A Wavelets Approach of CO2-GDP Relation in the NAFTA Zone

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Abstract

This paper reviews the relationship between CO2 emissions and gross domestic product per capita for the NAFTA countries. Previews research has been done in applying cointegration analysis mostly for developed economies using the global approach of time series analysis. The novelty in this paper is that it applies the wavelet transform which decomposes each time series into a time-scale basis and at each scale it is estimated the correlation. Empirical results show a non-homogenous relationship in which a higher degree of association governs most of the time-scales; and one explanation would be that the 'business cycle of technology' contributes to the higher relationship between the economic activity and CO2 emissions. The importance of these results concerns to proper environmental and economic policies formulations that may reduce contaminant emissions without giving up a regional sustainable economic growth in which the economic sphere is circumscribed into the biosphere.

JEL classification: *C63, Q53, Q56.*
Keywords: *wavelet transform, CO2 emissions, sustainable economic growth.*

1. Introduction

Kuznets (1955) addressed the question whether income inequality may increase or decrease as economic growth was attained by a country. Such relationship has been conceived as an inverted U-shaped form in which inequality increases at the first stages of economic growth and then it decreases at a mature stage when a certain level of income has been achieved. Since then, a similar relationship has been conceived between environmental pollution and economic growth, and it has been named as the Environmental Kuznet Curve (EKC) originally stated by Panayotou (1993). The EKC hypothesis proposes that the environmental quality decreases at early stages of increasing income and at a certain level of income it recovers again.

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In this sense, evidence of the EKC has been in testing the relation between carbon dioxide emissions and income, most of them motivated by Meadows et al. (1972) who informed that the economic activity negatively impacts the environment since greater production requires greater quantities of inputs such as fossil fuels. As a consequence, more contaminant emissions are generated which diminish the environmental quality. Then, the environment is at risk by the proper development. So, to avoid damaging the environment it would be needed to grow at a zero rate.

The main purpose in this paper is to review the relationship between carbon dioxide emissions and GDP/capita for the NAFTA countries based on the wavelet analysis against the global approach of time series analysis. Previews research has been performed by applying cointegration analysis on the whole series, but this paper decomposes the original series into different scales each one related to different timing frames. The decomposition is performed by using the maximal overlap discrete wavelet transform and the Haar wavelet as the filtering function. By this, it is possible to "denoise" the original series and look at detail patterns as described by Graps (1995) who states that by wavelets it is possible to look at the whole forest meanwhile looking at its details (trees).

The first applications of wavelet theory in economic and financial phenomena analysis are done by Ramsey and Lampart (1999), who examine the relationship between economic data such as personal income-spending, M1 and M2 monetary aggregates. T-bills interest rates and durable, non-durable and consumption goods and services. Aguiar, Azevedo and Soares (2007) analize the effects of interest rate changes on industrial production, inflation, and M1 and M2 monetary aggregates. Lee (2004) studies price and volatility spillovers between US and Korean stock markets. Sharkasi, Ruskin and Crane (2005) investigate the relationships among the U.K., the U.S., Irish, Portuguese, Brazilian, Japanese and Hong Kong stock markets. Tellez and Lopez (2010) analyze the correlation among Latin American and the United States stock markets.

A first attempt in analyzing energy and economic growth is found in Cifter and Ozun (2007) who analyze the relationship between energy consumption and gross national product in Turkey. Their main findings are that there is a bidirectional information flow between both series in the short run but in the long run the direction runs from GNP to energy consumption. Also an asymmetric pattern (non-homogeneus behaviour) was found at each resolution level where the maximum correlation was captured in a time framing between 5-8 years.

The remaining of this paper is organized as follows. Section two describes the wavelet analysis, in section three data and results are given. Finally, section four gives the concluding remarks.

2. Wavelet Analysis

Wavelets (small waves) are functions of special structure described by basis functions and represented by successive approximation series which allow the decomposition of signals at different scales and resolutions, against the Fourier analysis which only gives frequency components but cannot show their time localization. Wavelets have the property to concentrate their energy along time to allow the analysis of temporal, non-stationary and time-varying phenomena.

Wavelets analysis has its nearest foundations on the Short Time Fourier Transform (STFT), this one based on the Fourier Transform which main purpose is to represent a complex function by a weighted sum of simple functions obtained from a more one known as prototype or basis function. The Fourier transform has the characteristic to allow global analysis of a signal, since the terms *cos(wt)* and *sin(wt)* represent global functions, and by this is meant that the Fourier transform is perfect compactly supported in the frequency domain but not in the time domain,

$$\hat{f}(\omega) = \lim_{T\to\infty} \hat{f}_T(\omega_r) = \int_{-\infty}^{\infty} f(t)e^{i\omega t}dt, \tag{1}$$

The STFT works by taking out the required portion of the spectrum and then applying on it the Fourier transform. Nevertheless, the function used to remove the required portion of the signal is a constant window-function which cannot face the problem when signals have very high frequency components with short time spans, and low frequency components with long time spans. In the other side, the wavelet transform allows time-frequency analysis since it works with a time-varying window function: it is possible to increase the window-function radius along time whenever frequencies are decreasing, and decrease it whenever frequencies are increasing.

Given the importance of the window-function described above, the wavelet transform is the convolution of a wavelet function and the original signal, where the former depends on two parameters: 1) translation (localization), which represents movements of the wavelet function along axis time; 2) dilation, which allows expansion and contraction of the wavelet function, so high and low frequencies of the original signal can be captured. The wavelet transform works in an opposite direction to the STFT, it first decomposes the original signal in frequency bands and then analyzes it over time, as follows:

$$W(a,b) = \frac{1}{\sqrt{a}} \int f(t) \psi^* \left(\frac{t-b}{a} \right) dt, \tag{2}$$

Also the original signal can be reconstructed (inverse wavelet transform) as

$$f(t) = \frac{C_\psi}{a^2} \int_{a>0} \int_b W(a,b) \psi^* \left(\frac{t-b}{a} \right) dadb, \tag{3}$$

where $a>0$ and b, are the dilation and localization parameters, respectively; is the mother wavelet, is a constant which depends on , and $W(a,b)$ is the continuous wavelet transform (CWT). Also, the wavelet transform can be represented as the inner product

$$W(a,b) = \langle x, \psi_{a,b} \rangle, \tag{4}$$

Since the CWT is a function that depends on two continuous parameters, it may result in redundant information (a varying-number of coefficients with a fewer number of scales). This problem is solved by *discretizing* the parameters a and b using the multiresolution analysis: low-pass and high-pass filters are applied iteratively and then sampled down by two as a cascade. This process results in the Discrete Wavelet Transform (DWT). If a and b are represented by 2^{-j} and $k2^{-j}$, respectively, then the integral (2) becomes

$$W\left(k2^{-j}, 2^{-j}\right) = 2^{j/2} \int_{-\infty}^{\infty} f(t) \psi\left(2^j t - k\right) dt, \tag{5}$$

which is approximated as

$$W\left(k2^{-j}, 2^{-j}\right) \approx 2^{j/2} \sum_n f(n) \psi\left(2^j n - k\right). \tag{6}$$

Given *a* and *b* discretized, and a function *f(t)* in $\mathbf{L^2(R)}$ which can be represented by a sequence of mother and father wavelet functions, respectively

$$\psi_{j.k}(t) = 2^{-j/2}\psi\left(2^{-j}t - k\right) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right), \tag{7}$$

$$\phi_{j.k}(t) = 2^{-j/2}\phi\left(2^{-j}t - k\right) = 2^{-j/2}\phi\left(\frac{t-2^j k}{2^j}\right). \tag{8}$$

the function *f(t)* can be written as

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\phi_{J,k}(t) + \sum_k d_{J-1}, k\psi_{J-1}, k(t)$$
$$+ \ldots + \sum_k d_{1,k}\psi_{1,k}(t), \tag{9}$$

where the coefficients $s_{J,k}$, $d_{J,k}$, ..., $d_{1,k}$ are the wavelet transform coefficients contained in a column vector **W** with length *N*, where the first elements $N\text{-}N/2^J$ are the wavelet coefficients and the last elements $N/2^J$ are the scaling coefficients. The coefficients $s_{J,k}$ are the smooth coefficients which represent the underlying smooth behaviour of the signal at the coarse scale 2^J; $d_{J,k}$, are the detail coefficients named as crystals which represent deviations from the smooth behavior which describe deviations at the coarse scale, and $d_{J-1,k}$, ..., $d_{1,k}$, are progressively finer scale deviations.

Therefore, the function *f(t)* can be represented in terms of its detail series at different resolutions

$$D_j(t) = \sum_k d_{j,k}\psi_{J,k}(t) \text{ for all } \quad j = 1,2,\ldots,J. \tag{10}$$

and smooth variations

$$S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t), \tag{11}$$

which yields

$$f(t) = S_J(t) + D_J(t) + D_{J-1}(t) + \cdots + D_1(t), \tag{12}$$

named as the multiresolution decomposition (MRD).

The modified version of the DWT is the maximal overlap wavelet transform (MODWT) or non-decimating DWT, which is a highly redundant nonorthogonal transform resulting in new column vectors $\widetilde{W}_1, \widetilde{W}_2, \widetilde{W}_J$, which each column contains the MODWT wavelet coefficients associated with changes in **X** on a scale $\lambda_j = 2^{j-1}$.

3. Data and Results

3.1 Data

This paper uses annual carbon dioxide (CO_2) emissions from energy consumption and gross domestic product per capita (GDP/capita) from Canada, USA and Mexico. Annual data are transformed into log-returns as follows

$$\mathrm{Re}t_t = \ln\frac{P_t}{P_{t-1}}. \tag{13}$$

The whole data consisted of 29 observations from 1980 to 2008, obtained from the Energy Information Administration and International Monetary Fund statistics. Table 1 shows the descriptive statistics of each series in a global approach, from which it is observed by the skweness statistic that all countries have shown more negative (percentage changes) values than positive ones. Kurtosis values are not far from three which show that data are almost Normal distributed. Finally based on the Jarque-Bera test of normality, it can be rejected that CO_2 emissions and GDP/capita are Normal distributed.

Table 1. Descriptive statistics of carbon dioxide (CO_2) emissions and GDP/capita for each country

	CO2 emissions			GDP/capita		
	Canada	**USA**	**Mexico**	**Canada**	**USA**	**Mexico**
Mean	0.008004	0.007103	0.021952	0.015649	0.018791	0.009267
Median	0.015965	0.009425	0.029777	0.017489	0.021699	0.01871
Maximum	0.071175	0.054835	0.103358	0.047059	0.060637	0.058076
Minimum	-0.045806	-0.053724	-0.078656	-0.040974	-0.029183	-0.081899
Std. Dev.	0.033389	0.023375	0.041306	0.021348	0.017582	0.033432
Skewness	-0.122084	-0.348226	-0.480779	-0.905549	-0.622183	-1.028451
Kurtosis	1.924073	3.382318	2.894282	3.770404	4.349241	3.684352
Jarque-Bera	1.420108	0.736414	1.091731	4.519195	3.930381	5.48238
Probability	0.491618	0.691974	0.57934	0.104392	0.140129	0.064494
Sum	0.224125	0.198886	0.614656	0.438175	0.52614	0.259486
Sum Sq. Dev.	0.030099	0.014752	0.046068	0.012305	0.008346	0.030178
Observations	28	28	28	28	28	28

Table 2 shows the correlation between CO_2 emissions from energy consumption and GDP/capita, in which Mexico and the United States show higher correlations than Canada. Also it is shown the correlations of CO_2 from petroleum consumption and GDP/capita, which are positively related and the highest ones showed were between

Mexico and the United States.[2] In a "cross-country CO2 emissions correlation" analysis shown in Table 3, Canada and the United States show the highest correlation of 0.78 which can be interpreted as the two-NAFTA countries with a deeper economic integration than Mexico.[3]

Table 2: Correlation between GDP/capita and CO2 emissions from (a) energy consumption, and (b) petroleum consumption.

(a)		(b)	
	Energy		**Petroleum**
Canada	0.5354	**Canada**	0.5010
Mexico	0.7394	**Mexico**	0.6804
United States	0.7320	**United States**	0.6818

Nevertheless, as shown by Granger causality tests, only Mexico showed a statistical significance that information flows form GDP/capita to CO2 emissions from energy consumption. So, besides CO2 emissions and GDP/capita are highly correlated in the United States and Mexico, only Mexico is showing causality in the Granger standpoint of view that GDP/capita is really affecting CO2 emissions from energy consumption. In this sense and considering other factors that may be causing CO2 emissions, Mexico should be working hard in defining an environmental-economic policy to let a sustainable economic growth without threatening the environment any time the environment is at risk with greater CO2 emissions.

Table 3: Cross-country correlation of CO2 emissions from (a) energy consumption and (b) from petroleum.

(a)

	Canada	**Mexico**	**United States**
Canada	1	0.0481	0.7765
Mexico		1	0.0478
United States			1

(b)

[2] See Appendix A and B for graphs relating to CO2 and GDP/capita for each country.

[3] The GDP/capita correlation between Canada and the United States showed to be 0.86, against CAN-MEX of 0.18 and MEX-USA of 0.16. So at least and shown by these results, it would be questionable the benefits of Mexico in the free-trade zone, but also the opportunity to seek environmental policies without affecting Canada and the United States.

	Canada	Mexico	United States
Canada	1	0.0769	0.6556
Mexico		1	-0.0686
United States			1

3.2 Results

The decomposition process based on equation (12) is done in 6 resolution levels or *crystals*, each level related to a timing frame. For instance, the highest frequency components are captured in the first level D_1 which occur between 1 and 2 years; D_2 captures the next frequency components occurring in a time-frame between 2 and 4 years, and successively up to D_6 which captures the lowest frequency components of the original data occurring in a time-frame between 16 and 32 years. [4]

Figure 1 shows the decomposition of CO2 emissions from energy consumption of Mexico in which the lowest scale D_1 is capturing the highest frequency components related to movements in the very short-run of CO2 emissions, and D_6 is capturing the lowest frequency components related to movements in the long-run of CO2 emissions. As can be shown, the wavelet transform has "denoised" the original signal and it is observed the "business cycle" pattern at level D_6 of CO2 emissions.

Then at each level of decomposition D_j*(for j=1, 2, 3, 4, 5, 6)*, it is estimated the wavelet correlation between the CO2 emissions and GDP/capita for each country. The wavelet correlation is expressed as

$$\rho_X\left(\lambda_j\right)=\frac{\gamma_X\left(\lambda_j\right)}{\upsilon_1\left(\lambda_j\right)\upsilon_2\left(\lambda_j\right)}, \tag{14}$$

where $\upsilon_1^2\left(\lambda_j\right)$, $\upsilon_2^2\left(\lambda_j\right)$ are the wavelet variances of the series $x_{1,t}$ and $x_{2,t}$ related to λ_j scale. For instance, the correlation between CO2 emissions and GDP/capita for Mexico at resolution level D_1 is expressed as:

$$\rho_{CO2,GDP}^{MX}\left(\lambda_1\right)=\frac{\gamma_{CO2,GDP}\left(\lambda_1\right)}{\upsilon_{CO2}\left(\lambda_1\right)\upsilon_{GDP}\left(\lambda_1\right)}.$$

[4] See Appendix D and E for the decomposition series of the three countries.

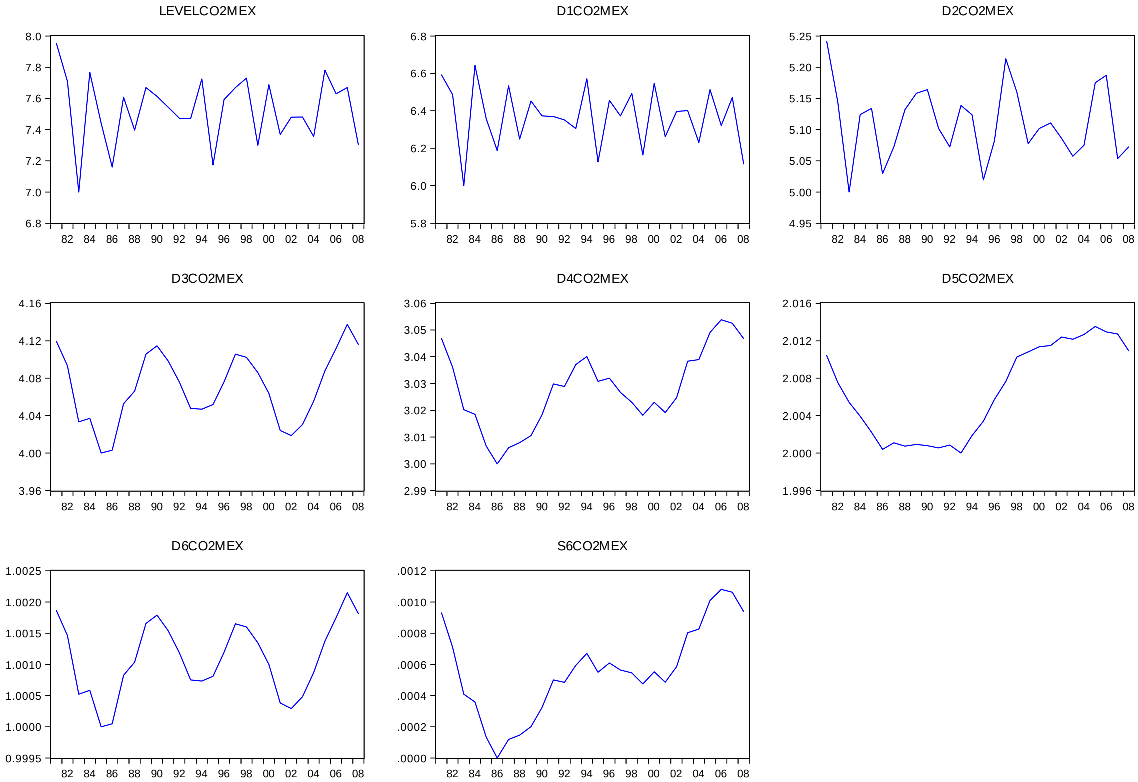

Figure 1: Decomposition of Mexico's CO2 emissions using the Haar wavelet and 6 levels of resolution.

Table 4 shows the correlations for each country at six levels of decomposition. It is observed that Canada shows the lowest correlation values at each level of resolution against the United States and Mexico. But, in time spans between 2-4, 4-8 years and 32 years and above it shows higher degrees of association than the other resolution levels. A first attempt in interpreting these results for Canada is that the economic activity is less correlated with air pollution (CO2 emissions) in the very short-run and when time passes this relationship breaks-down due to the incorporation of new (clean) technology in production processes. New and clean technology means low-carbon emissions that could be related with goods and services produced with highly recyclable materials. But in the long run, the correlation increases as that in the medium run which indicates that Canada has reached its final cycle in the technology adopted and it is time to change the production process.

So, the decomposed degree of association shows that in the short run besides a low degree of association between the economic activity and CO2 emissions, because of new technology the production process doesn't contributes significantly to CO2 emissions.

Table 4: Correlation between CO_2 emissions and GDP/capita at six levels of resolution for Canada, the United States, and Mexico.

	Canada	United States	Mexico
Original	0.5354	0.7320	0.7394
D1	0.3517	0.6588	0.7678
D2	0.6073	0.7984	0.7158
D3	0.7274	0.8079	0.8868
D4	0.4285	0.6416	0.6438
D5	0.3249	0.9724	0.7324
D6	0.7274	0.8079	0.8869
S6	0.3755	0.7199	0.6630

But once technology has reached its maximum performance it is needed an improvement in the production process; at this level industrial activity is highly correlated with CO_2 emissions. Then, improvements to production processes are done and this may reduce its contribution to CO_2 emissions, even though CO_2 emissions are still increasing explained by other means. Again (crystal D6 or long run) when production processes have reached its maximum technological profitability, economic activity is highly related and contributing to CO_2 emissions.

The interpretation for the United States and Mexico is something similar, but in this two cases as time span increases the correlation between CO_2 emissions and GDP/capita considerably increases and can be interpreted as if CO_2 emissions is always subjected to economic growth in this two countries. In these cases, the global correlation is fully fed by the higher degrees of association in the medium and long-run, as compared to Canada which global correlation is influenced by the low correlations in the short term. So Canada can be considered a low-carbon economy which can lower its CO_2 emissions without giving up its sustainable economic growth. In the other side, the United States and Mexico should take care in their environmental policies or low-carbon emissions compromises since an increase or decrease in their GDP/capita in the short and long-run is always showing a high degree of association. In this sense as stated by Meadows et al (1972), in their attempt to avoid environmental damages both countries should grow at a zero rate.

4. Conclusions

The benefits of applying the wavelets approach against the global time series analysis is that it allows to see the whole forest while analyzing specific trees. In this case it is possible to look at the whole data while comparing differences among different time spans such as short, medium and long run, in the relationship between economic phenomena data. Also it is useful to apply the wavelet transform to economic time series since most of them have shown temporal, non-stationary and time-varying facts.

This paper applied the wavelet transform to analyze the correlation between CO2 emissions from energy consumption and GDP/capita for the NAFTA countries. Firstly, based on a global approach, Canada showed the lowest degree of association and when it was applied the wavelet approach it was observed that this global low-correlation was fed by low degrees of association in the medium term. It was interpreted as a first attempt that it would be related to the “business cycle” of technology which as time passes, the adoption of clean and new technology is resulting in a low-carbon economy without giving up a sustainable economic growth.

The United States and Mexico were the countries which showed a higher correlation between CO2 emissions and GDP/capita, and any attempt to lower carbon emissions would result in giving up its economic growth. Also, it would be interpreted as if the United States and Mexico should need a longer time to achieve a sustainable economic growth without threatening the environment or producing with low CO2 emissions.

The importance of this paper concerns to environmental and economic policies formulations which may seek to achieve a sustainable development in which the economic activity is being considered as circumscribed in the biosphere. By means of effective policies, the NAFTA countries could work jointly in achieving common environmental results by aligning their business cycles. This paper used GDP/capita as an economic variable that could explain CO2 emissions, but other factors or transmission mechanisms may be causing a high or low degree of association between CO2 emissions and GDP/capita. Also, causality tests can be performed at each resolution level in such a way to identify feedback relationships.

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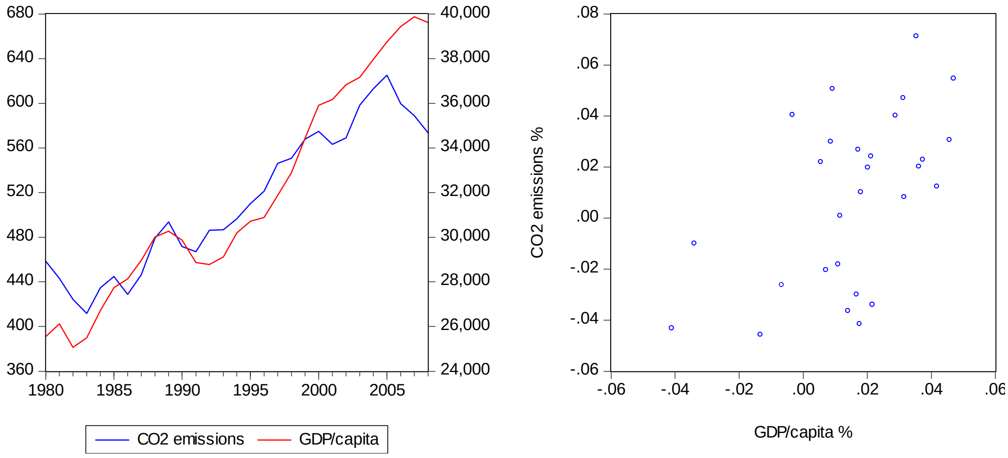
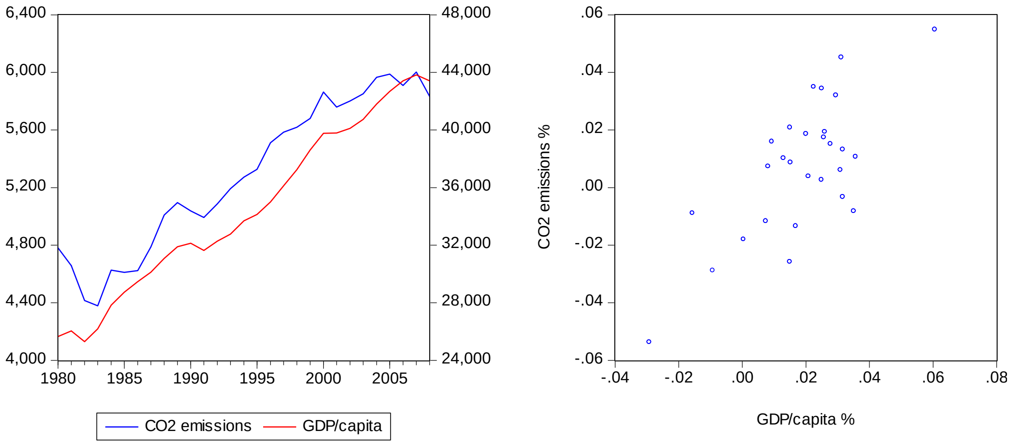
Appendix

A. Carbon Dioxide (CO2) emissions from energy consumption and GDP/capita.

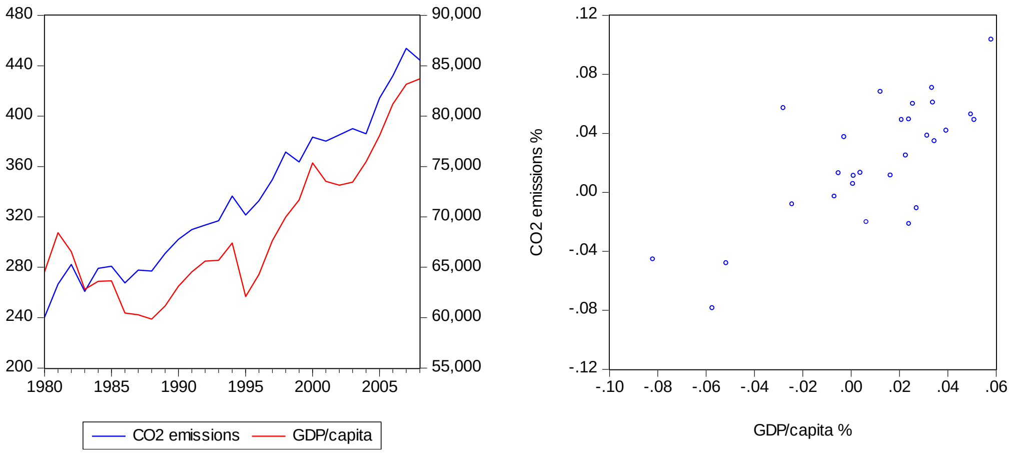
(a) Canada



(b) United States

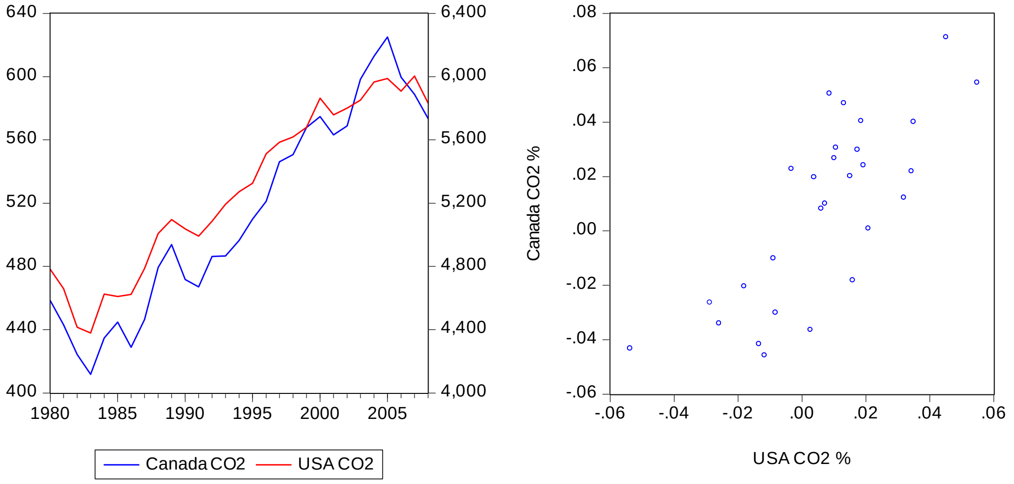


(c) Mexico

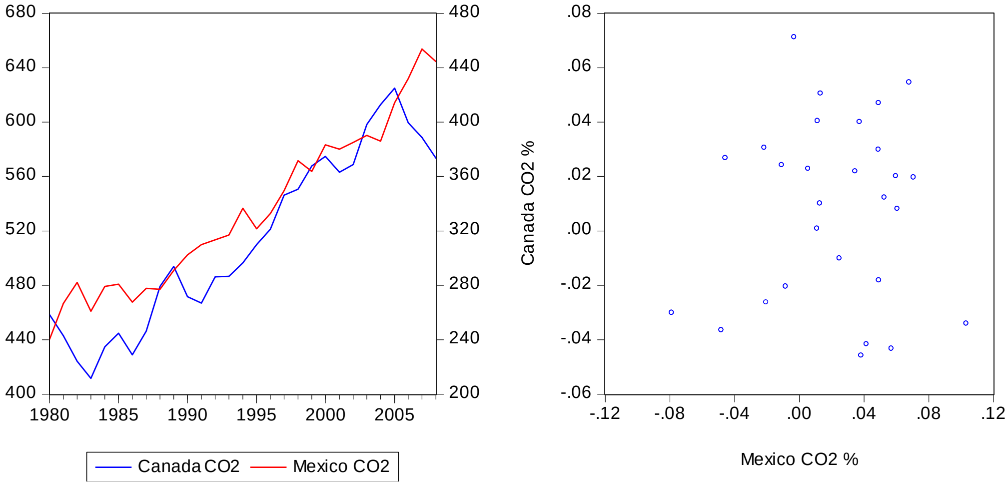


B. Cross-country correlation of carbon dioxide (CO_2) emissions from energy consumption.

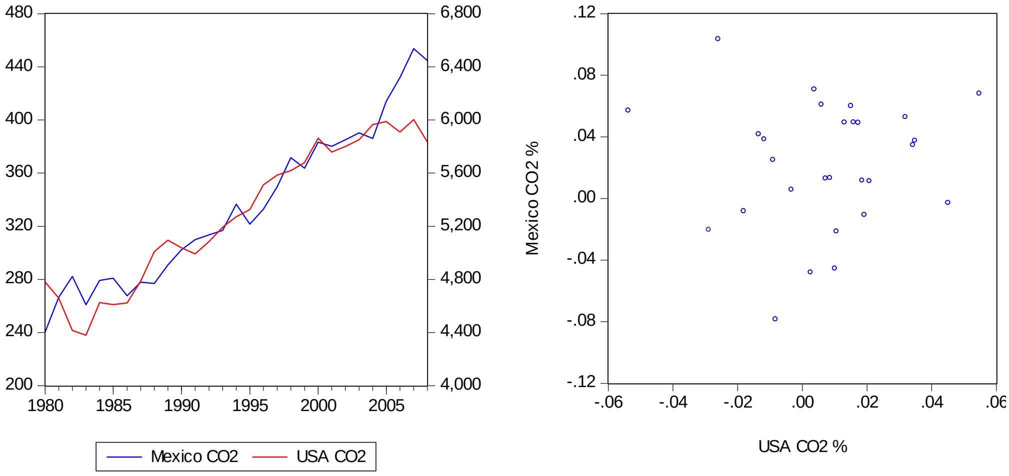
(a) Canada and United States



(b) Canada and Mexico



(c) United States and Mexico



C. Granger causality tests.

(a) Canada

Pairwise Granger Causality Tests
Sample: 1980 2008
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
LOGGDPCAPCANADA does not Granger Cause LOGCO2ECANADA	26	1.0422	0.3702
LOGCO2ECANADA does not Granger Cause LOGGDPCAPCANADA		0.3437	0.7131

(b) United States

Pairwise Granger Causality Tests
Sample: 1980 2008
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
LOGGDPCAPUSA does not Granger Cause LOGCO2EUSA	26	1.52896	0.2399
LOGCO2EUSA does not Granger Cause LOGGDPCAPUSA		1.23534	0.311

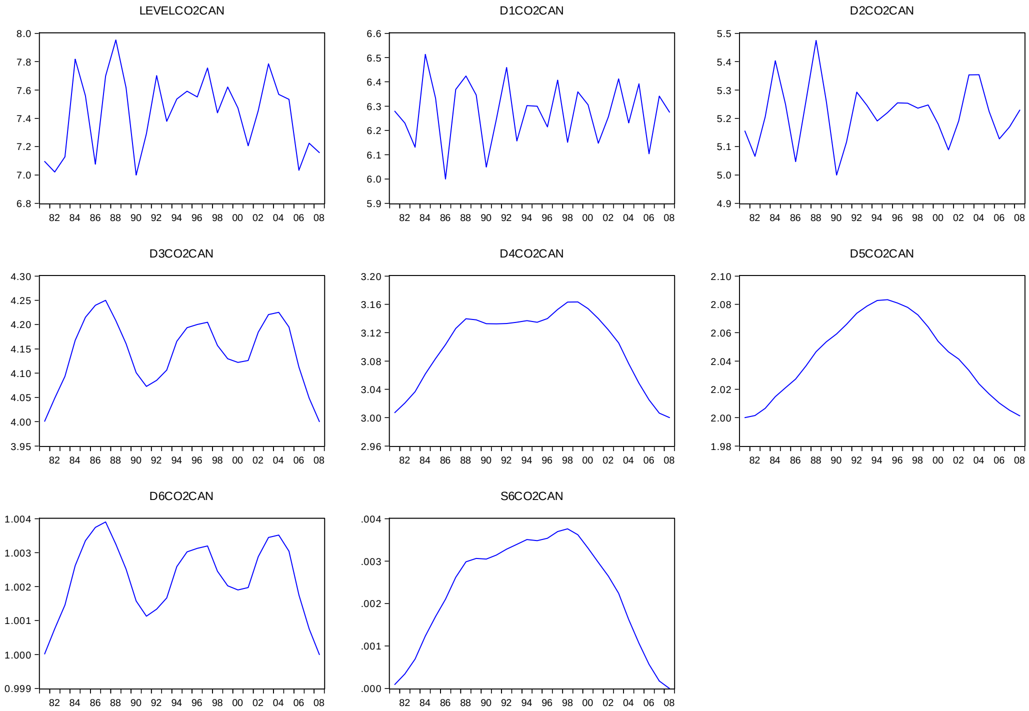
(c) Mexico

Pairwise Granger Causality Tests
Sample: 1980 2008
Lags: 2

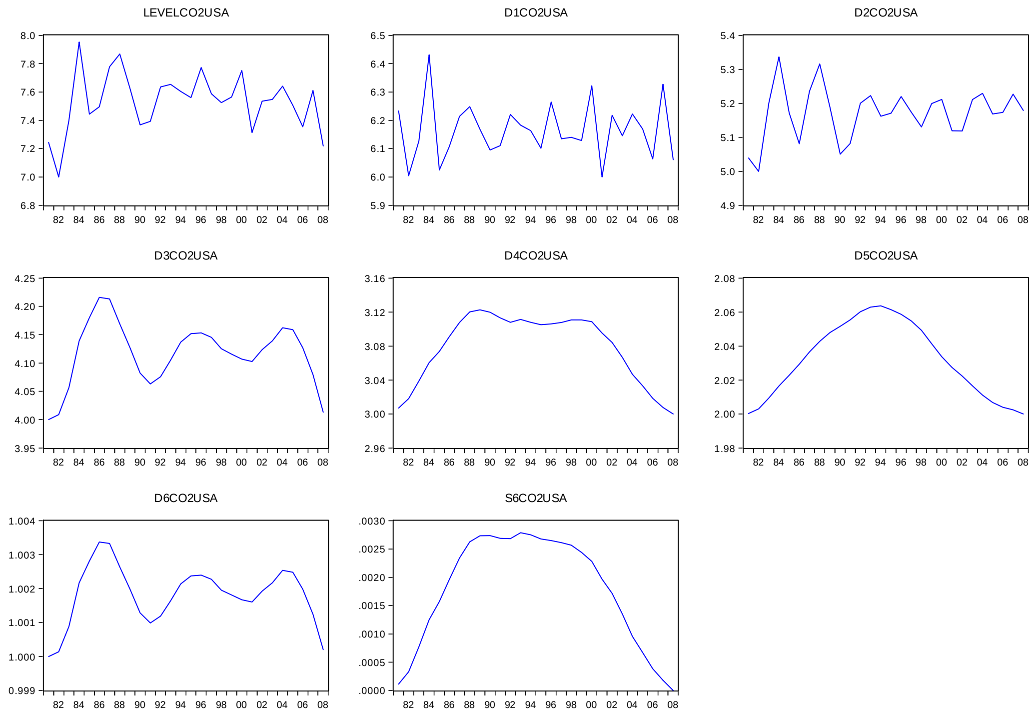
Null Hypothesis:	Obs	F-Statistic	Prob.
LOGGDPCAPMEXICO does not Granger Cause LOGCO2EMEXICO	26	4.14055	0.0305
LOGCO2EMEXICO does not Granger Cause LOGGDPCAPMEXICO		2.73519	0.088

D. Decomposition of CO2 emissions for (a) Canada and (b) the United States.

(a) Canada

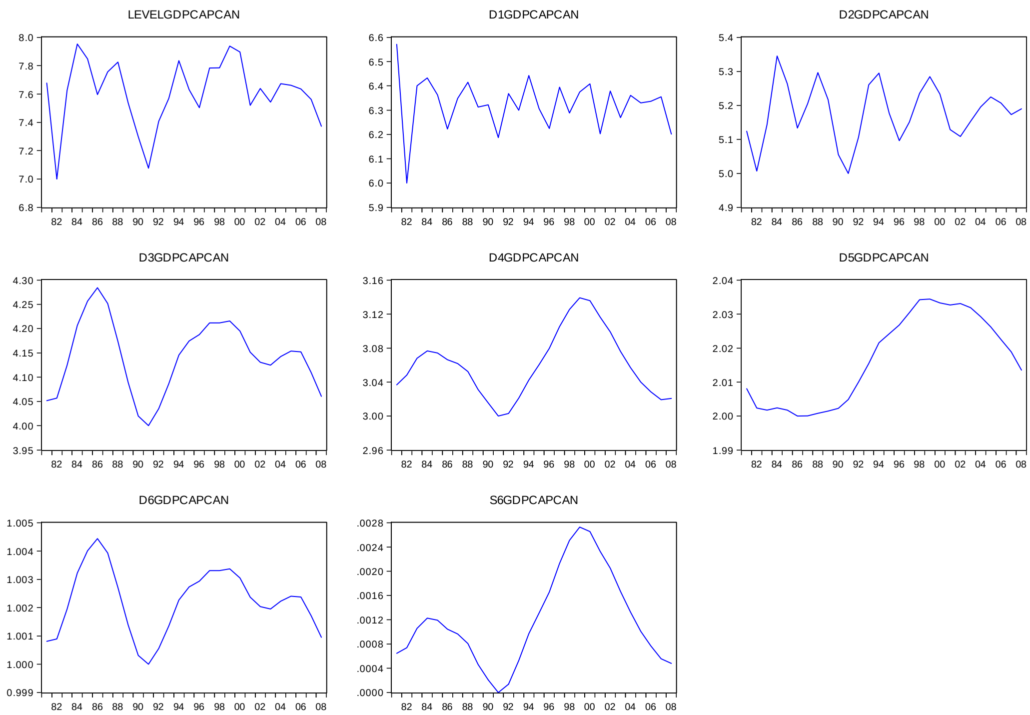


(b) United States

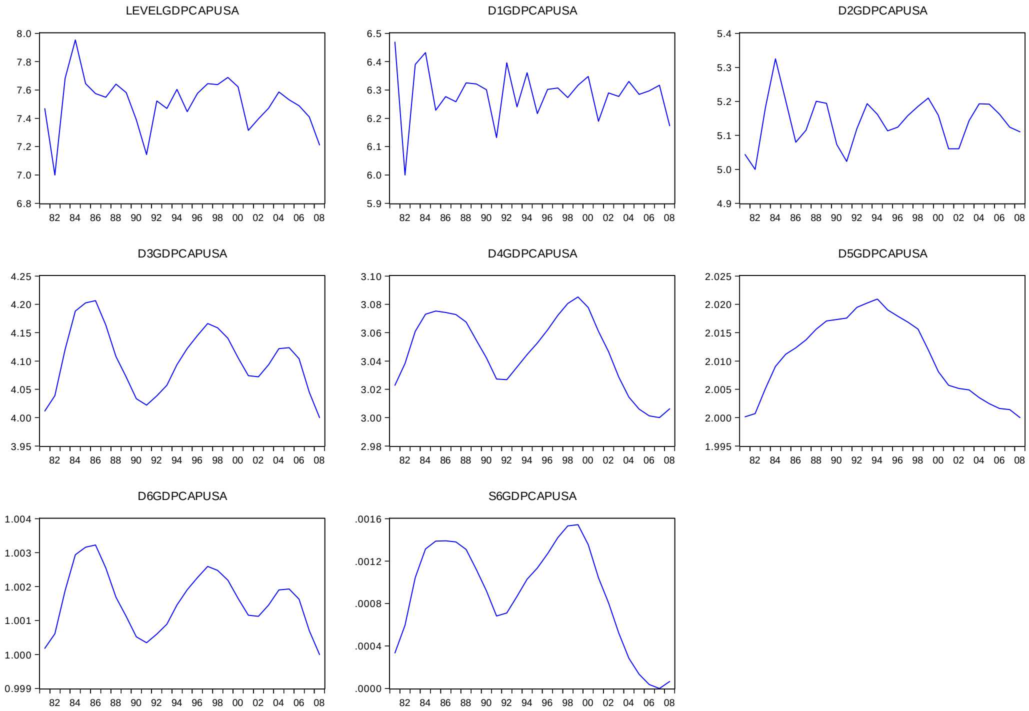


E. Decomposition of GDP/capita for (a) Canada, (b) the United States, and (c) Mexico.

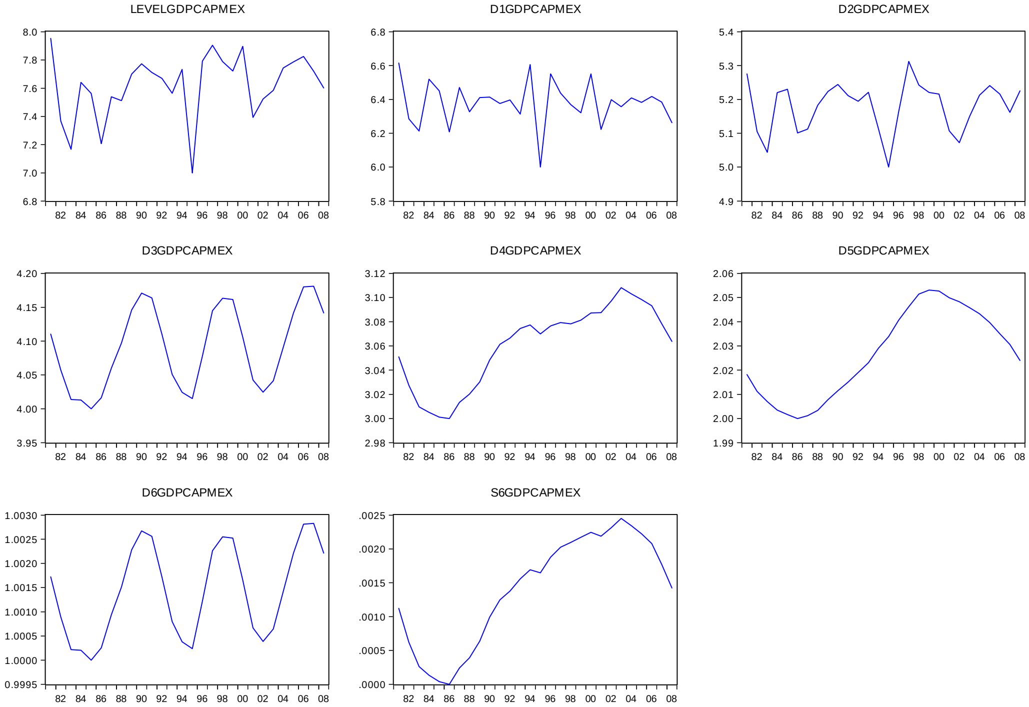
(a) Canada



(b) United States


LEVELGDPCAPUSA
D1GDPCAPUSA
D2GDPCAPUSA
D3GDPCAPUSA
D4GDPCAPUSA
D5GDPCAPUSA
D6GDPCAPUSA
S6GDPCAPUSA

(c) Mexico


LEVELGDPCAPMEX
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S6GDPCAPMEX