0.$$

Thus, the addition of the new tie necessarily decreases actor *i*'s network constraint and hence increases his utility. Similarly, the constraint decreases for actor *r*. This completes the proof.

Proof of theorem 2. Removing one or more ties is not an option in a complete bipartite network because that would create a shortest path longer than 2, and hence it cannot be an improvement by corollary 1. Therefore, we need to consider only conditions under which group members create a tie within their group. Assume the groups have size *k* and *l* with $k \le l$ and $k + l = n$. The constraint in the complete bipartite network equals $\frac{1}{k}$ for actors in the group of size *l* and $\frac{1}{l}$ for actors in the group of size *k*. Creating a tie in the larger group of *l* actors changes the constraint of the two actors involved in that tie to

$$\frac{k}{(k+1)^2}\left[1+\frac{1}{(k+1)}\right]^2 + \frac{1}{(k+1)^2}\left[1+\frac{k}{l}\right]^2 = \frac{1}{(k+1)^2}\left[\frac{k(k+2)^2}{(k+1)^2}+\frac{(l+k)^2}{l^2}\right],$$

because these actors now have one common neighbor with all the actors in the group of size *k* and *k* common neighbors with each other. In order for the network to be pairwise Nash, this expression must be larger than $\frac{1}{k}$, or

$$\left[\frac{k(k+2)^2}{(k+1)^2}+\frac{(l+k)^2}{l^2}\right]>\frac{(k+1)^2}{k}\Leftrightarrow k^2l^2(k+2)^2+k(k+1)^2(k+l)^2>(k+1)^4l^2$$
$$\Leftrightarrow k(k+1)^2(k+l)^2>(2k^2+4k+1)l^2. \quad (2)$$

Thus, if $k = 1$, $4(l+1)^2 > 7l^2 \Leftrightarrow l < 4$ should hold. Therefore, stars are stable only if there are fewer than 4 peripheral actors. If $k > 1$, then the inequality above is always implied by $k(k+1)^2 > 2k^2+4k+1 \Leftrightarrow k^3-3k-1>0$, and this condition is always fulfilled for $k > 1$. The same expression should hold for actors in the small group, but then with k and l reversed:

$$\left[\frac{l(l+2)^2}{(l+1)^2}+\frac{(l+k)^2}{k^2}\right]>\frac{(l+1)^2}{l} \quad (3)$$

Inequality (3) is satisfied for any $l > 1$, $l \geq k \geq 1$, by reasons of symmetry, because (2) holds for all $k > 1$. Note that the case $l = k = 1$ is irrelevant because no tie can be added. This completes the proof.

Proof of theorem 3. Consider an actor i from the smaller group of $k \leq l$ actors. The network constraint of i can be lower in another network only if he has more than l ties because $1/l$ is the minimal constraint one can have with l ties. Let $a \leq k - 1$ be this additional number of ties of actor i, let j be the index for neighbors of i, and q the index for actors that i and j share as neighbors, let π_j indicate the proportion of ties of j with other neighbors of i, and $\bar{\pi}_j$ the average of all $l + a$ π_j's. Then, for i to have a lower network constraint, the following inequality must hold:

$$\frac{1}{l}>\frac{1}{(l+a)^2}\sum_j\left[1+\sum_q\frac{1}{d_q}\right]^2\geq\frac{1}{(l+a)^2}\sum_j\left[1+\sum_q\frac{1}{d_q}\right]=\frac{1}{(l+a)^2}\sum_j\left[1+\sum_q\frac{1}{d_j}\right]$$

$$= \frac{1}{(l+a)^2} \sum_j \left[1+\pi_j\right] = \frac{1}{(l+a)} \left[1+\overline{\pi}_j\right] \Rightarrow \pi_j < \frac{a}{l} \text{ for some } j.$$

The most difficult step in the derivation above is the first equality, which is implied by the fact that for each *j* the number of times $1/d_j$ should be added due to closed triads is equal to number of common neighbors *j* has with *i*.

Note that for each *j*, to be at least as well off in the new network as in the complete bipartite network considered, his degree d_j must be at least *k*. Only $k - a - 1$ of *j*'s connections can be to actors whom actor *i* is not connected to, thereby excluding *i* himself. For each *j*, π_j may therefore be no less than a/k:

$$\pi_j \geq \frac{d_j - k + a}{d_j} \geq \frac{a}{k} \geq \frac{a}{l} \text{ for all } j.$$

We have reached a contradiction. Thus, to improve *i*'s network constraint, at least one actor *j* must be given fewer than *k* neighbors, and this actor is consequently strictly worse off in the new network than in the complete bipartite network considered.

A Pareto-improvement must therefore leave the network constraints of all actors with *l* ties unchanged and give them precisely *l* ties. This can be done only by keeping all ties that connect such an actor with an actor with *l* ties, or in the case of $k = l$, by switching to a network that contains a different *k* by *l* complete bipartite network. The only remaining possibility for a Pareto-improvement is then the addition of one or more ties between actors with *k* ties. But any such addition involves the creation of triangles and hence increases the network constraint of all actors with *l* ties. This renders the assumed Pareto-improvement impossible and completes the proof.

Proof of theorem 4. *If*. Consider an actor i from the group of k actors. We know from the proof of theorem 3 that we cannot make this actor better off without letting one of his neighbors have a degree lower than k. But leaving the ties that do not involve actor i unchanged, all his neighbors have at least degree k. In the even case, in which $k = l$, by symmetry, this impossibility of unilateral improvement extends to actors of the group of size l. The single remaining possibility for unilateral improvement is therefore a permitted decrement of the constraint of an actor i from the group of l actors in the uneven case, in which $k = l - 1$. Let $0 \leq b_k \leq k$ be the number of ties actor i has with actors from the group of size k in the new network, and let $0 \leq b_l \leq k$ be the number of ties he has with actors from the group of size l. Again, leave the ties that do not involve actor i unchanged. Because these ties constitute the balanced bipartite network with $k = l - 1$ actors in each group, the following inequality must hold:

$$\frac{1}{k} > \frac{1}{(b_k+b_l)^2}\left(b_k\left[1+\frac{b_l}{k+1}\right]^2 + b_l\left[1+\frac{b_k}{k+1}\right]^2\right) = \frac{b_k(b_l+k+1)^2 + b_l(b_k+k+1)^2}{(b_k+b_l)^2(k+1)^2} \Leftrightarrow$$

$$b_l^2(k+1)^2 + 2b_kb_l(k+1)^2 + b_k^2(k+1)^2 > b_kk(k+1)^2 + 2b_kb_lk(k+1) + b_kb_l^2k + b_lk(k+1)^2 + 2b_kb_lk(k+1) + b_lb_k^2k \Leftrightarrow$$

$$b_l^2(k+1)^2 - 2b_kb_l(k-1)(k+1) + b_k^2(k+1)^2 > b_kk(k+1)^2 + b_kb_l^2k + b_lk(k+1)^2 + b_lb_k^2k \Leftrightarrow$$

$$\left[b_k(b_k-k) + b_l(b_l-k)\right](k+1)^2 > b_kb_l\left[(b_k+b_l)k + 2(k-1)(k+1)\right].$$

The left-hand side of this last inequality is never strictly positive, and the right-hand side is never strictly negative. Hence, it cannot be satisfied.

Only if. If $l - k > 1$, an actor from the larger group of l actors can delete all his ties with actors from the smaller group of k actors and add $l - 1$ ties to the other actors from the larger group of l actors. By doing so, she decreases her constraint from $\frac{1}{k}$ to $\frac{1}{l-1}$. By permitting this change, the $l-1$ actors see their constraint fall from $\frac{1}{k}$ to $\frac{1}{k+1}$. This completes the proof.

Proof of theorem 5. Denote n_2 a factor of n. Then for a complete n/n_2-partite network with equal groups of n_2 size, the following inequality should hold such that no one wants to severe a tie (note that we need only one equation because all actors have regularly equivalent positions).

$$\frac{1}{(n-n_2)^2}(n-n_2)\left[1+\frac{n-2n_2}{n-n_2}\right]^2-\frac{1}{(n-n_2-1)^2}\left[(n-2n_2)\left[1+\frac{n-2n_2-1}{n-n_2}\right]^2+(n_2-1)\left[1+\frac{n-2n_2}{n-n_2}\right]^2\right]<0\Leftrightarrow$$

$$\frac{(2n-3n_2)^2}{(n-n_2)^3}-\frac{1}{(n-n_2-1)^2}\left[\frac{(n-2n_2)(2n-3n_2-1)^2+(n_2-1)(2n-3n_2)^2}{(n-n_2)^2}\right]<0\Leftrightarrow$$

$$\frac{(2n-3n_2)^2}{(n-n_2)^3}-\frac{1}{(n-n_2-1)^2}\left[\frac{(n-n_2-1)(2n-3n_2)^2-(n-2n_2)(4n-6n_2-1)}{(n-n_2)^2}\right]<0\Leftrightarrow$$

$$\frac{(n-2n_2)(4n-6n_2-1)}{(n-n_2-1)^2(n-n_2)^2}-\frac{(2n-3n_2)^2}{(n-n_2-1)(n-n_2)^3}<0\Leftrightarrow$$

$$(n-2n_2)(4n-6n_2-1)(n-n_2)-(n-n_2-1)(2n-3n_2)^2<0\Leftrightarrow$$

$$4(n-2n_2)(n-n_2)(n-\frac{3}{2}n_2-\frac{1}{4})-4(n-n_2-1)(n-\frac{3}{2}n_2)^2<0$$

which is always true because

$$(n-2n_2)(n-n_2)<(n-\frac{3}{2}n_2)^2 \text{ and } (n-\frac{3}{2}n_2-\frac{1}{4})<(n-n_2-1) \text{ if } n_2\geq 2.$$

For no actor to benefit from adding any of his equivalent potential ties in this complete n/n_2-bipartite network with equal-sized groups, the following inequality must hold:

$$\frac{1}{(n-n_2)^2}(n-n_2)\left[1+\frac{n-2n_2}{n-n_2}\right]^2-\frac{1}{(n-n_2+1)^2}\left[(n-n_2)\left[1+\frac{1}{n-n_2+1}+\frac{n-2n_2}{n-n_2}\right]^2+\left[1+\frac{n-n_2}{n-n_2}\right]^2\right]<0\Leftrightarrow$$

$$\frac{(2n-3n_2)^2}{(n-n_2)^3}-\frac{[(n-n_2+1)(2n-3n_2)+(n-n_2)]^2+4(n-n_2+1)^2(n-n_2)}{(n-n_2+1)^4(n-n_2)}<0 \Leftrightarrow$$

$$(2n-3n_2)^2(n-n_2+1)^4-\left([(n-n_2+1)(2n-3n_2)+(n-n_2)]^2+4(n-n_2+1)^2(n-n_2)\right)(n-n_2)^2<0 \Leftrightarrow$$

$$x^4(7-6n_2)+x^3(12-18n_2)+x^2(4-16n_2)-4xn_2+2x^3n_2^2+5x^2n_2^2+4xn_2^2+n_2^2<0, \text{ where } x=n-n_2.$$

Since $n_2 < x = n - n_2$, the equation above is implied by (replacing n_2^2 by xn_2)

$$x^4(7-4n_2)+x^3(12-13n_2)+x^2(4-12n_2)-3xn_2<0,$$

which is true because $n_2 \geq 2$ and $x > 0$. This completes the proof.

References

Bala, Venkatesh and Sanjeev Goyal. 2000. "A Noncooperative Model of Network Formation." *Econometrica* 68: 1181-1229.

Barabasi, Albert-László and Réka Albert. 1999. "Emergence of Scaling in Random Networks." *Science* 286: 509-512.

Bian, Yanjie. 1994. *Work and Equality in Urban China*. Albany, NY: State University of New York Press.

Bonacich, Phillip. 2004. "Social Capital in Exchange Networks: A Simulation Study of Network Evolution." Pp. 104-115 in *Creation and Returns of Social Capital : A New Research Program*, edited by Henk Flap and Beate Völker. New York: Routledge.

Breiger, Ronald L., Kathleen M. Carley, and Philippa E. Pattison. 2003. *Dynamic Social Network Modeling and Analysis: Workshop Summary and Papers*. Washington: The National Academies Press.

Burt, Ronald S. 1992. *Structural Holes. The Social Structure of Competition*. Cambridge, MA: Harvard University Press.

Burt, Ronald S. 1997. “The Contingent Value of Social Capital.” *Administrative Science Quarterly* 42: 339-365.

Burt, Ronald S. 1998. “The Gender of Social Capital.” *Rationality and Society* 10: 5-46.

Burt, Ronald S. 2000. “The Network Structure of Social Capital.” Pp. 345–423 in *Research in Organizational Behavior* 22, edited by Robert I. Sutton and Barry M. Staw. Greenwich: Elsevier Science.

Burt, Ronald S. 2001. “Bridge Decay.” *Social Networks* 24 (4): 333-363.

Burt, Ronald S. 2002. “The Social Capital of Structural Holes.” Chapter 7 in *New Directions in Economic Sociology*, edited by Mauro. F. Guillén, Randall Collins, Paula England, and Marshall Meyer. New York: Russell Sage.

Burt, Ronald S. 2004. “Structural Holes and Good Ideas.” *American Journal of Sociology* 110: 349-399.

Burt, Ronald S. 2005a. *Brokerage and Closure: An Introduction to Social Capital*. Oxford: Oxford University Press.

Burt, Ronald S. 2005b. “Second-Hand Brokerage.” Paper presented at the 2005 meetings of the Academy of Management.

Burt, Ronald S., Miguel Guilarte, Holly J. Raider, and Yuki Yasuda. 2002. “Competition, Contingency, and the External Structure of Markets.” *Advances in Strategic Management* 19: 167-217.

Burt, Ronald S., Robin M. Hogarth, and Claude Michaud. 2000. “The Social Capital of French and American Managers.” *Organization Science* 11: 123-147.

Buskens, Vincent. 2002. *Social Networks and Trust*. Boston, MA: Kluwer.

Byrne, Donn E. 1971. *The Attraction Paradigm*. New York: Academic Press.

Cálvo-Armengol, Antoni. 2004. “Job Contact Networks.” *Journal of Economic Theory* 115: 191-206.

Carley, Kathleen. 1991. “A Theory of Group Stability.” *American Sociological Review* 56: 331-354.

Cartwright, Dorwin, and Frank Harary. 1956. “Structural Balance: A Generalization of Heider’s Theory.” *Psychology Review* 63: 277-293.

Cohen, Jere M. 1977. “Sources of Peer Group Homogeneity.” *Sociology of Education* 50: 227-241.

Coleman, James S. 1988. “Social Capital in the Creation of Human Capital.” *American Journal of Sociology* 94: 95-120.

Cook, Karen S., and Richard M. Emerson. 1978. “Power, Equity, and Commitment in Exchange Networks.” *American Sociological Review* 43: 721-739.

Doreian, Patrick and Andrej Mrvar. 1996. “A Partitioning Approach to Structural Balance.” *Social Networks* 18: 149-168.

Doreian, Patrick and Frans N. Stokman (Eds.). 1997. *Evolution of Social Networks*. Amsterdam: Gordon and Breach Publishers.

Dutta, Bhaskar and Matthew O. Jackson (Eds.) 2003. *Networks and Groups. Models of Strategic Formation*. Berlin: Springer.

Fararo, Thomas J. and John Skvoretz. 1986. “E-state Structuralism – A Theoretical Method.” *American Sociological Review* 51: 591-602.

Flap, Henk. 2003. “Creation and Returns of Social Capital.” Pp 3-23 in *Creation and Returns of Social Capital: A New Research Program,* edited by Henk Flap and Beate Völker. New York: Routledge.

Freeman, Linton C. 1979. "Centrality in Social Networks: I. Conceptual Clarification." *Social Networks* 1: 215-239.

Gabbay, Shaul M. 1997. *Social Capital in the Creation of Financial Capital*. Champaign, IL: Stipes.

Gilles, Robert P. and Sudipta Sarangi. 2004. "The Role of Trust in Costly Network Formation." Working paper.

Gould, Roger V. 2002. "The Origins of Status Hierarchies: A Formal Theory and Empirical Test." *American Journal of Sociology* 107: 1143-1178.

Goyal, Sanjeev and Sumit Joshi 2004. *Unequal Connections*. Working paper.

Goyal, Sanjeev and Fernando Vega-Redondo (2004). "Structural Holes in Social Networks." Working paper.

Granovetter, Mark S. 1995 [1974]. *Getting a Job*. Chicago, IL: University of Chicago Press.

Heider, Fritz. 1946. "Attitudes and Cognitive Organization." *Journal of Psychology* 21: 107-112.

Homans, George C. 1950. *The Human Group*. New York: Harcourt Brace.

Hummon, Norman P. and Patrick Doreian. 2003. "Some Dynamics of Social Balance Processes: Bringing Heider Back into Balance Theory." *Social Network* 25: 17-49.

Jackson, Matthew O. 2003. A Survey of Models of Network Formation: Stability and Efficiency. Forthcoming in *Group Formation in Economics; Networks, Clubs and Coalitions*, edited by Gabrielle Demange and Myrna Wooders. Cambridge: Cambridge University Press.

Jackson, Matthew O. and Asher Wolinsky 1996. "A Strategic Model of Social and

Economic Networks." *Journal of Economic Theory* 71: 44-74.

Kandel, Denise B. 1978. "Homophily, Selection, and Socialization in Adolescent Friendship Networks." *American Journal of Sociology* 84: 427-436.

Krackhardt, David. 1999. "The Ties That Torture: Simmelian Tie Analysis in Organizations." *Research in the Sociology of Organizations* 16: 183-210.

Leenders, Roger and Shaul M. Gabbay. 1999. *Corporate Social Capital and Liability*. Amsterdam: Kluwer.

Lin, Nan. 1999. "Social Networks and Status Attainment." *Annual Review of Sociology* 25: 467-487.

Lin, Nan, Karen S. Cook, and Ronald S. Burt. 2001. *Social Capital*. Chicago, IL: Aldine.

Lott, Albert J. and Bernice E. Lott. 1965. "Group cohesiveness as interpersonal attraction: a review of relationships with antecedent and consequent variables." *Psychological Bulletin* 64:259-309.

Macy, Michael W., James A. Kitts, and Andreas Flache. 2003. "Polarization in Dynamic Networks: A Hopfield Model of Emergent Structure." Pp. 162–73 in *Dynamic Social Network Modeling and Analysis: Workshop Summary and Papers*, edited by Ronald Breiger, Kathleen Carley, and Philippa Pattison, Washington, DC: The National Academy Press.

Mark, Noah. 1998. "Beyond Individual Differences: Social Differentiation from First Principles." *American Sociological Review* 63: 309-330.

McKay, Brendan D. 1990. "Nauty User's Guide (version 1.5)." Tech. Rpt. TR-CS-90-02, Dept. Computer Science, Australian National University.

Mehra, Ajay, Martin Kilduff, and Daniel J. Brass. 2001. "The Social Networks of High and Low Self-Monitors: Implications for Workplace Performance." *Administrative Science Quarterly* 46: 121-146.

Mizruchi, Mark S. and Linda B. Sterns. 2001. "Getting Deals Done: The Use of Social Networks in Bank Decision Making." *American Sociological Review* 66: 647-671.

Myerson, Robert B. 1991. *Game Theory: Analysis of Conflict*. Cambridge, MA: Harvard University Press.

Newcomb, Theodore M. 1961. *The Acquaintance Process*. New York: Holt, Rhinehart, and Whinston.

Podolny, Joel M. and James N. Baron. 1997. "Relationships and Resources. Social Networks and Mobility in the Workplace." *American Sociological Review* 62: 673-693.

Raub, Werner, and Jeroen Weesie. 1990. "Reputation and Efficiency in Social Interaction: An Example of Network Effects." *American Journal of Sociology* 96: 626-654.

Reagans, Ray E., Ezra W. Zuckerman, and Bill McEvily. 2004. "Two Holes in One: Information and Control in the analysis of Structural Autonomy." Working paper.

Robins, Garry, Philippa Pattison, and Jodie Woolcock (2005). "Small and Other Worlds: Global Network Structure from Local Processes." *American Journal of Sociology* 110: 894-936.

Schelling, Thomas C. 1969. "Models of Segregation." *American Economic Review, Papers and Proceedings* 59: 488-493.

Schelling, Thomas C. 1971. "Dynamic Models of Segregation." *Journal of Mathematical Sociology* 1: 143-186.

Skvoretz, John, Katherine Faust, and Thomas J. Fararo. 1996. "Social Structure, Networks, and E-State Structuralism Models." *Journal of Mathematical Sociology* 21 (1-2): 57-76.

Snijders, Tom A. B. 1996. "Stochastic Actor-Oriented Models for Network Change." *Journal of Mathematical Sociology* 21: 149-172.

Snijders, Tom A. B. 2001. "The Statistical Evaluation of Social Network Dynamics." Pp. 361–95 in *Sociological Methodology*, edited by Michael E. Sobel and Mark P. Becker. London: Basil Blackwell.

Snijders, Tom A.B. 2005. "Models for Longitudinal Network Data." Chapter 11 in *Models and Methods in Social Network Analysis,* edited by Peter J. Carrington, John Scott, and Stanley Wasserman. New York: Cambridge University Press.

Stokman, Frans N. and Patrick Doreian (Eds.). 2001. The Evolution of Social Networks. Part II. Special issue of *Journal of Mathematical Sociology*.

Talmud, Ilan. 1994. "Relations and Profits: The Social Organizations of Israeli Industrial Competition." *Social Science Research* 23: 109-135.

Van der Bunt, Gerhard G., Marijtje A. J. van Duijn, and Tom A. B. Snijders. 1999. "Friendship Networks Through Time: An Actor-Oriented Dynamic Statistical Network Model." *Computational and Mathematical Organization Theory* 5 (2): 167-192.

Verbrugge, Lois M. 1977. "The Structure of Adult Friendship Choices." *Social Forces* 56: 576-597.

Wang, Zhigang and Warren Thorngate. 2003. "Sentiment and Social Mitosis: Implications of Heider's Balance Theory." *Journal of Artificial Societies and Social*

Simulation 6(3).

Wasserman, Stanley and Katherine Faust. 1994. *Social Network Analysis*. Cambridge: Cambridge University Press.

Watts, Duncan J. 1999. "Networks, Dynamics, and the Small-World Phenomenon." *American Journal of Sociology* 105 (2): 493-527.

Watts, Duncan J. and Stephen H. Strogatz. 1998. "Collective Dynamics of Small-World Networks." *Nature* 393: 440-442.

Weesie, Jeroen and Henk Flap. 1990. *Social Networks Through Time*. Utrecht: ISOR.

Whitmeyer, Joseph M. "A Deductive Approach to Friendship Formation." *Journal of Mathematical Sociology* 26: 147-165.

Willer, David. 1999. *Network Exchange Theory*. New York: Greenwood.

Yasuda, Yuki. 1996. *Network Analysis of Japanese and American Markets*. Tokyo: Bokutaku-Sha.

Zeggelink, Evelien P.H., Frans N. Stokman, and Gerhard G. van der Bunt. 1996. "The Emergence of Groups in the Evolution of Friendship Networks." *Journal of Mathematical Sociology* 21: 29-55.

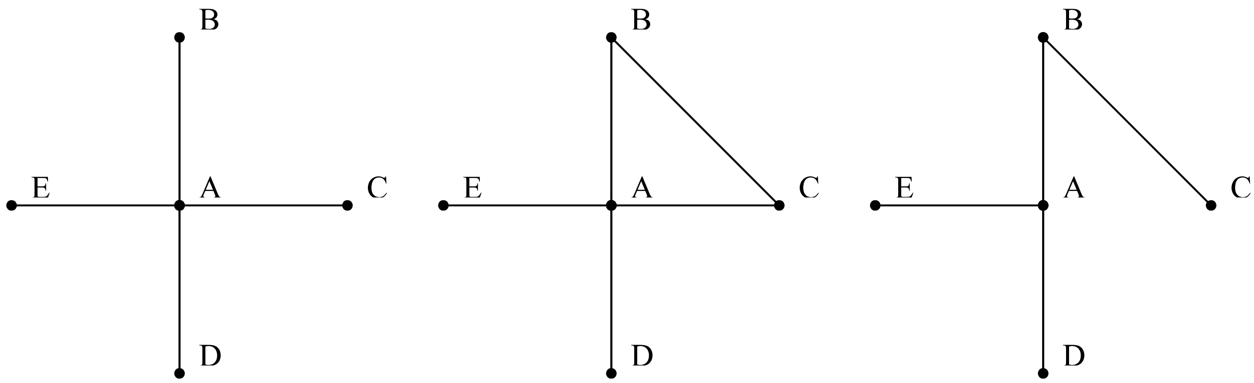
Table 1. Number of networks (found) for various stability criteria (note that we do not have any guarantee that we have all pairwise stable networks for networks with $n > 10$).

n	Non-isomorphic networks	Connected	Pairwise stable	Pairwise Nash	Unilaterally stable
2	2	1	1	1	1
3	4	2	1	1	1
4	11	6	2	2	1
5	34	21	2	2	2
6	156	112	4	3	1
7	1044	853	3	3	1
8	12346	11117	10	7	2
9	274668	261080	9	7	1
10	12.01e6	11.72e6	14	9	2
11	10.19e8	10.07e8	≥13	≥8	≥1
12	16.51e10	16.41e10	≥25	≥12	≥1
13	50.50e12	50.34e12	≥17	≥8	≥1
14	29.05e15	29.00e15	≥21	≥10	≥1
15	31.43e18	31.40e18	≥34	≥16	≥2
16	64.00e21	63.97e21	≥43	≥21	≥2

Table 2. Simulation results

n	Number of starting networks	Different pairwise stable networks in simulation	% Balanced complete bipartite	% Just unbalanced complete bipartite (difference in size between groups is 2 or 3)	Correlation density starting network – resulting network
2	2	1	1	n.a.	n.a.
3	4	1	1	n.a.	n.a.
4	11	2	.86	.14	.24
5	34	2	.81	n.a.	.17
6	156	4	.71	.18	.41
7	1044	3	.63	.03	.10
8	12346	10	.62	.15	.21
9	9292	9	.87	.01	.29
10	10070	11	.69	.23	.21
11	10898	12	.88	.04	.28
12	10930	23	.61	.31	.26
13	5078	15	.81	.05	.34
14	5700	18	.54	.30	.37
15	6358	31	.79	.06	.37
16	7062	39	.51	.30	.44
17	2346	35	.79	.06	.44
18	2666	42	.53	.32	.46
19	3006	44	.79	.07	.43
20	3366	56	.52	.34	.42
21	3746	59	.77	.08	.45
22	4146	73	.50	.35	.43
23	4566	86	.75	.09	.47
24	5006	107	.50	.33	.47
25	5466	110	.74	.10	.48

Figure 1. The constraint measure; three example networks.



$c_A = 4\,(¼ + 0)^2 = ¼$
$c_B = 1\,(1/1 + 0)^2 = 1$
$c_C = 1\,(1/1 + 0)^2 = 1$
$c_D = 1\,(1/1 + 0)^2 = 1$
$c_E = 1\,(1/1 + 0)^2 = 1$

$c_A = 2\,(¼ + 0)^2 + 2\,(¼ + ¼\,½)^2 = 13/32$
$c_B = (½ + ½\,¼)^2 + (½ + ½\,½)^2 = 61/64$
$c_C = (½ + ½\,¼)^2 + (½ + ½\,½)^2 = 61/64$
$c_D = 1\,(1/1 + 0)^2 = 1$
$c_E = 1\,(1/1 + 0)^2 = 1$

$c_A = 3\,(⅓ + 0)^2 = ⅓$
$c_B = 2\,(½ + 0)^2 = ½$
$c_C = 1\,(1/1 + 0)^2 = 1$
$c_D = 1\,(1/1 + 0)^2 = 1$
$c_E = 1\,(1/1 + 0)^2 = 1$

Figure 2. Some pairwise stable networks. Numbers are constraint values. Box, 4-star, Pentagon, Bag, and 10-actor balanced complete bipartite network are also unilaterally stable.